Suggested Answers

1. 10.0 g of ice at –10 °C are introduced into 100 g of water at 50 °C contained in a copper calorimeter with a mass of 50.0 g. Calculate the final temperature reached.

   (specific latent heat of ice = 0.34×10^6 J kg^{-1}
   specific heat capacity of ice = 2.09×10^3 J kg^{-1} K^{-1}
   specific heat capacity of copper = 380 J kg^{-1} K^{-1}
   specific heat capacity of water = 4200 J kg^{-1} K^{-1})

   [38.1°C]

   Assume the heat exchange with the surrounding is negligible. Let the final temperature be θ.

   heat absorbed (by ice from -10°C to 0°C + by ice to melt + by water from melted ice to reach θ)
   = heat supplied (by water to reach θ + by calorimeter to reach θ)

   $$\left(10.0\times10^{-3}\right)\left(2.09\times10^{3}\right)(10)+\left(10.0\times10^{-3}\right)\left(0.34\times10^{6}\right)+\left(10.0\times10^{-3}\right)(4200)(\theta-0)$$
   $$=\left(100\times10^{-3}\right)(4200)(50-\theta)+\left(50.0\times10^{-3}\right)(380)(50-\theta)$$

   $$\Rightarrow 2.09\times10^{2}+3.40\times10^{3}+42\theta=4.39\times10^{2}(50-\theta)$$
   $$\Rightarrow \theta = 38.13^\circ\text{C}$$

2. 1 kg of vegetables, having a specific heat capacity 2200 J kg^{-1} K^{-1}, at a temperature of 373 K are plunged into a mixture containing some ice and 1 kg of water at 273 K. After all the ice has melted, the final temperature of the entire mixture is 300 K. Calculate the mass of ice originally present. Assume no heat loss to the container and the surroundings.

   Specific latent heat of fusion of ice = 3.34×10^5 J kg^{-1}

   [0.105 kg]

   (Assuming the ice point is at 273 K.)

   Let the mass of ice be *m* kg.

   heat supplied by vegetables = heat absorbed by (ice to melt + water from melted ice + 1 kg of water)
   $\Rightarrow$
   $$(1)(2200)(373-300)=(m)\left(3.34\times10^{5}\right)+(m+1)(4200)(300-273)$$
   $$\Rightarrow m = 0.105 \text{ kg}$$

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3. A thermally insulated vessel containing liquid water and water vapour is connected to a vacuum pump which removes water vapour continuously. When the temperature reaches 0 °C, the vessel contains 110 g of liquid water. What mass of ice has been formed when no liquid remains?

   Specific latent heat of fusion of water = 3.40×10^5 J kg^{-1};
   Specific latent heat of vaporisation of water [1]= 2.52×10^6 J kg^{-1} (at 0.01 °C)

   [96.9 g]

   Let m_{ice} (in kg) be the mass of the ice formed, then the mass of vapor m_{vapor} produced would be $(0.110 - m_{ice})$ kg

   By conservation of energy,
   heat given off in freezing water to ice = heat required to boil water close to 0°C

$$m_{ice} L_f = m_{vapor} L_v = (0.110 - m_{ice}) L_v$$
$$m_{ice}(L_f + L_v) = 0.110 L_v$$
$$m_{ice} = \frac{0.110 L_v}{(L_f + L_v)}$$
$$= \frac{0.110 \times 2.52 \times 10^6}{3.40 \times 10^5 + 2.52 \times 10^6}$$
$$= 0.0969 \text{ kg}$$
$$= 96.9 \text{ g}$$

4. A lead bullet is fired with a muzzle velocity of 320 m s^{-1} and becomes immediately embedded in clay. Estimate the fraction of the mass of the bullet which melts, assuming that all the heat generated remains as internal energy in the lead.

   Given that the muzzle temperature of the bullet is 30 °C,
   a. melting point of lead is 330 °C,
   b. the specific heat capacity of lead is 130 J kg^{-1} K^{-1} and
   c. the specific latent heat of fusion of lead is 2.1×10^4 J kg^{-1}.

   [0.58]

$$\text{heat supplied to (increase bullet's temperature} + \text{melt } m' \text{ of lead)} = \text{loss in kinetic energy}$$
$$m(130)(330 - 30) + m'(2.1\times10^4) = \frac{1}{2} m (320^2)$$
$$21000m' = 12200m$$
$$\frac{m'}{m} = 0.581$$

[1] The heat of vaporization diminishes with increasing temperature.

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5. Cooling water enters the heat exchanger in the turbine hall of a nuclear power station at 6.0°C and leaves at 14.0°C. The rate of heat removal by the water is 6.7×10^9 J per minute. The specific heat capacity of water is 4200 J kg^{-1} K^{-1}. Determine the mass of water flow per unit time. [3320 kg s^{-1}]

By conservation of energy,

rate of heat gain by water = rate of heat loss by heat exchanger

$$\frac{mc\Delta T}{t} = \frac{6.7\times10^9}{60}$$

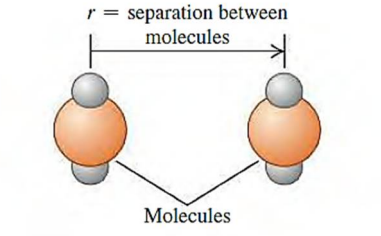
$$\Rightarrow \frac{m}{t} = \left(\frac{6.7\times10^9}{60}\right)\left(\frac{1}{4200(8.0)}\right) = 3320 \text{ kg s}^{-1}$$

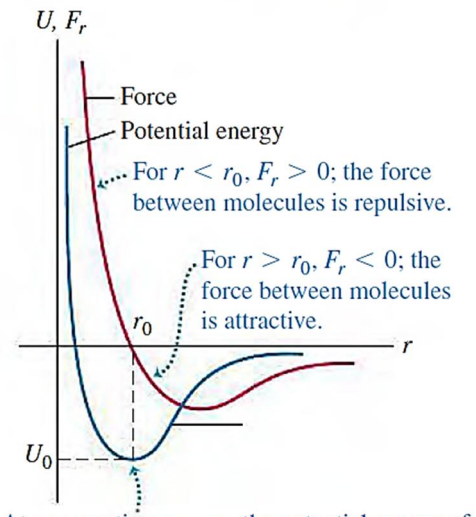
6. (a) Using the concept of internal energy, compare the internal energy per unit mass of water and water vapour at the same temperature.

(a) The internal energy U of a system is determined by the state of the system, i.e. U of system is a function of state coordinates p, V and T. It can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of the system.

At the same temperature, both water and water vapour molecules have the same kinetic energy of random motion. When unit mass of water turns into vapour, it does work in separating the molecules, against their mutual attractions. This amount of energy is a measure of the increase in potential energy of the molecules of water in gaseous state over that in the liquid state, at the same temperature. Thus the internal energy per unit mass of water vapour is greater than that of water.

How the force between molecules and their potential energy of interaction depend on their separation r.





At a separation $r = r_0$, the potential energy of the two molecules is minimum and the force between the molecules is zero.

The force between molecules in a gas varies with the distance r between molecules somewhat as shown.

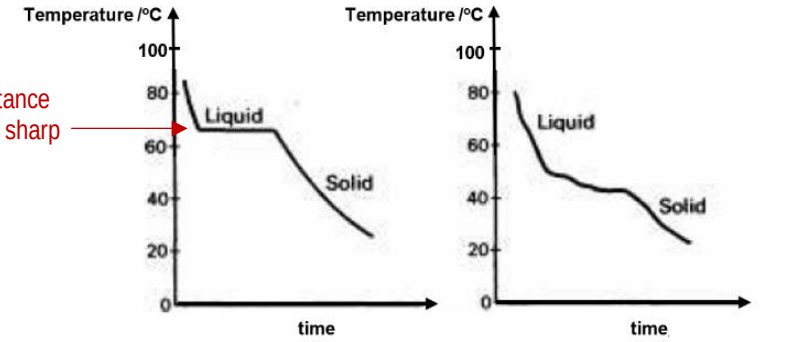
When molecules are far apart, the intermolecular forces are very small and usually attractive. As a gas is compressed and its molecules are brought closer together, the attractive forces increase. The intermolecular force becomes zero at an equilibrium spacing r_0, corresponding roughly to the spacing between molecules in the liquid and solid states. In liquids and solids, relatively large pressures are needed to compress the substance appreciably.

(b) Explain, using a simple kinetic model for matter,
   (i) why the temperature of a pure substance does not change when it experiences a change of state.
   (ii) the *specific latent heat of vaporisation*[2] of water ($L_v = 2.26 \times 10^6$ J kg^{-1}) is greater than the *specific latent heat of fusion* ($L_f = 3.34 \times 10^5$ J kg^{-1}).

[2] The specific latent heat of fusion (or vaporisation) of a material is the energy required to change the state of 1 kg of the material from solid to liquid (or liquid to vapour) without change of temperature.

(b) (i) During melting and boiling, all the heat energy absorbed by the pure substance is used to change the molecular structures by breaking the bonds and increase the potential energy of the molecules only. The kinetic energy of the molecules does not change, and hence the temperature of substance does not increase. An impure substance is a mixture of several substances, which melt or boil at slightly different temperatures. The heat absorbed is used to change the molecular structures of some types of molecules present in the substance, and to change the molecular kinetic energy of other types of molecules, so the temperature changes and the substance has no definite melting/boiling point.

Example: Pure substance (Naphthalene) has a sharp melting point.

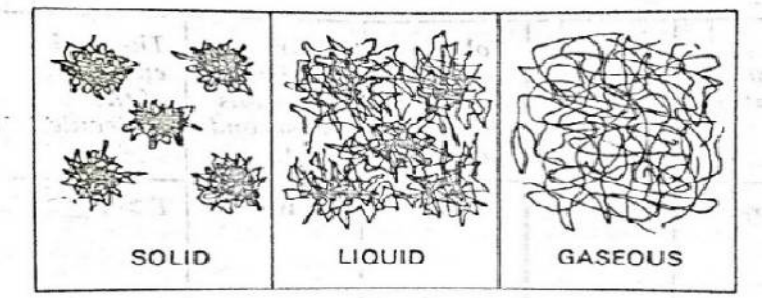


Naphthalene (pure crystalline solid) *Paraffin wax (mixture of several waxes)*

(ii) To melt a solid, work must be done to separate some of the molecules against their mutual attractions (i.e. molecular bonds[3] are broken), so that the structure no longer has any rigidity. Once a liquid has been formed from a solid the number of broken bonds is constant, on average. When a liquid evaporates, all the remaining bonds must be broken. Since melting means the breaking of relatively few bonds we expect the specific latent heat of melting to be less than that of vaporization. The relative sizes of L_f and L_v indicates the proportion of the molecular bonds broken when the solid melts.

There is nearly always expansion when a solid melts (ice is the only common exception) and always an expansion when a liquid evaporates. This expansion means that the atmosphere must be pushed back to make room, and energy is needed for this external work done. However, this energy is, even for the vaporization, a [4]small fraction of the total latent heat. Latent heat must be supplied mainly to break molecular bonds.

Models of particle behaviour in different phases:



solid: molecules vibrate about a fixed site in the lattice	liquid: molecules vibrate within ordered clusters which have some restricted translational motion	gas: totally random translational motion

[3] The bonds are not real, like chains or ropes or even springs, but it is convenient to use this phrase to describe the electrical links holding the molecules together.
[4] Refer to comments on 'external latent heat' for Question 7.

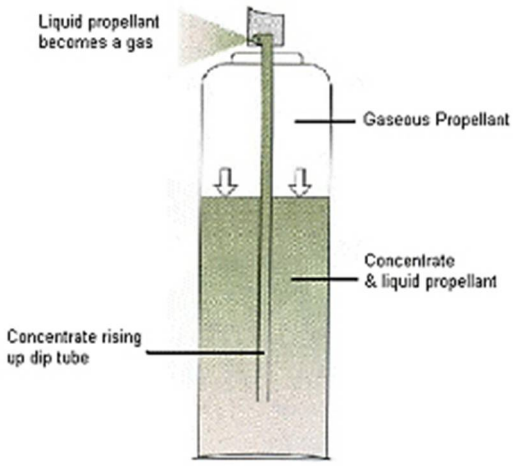
(c) Explain, using First Law of Thermodynamics,
(i) the temperature of a fixed mass of gas increases when it is compressed quickly in an insulated cylinder using a piston,
(ii) the temperature of an insecticide spray canister drops when it is used.

(c) (i) When the piston compresses the gas, work is done on the gas (W is +ve). As this process happens quickly, there is no heat transfer between the gas and its surroundings. (Q = 0). Using the First Law of Thermodynamics, $\Delta U = Q + W$ gives ΔU +ve. This implies that there is an increase in internal energy. Hence, the temperature increases.

Or using Kinetic Model to explain*: the moving piston transfers momentum to the gas molecules and hence the mean random kinetic energy of the molecules increases. Since temperature of the gas is proportional to the mean random kinetic energy, the temperature of the gas would increase.*

(ii) The first law of thermodynamics states that the increase in internal energy of a system equals the heat supplied to the system plus the work done on the system: $\Delta U = Q + W$.

When the insecticide spray canister is used momentarily, it means its trigger is pressed for a very short duration of time. The compressed gas expands rapidly. This rapid expansion is essentially an adiabatic process, meaning there's negligible heat exchange with the surroundings in such a short time frame. Therefore, Q = 0.



As the gas expands, it does work on the surroundings (pushing the insecticide out of the nozzle). This means work done on the gas is negative, so $W < 0$.

Substituting Q = 0 and $W < 0$ into the first law equation, ΔU is negative. A negative change in internal energy means the internal energy of the gas decreases. Since internal energy is directly proportional to the (thermodynamic) temperature, a decrease in internal energy corresponds to a decrease in temperature of the gas. Consequently, the canister feels cold to the touch momentarily after use.

While the first law of thermodynamics provides a quantitative explanation, the following approach offers an intuitive and qualitative explanation that is still accurate:

The rapid release of the pressurized liquid triggers a rapid phase change from liquid to gas, a process known as evaporation. This phase change requires energy, which is absorbed from the remaining liquid. As a result, the average kinetic energy of the remaining liquid molecules decreases, leading to a drop in temperature.

This phenomenon can be understood in simpler terms: the most energetic molecules, those with the highest kinetic energy, are the ones that escape into the gas phase. This leaves behind the slower, less energetic molecules, resulting in a lower average kinetic energy and, consequently, a lower temperature.

7. At a temperature of 100°C and a pressure of 1.01×10^5 Pa, 1.00 kg of steam occupies 1.67 m^3 but the same mass of water occupies only 1.04×10^{-3} m^3. The specific latent heat of vaporization of water at 100°C is 2.26×10^6 J kg^{-1}. For a system consisting of 1.00 kg of water changing to steam at 100°C and 1.01×10^5 Pa, determine
   (a) the heat supplied to the system, [2.26×10^6 J]
   (b) the work done by the system, [1.69×10^5 J]
   (c) the increase in internal energy of the system. [2.09×10^6 J]

(a) $Q = mL = (1.00 \text{ kg})(2.26 \times 10^6 \text{ J kg}^{-1}) = 2.26 \times 10^6$ J
(b) work done by system $= p\Delta V = (1.01 \times 10^5)(1.67 - 1.04 \times 10^{-3}) = 1.69 \times 10^5$ J
(c) $\Delta U = Q + W = 2.26 \times 10^6 + (-1.69 \times 10^5) = 2.09 \times 10^6$ J

Comments: *Latent heat and internal energy*

When **1.00 kg** of water turns into steam, it expands by 1.66896 m^3, in doing so, it does work against the atmospheric pressure. The heat equivalent of this work is that part of the latent heat which must be supplied to the water to make it overcome atmospheric pressure as it evaporates; it is called the '*external* latent heat'. The rest of the specific latent heat – the *internal* part – is the equivalent of the work done in separating the molecules, against their mutual attractions. This amount of energy, then, is a measure of the increase in potential energy of the molecules of water in the gaseous state over that in the liquid state, at the same temperature.

The '*external* latent heat' is therefore $l_{ext} = 1.69 \times 10^5$ J kg^{-1}
The '*internal* latent heat' is $l_{in} = L - l_{ext} = 2.26 \times 10^6 - 1.69 \times 10^5 = 2.09 \times 10^6$ J kg^{-1}
Note that the *external* part of the specific latent heat is much less than the *internal* part. As a fraction of the total specific latent heat, it is only about $(1.69 \times 10^5)/(2.26 \times 10^6) = 7\%$.

For one mole of water, 0.018 kg, the number of molecules is 6.02×10^{23}. Hence, from above, the gain in potential energy of a molecule of water when changing from liquid at 100°C to vapour at 100°C is $(0.018 \times 2.09 \times 10^6) / (6.02 \times 10^{23}) = 6.2 \times 10^{-20}$ J $molecule^{-1}$.

8. An ideal gas of volume 1.5×10^{-3} m^3 and at pressure 1.0×10^5 Pa is supplied with 70 J of energy. The volume increases to 1.7×10^{-3} m^3, the pressure remaining constant. Calculate the change in internal energy of the gas. [+50 J]

$Q = +70$ J

$|W| = p\,|\Delta V| = (1.0 \times 10^5)(1.7 \times 10^{-3} - 1.5 \times 10^{-3}) = 20$ J

Since the process is constant pressure expansion, work is done by the gas, $W = -20$ J
First law of thermodynamics:
$\Delta U = Q + W = +70 - 20 = 50$ J

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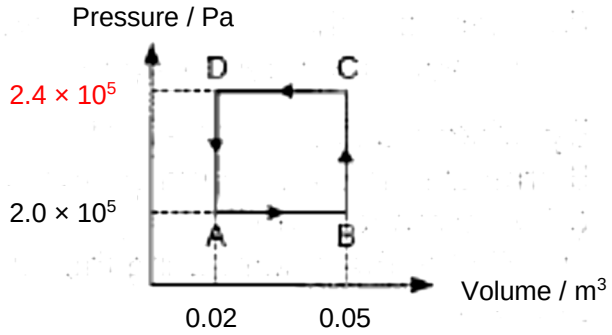
9. Consider a system that is taken along the paths shown in the *P*-*V* diagram below. The internal energy of the system at A and at C is 30 kJ and 90 kJ respectively.



(a) Calculate the internal energy of the system at point B, if 51 kJ of heat is absorbed by the system in going from A to B.

(b) Find the amount of heat that enters the system along the path from B to C.

[75 kJ, 15 kJ]

(a) $|W_{AB}| = p\,|\Delta V_{AB}| = (0.03) \times (2.0 \times 10^5) = 6000$ J

AB is a constant-pressure expansion process, work is done by the system, so W is negative: $W = -6000$ J

$\Delta U_{AB} = Q + W = 51000 + (-6000) = 45000$ J

$\Delta U_{AB} = U_B - U_A$

➔ $U_B = U_A + \Delta U_{AB} = 30000 + 45000 = 75$ kJ

(b) $\Delta U_{BC} = U_C - U_B = 90000 - 75000 = 15$ kJ

As BC is a constant-volume process, $W = 0$ J

$\Delta U = Q + W$ ➔ 15 kJ = Q + 0 ➔ Q = 15 kJ.

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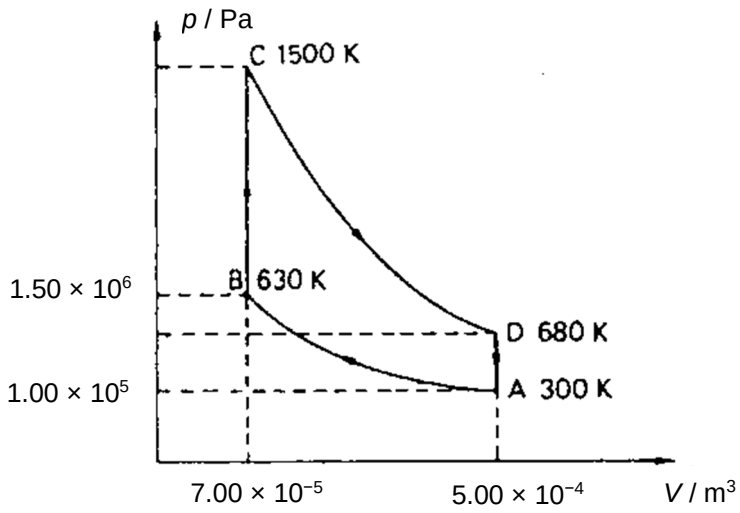
10. The figure below shows some details concerning the behaviour of a fixed mass of a gas (assumed to be [5]ideal) in a petrol engine.



(a) Use the equation of state for an ideal gas to find the number of moles in the fixed mass of gas.

(b) In the change from state B to state C, the temperature of the gas rises from 630 K to 1500 K. The molar heat capacity of the gas at constant volume is 21 J mol^{-1} K^{-1}. Calculate the thermal energy supplied in this change.

(c) How much work is done by the gas in the change from state B to C?

(d) In the change from state C to D the gas expands to its original volume. The temperature at state D is 680 K. Calculate the pressure at state D.

[0.0201 mol, 367 J, 0 J, 2.27×10^5 Pa]

(a) Consider state B:

$pV = nRT \rightarrow n = pV / RT = (1.50 \times 10^6)(7.00 \times 10^{-5}) / (8.31)(630) = 0.0201$ mol

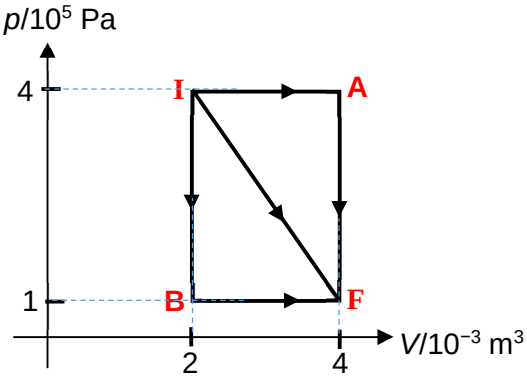
(b) $Q = n c_{m,v} \Delta T = (0.0201)(21)(1500 - 630) = 366$ J

(c) BC is a constant volume process. Area between graph of p and the V-axis is zero. Hence no work is done by the system during process BC, $W = 0$.

(d) $pV = nRT \rightarrow p_D = nRT_D / V_D = (0.0201)(8.31)(680)/(5.00 \times 10^{-4}) = 2.27 \times 10^5$ Pa

[5] The question does not mention what type of ideal gas, e.g. monatomic, diatomic, etc. So it is more appropriate to use 1st Law of Thermodynamics to calculate ΔU. The answer is consistent with the use of $\Delta U = (5/2)\, p\, \Delta V$, assuming ideal diatomic gas molecules.

11. A gas expands from **I** to **F** along the three paths indicated in the figure below.



(a) Calculate the work done **on** the gas along the path

(i) **IAF** [–800 J]

(ii) **IF** [–500 J]

(iii) **IBF** [–200 J]

(b) When the gas expands from **I** to **F** along the diagonal path, the energy added to the gas by heating is 418 J.

(i) What is the change in internal energy of the gas? [–82 J]

(ii) How much energy must be added to the gas by heating along the indirect path **IAF** to give the same change in internal energy? [718 J]

(a) (i) area under **IAF** = $p\Delta V = (4 \times 10^5)(4 \times 10^{-3} - 2 \times 10^{-3}) = 800$ J

work done **on** gas = – 800 J

(ii) area under **IF** = $\frac{1}{2}(1+4)\times 10^5 \times (2 \times 10^{-3}) = 500$ J

work done **on** gas = – 500 J

(iii) area under **IBF** = $(1 \times 10^5)(4 \times 10^{-3} - 2 \times 10^{-3}) = 200$ J

work done **on** gas = – 200 J

(b) (i) $\Delta U = Q_{IF} + W = 418 + (-500) = -82$ J

(ii) $\Delta U = Q_{IAF} + W$

ΔU for process **IF** and process **IAF** are the same because U is a function of state; the processes have the same final and initial state. Thus

$-82 = Q_{IAF} - 800 \Rightarrow Q_{IAF} = 718$ J

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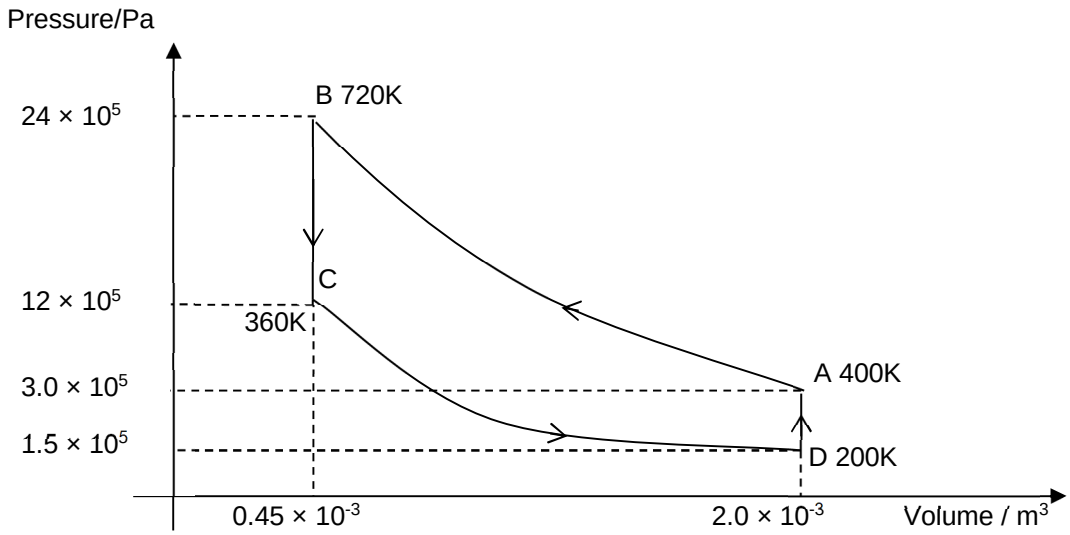
12. A fixed mass of gas in a heat pump undergoes a cycle of changes of pressure, volume and temperature as illustrated in the graph below. The gas is assumed to be ideal[6].



The table below shows the increase in internal energy which takes place during each of the changes A to B, B to C and C to D. It also shows that in both sections A to B and C to D, no heat is supplied to the gas. (Fill out the table in the sequence indicated.)

	***Increase** in internal energy* /J	*Heat supplied **to** gas* /J	*Work done **on** gas* /J
A to B	1200	0	❶ $W = \Delta U - Q$ = 1200 – 0 = **1200**
B to C	–1350	❹ $Q = \Delta U - W$ = −1350 – 0 = **−1350**	❸ **0** (since $\Delta V = 0$)
C to D	–600	0	❷ $W = \Delta U - Q$ = – 600 – 0 = **– 600**
D to A	In a cycle, net change in ❻ internal energy is zero since the system returns to the original state. Since net $\Delta U = 0$, $\Delta U_{D\to A}$ = 0 – 1200 – (–1350) – (–600) = **750**	❼ $Q = \Delta U - W$ = 750 – 0 = **750**	❺ **0** (since $\Delta V = 0$)

… End …

[6] It may not be monatomic for which $U = \frac{3}{2}nRT$. Here the ideal gas should be diatomic with $U = \frac{5}{2}nRT$ (not in exam syllabus).

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