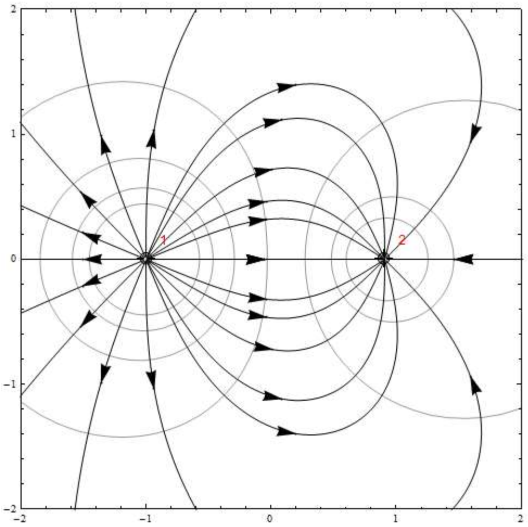
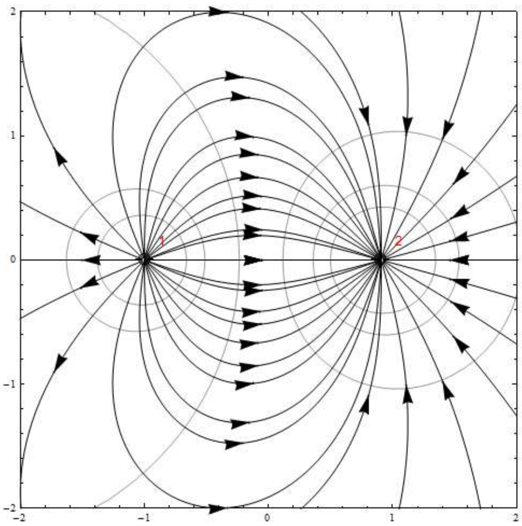
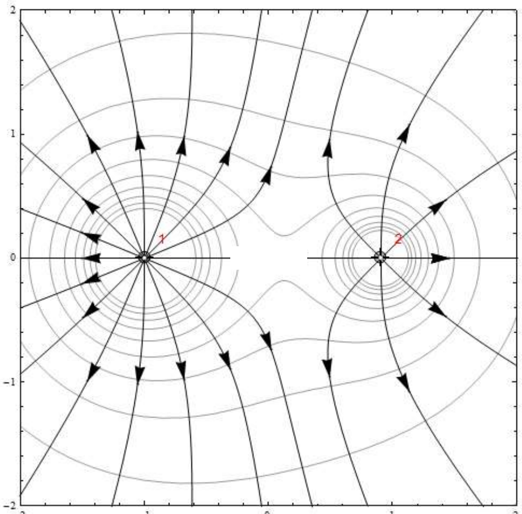
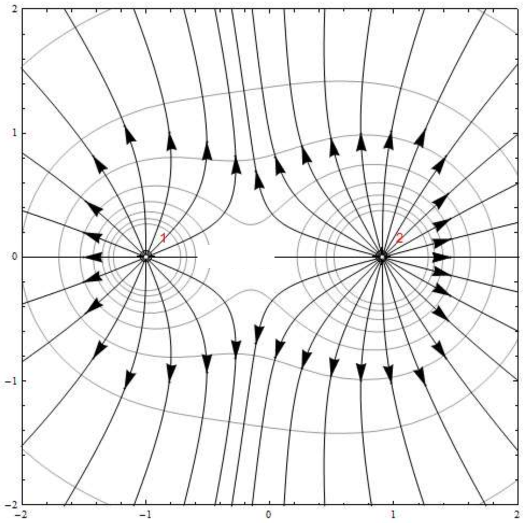
	Concept of an electric field	
1	(a)	 Q_1 = +2.0 μC Q_2 = –1.0 μC
	(b)	 Q_1 = +2.0 μC Q_2 = –3.0 μC
	(c)	 Q_1 = +2.0 μC Q_2 = +1.0 μC

(d)

Q_1 = +2.0 μC



Q_2 = +3.0 μC

2

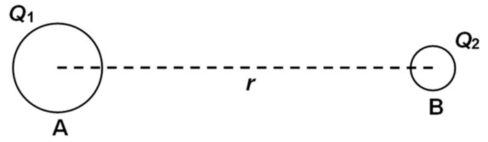
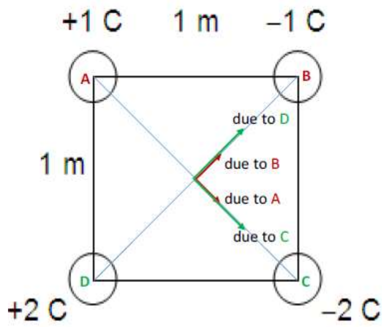
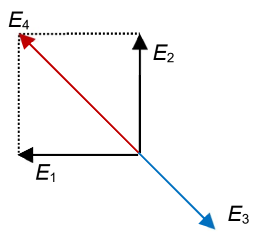
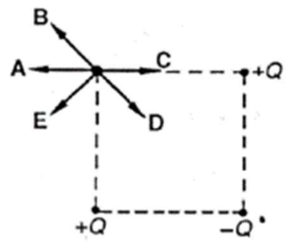
	Electric	**Gravitational**
constant	$\dfrac{1}{4\pi\varepsilon_0}$	G
field strength	$E \equiv \dfrac{F}{q}$	$g \equiv \dfrac{F}{m}$
field strength of a point charge Q or point mass M	$E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$	$g = G\dfrac{M}{r^2}$
force between point charges (masses)	$F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q_1Q_2}{r^2}$ (Coulomb's law)	$F = G\dfrac{m_1m_2}{r^2}$ (Newton's law of gravitation)
potential	$V \equiv \dfrac{W}{q}$	$\phi \equiv \dfrac{W}{m}$
potential of a point charge Q or point mass M	$V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$	$\phi = -\dfrac{GM}{r}$
field strength at a point is numerically equal to the potential gradient at that point	$E = -\dfrac{dV}{dr}$	$g = -\dfrac{d\phi}{dr}$

Point charges

3 (2012 P1 Q26)
If the distance from a charged particle is doubled, how do the new values of field strength and potential due to the charge compare to the original values?

	field strength	potential
A	1/4	1/4
B	1/4	1/2
C	1/2	1/4
D	1/2	1/2

Based on $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$ and $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r}$.

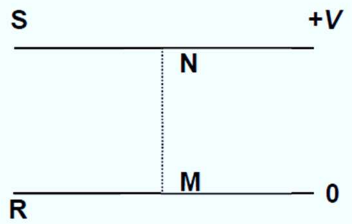
4	(a)	$F = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
	(b)	$V_A = \dfrac{Q_2}{4\pi\varepsilon_0 r}$ 
	(c)	$U_{AB} = \dfrac{Q_1Q_2}{4\pi\varepsilon_0 r}$
	(d)	$V_B = \dfrac{Q_1}{4\pi\varepsilon_0 r}$
	(e)	$V = \dfrac{Q_1}{4\pi\varepsilon_0 \frac{r}{2}} + \dfrac{Q_2}{4\pi\varepsilon_0 \frac{r}{2}} = \dfrac{Q_1+Q_2}{2\pi\varepsilon_0 r}$
	(f)	Q_1 and Q_2 are point charges; The separation between the charges is much larger than the dimensions of the charges; The charges are uniformly distributed on the object surfaces.
5	(a)	(vector sum of E fields due to +1C & $-$1C, to the right) + (vector sum of E fields due to +2C & $-$2C, to the right) $2\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{1}{2(0.5)^2}\cos 45^\circ\right) + 2\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{2}{2(0.5)^2}\cos 45^\circ\right)$ $= 3\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{1}{(0.5)^2}\cos 45^\circ\right)$ = 7.64×10^{10} N C^{-1} towards the right. 
	(b)	$\dfrac{(9\times10^9)(+1)}{\sqrt{0.5}} + \dfrac{(9\times10^9)(-1)}{\sqrt{0.5}} + \dfrac{(9\times10^9)(2)}{\sqrt{0.5}} + \dfrac{(9\times10^9)(-2)}{\sqrt{0.5}}$ = 0 J C^{-1}
6	Electric field strength at the fourth corner due to the two +Q charges are E_1 and E_2, with magnitude $\dfrac{Q}{4\pi\varepsilon_o r^2}$. E_4 is the resultant of E_1 and E_2, hence $E_4 = \dfrac{Q\sqrt{2}}{4\pi\varepsilon_o r^2}$. Electric field strength due to the $-Q$ charge is $E_3 = \dfrac{Q}{4\pi\varepsilon_o (r\sqrt{2})^2} = \dfrac{Q}{8\pi\varepsilon_o r^2}$. Since E_3 is less than E_4, the direction is B.   The magnitude of E_4 and E_3 is $\dfrac{Q\sqrt{2}}{4\pi\varepsilon_o r^2} - \dfrac{Q}{8\pi\varepsilon_o r^2} = \dfrac{(2\sqrt{2}-1)Q}{8\pi\varepsilon_o r^2} = 8.22\times10^9$ N C^{-1}	

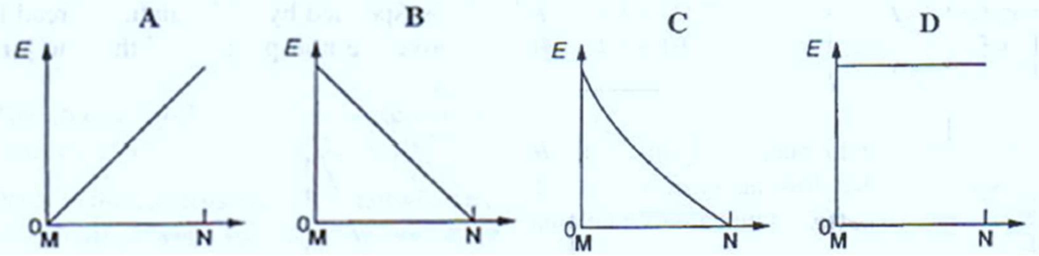
Uniform electric fields

7

Two horizontal conducting plates **R** and **S** are a fixed distance apart. Plate **S** is at a potential $+V$ with respect to plate **R**. **MN** is a line perpendicular to the plates.

Which graph shows how the magnitude E of the electric field strength varies along the line **MN**? [1]





(Answer: D)
Electric field strength between parallel plates is constant throughout.

8

(2007 P1 Q24)

The diagram shows two charged horizontal metal plates X and Z. The magnitudes of the charges on the plates are equal. Between the plates, suspended stationary in the uniform electric field, is a charged sphere Y. What could be the signs of the charges on the three components?

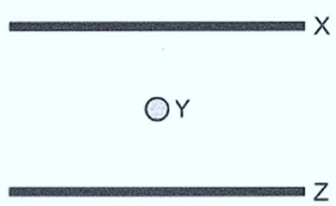
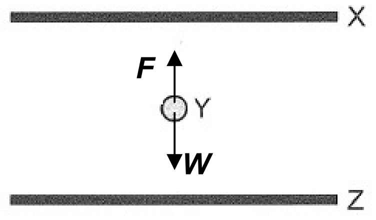


	plate X	sphere Y	plate Z
A	negative	positive	negative
B	negative	positive	positive
C	positive	positive	negative
D	positive	negative	positive

(Answer: B)

The weight acts downwards and has to be considered. The sphere is suspended stationary, therefore it is in equilibrium.



To counter the weight, we need an upward force. Find the combination of charges and polarity that achieves this. X and Y has to be oppositely-charged to attract, and Y and Z should be like-charged to repel.

<table>
<tr><td>9</td><td>(a)</td><td>

Apply $E = \dfrac{V}{d}$

$E = \dfrac{10\text{ V}}{(5.0\times10^{-2})\text{ m}}$

$= 200\text{ V m}^{-1}$ (vertically downwards)

</td></tr>
<tr><td></td><td>(b)</td><td>

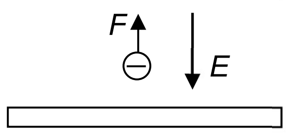
Magnitude of electric force, $F = qE$

$= (2.0 \times 10^{-6})(200)$

$= 4.0 \times 10^{-4}$ N

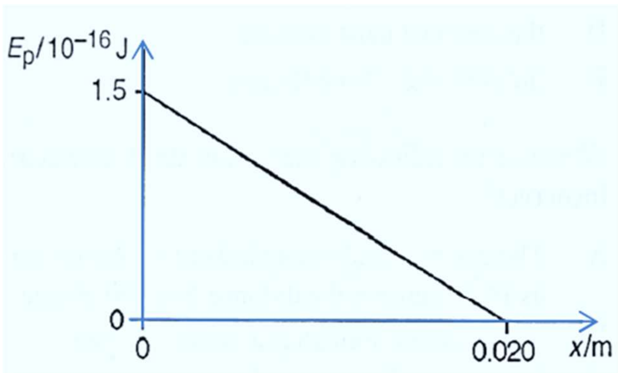
Direction of electric force: vertically upwards

</td></tr>
<tr><td></td><td>(c)</td><td>



Force on negative charge acts towards the plate with a higher electric potential. Notice the electric force exerted on a negative charge acts opposite to the direction of electric field between the plates.

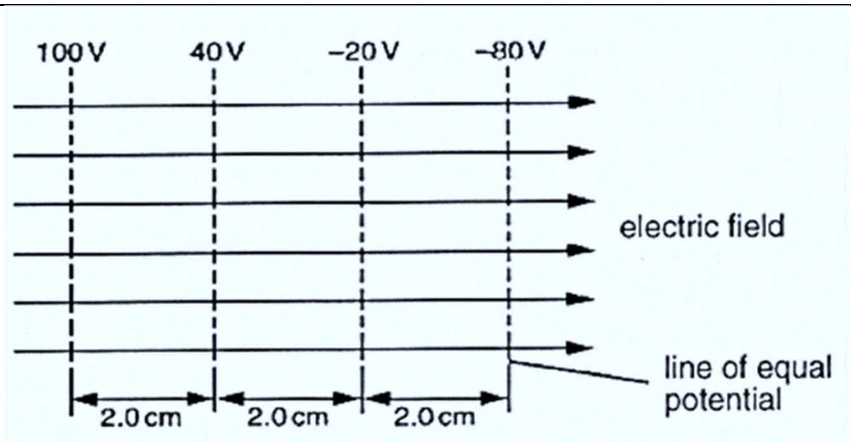
</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>10</td><td colspan="2">



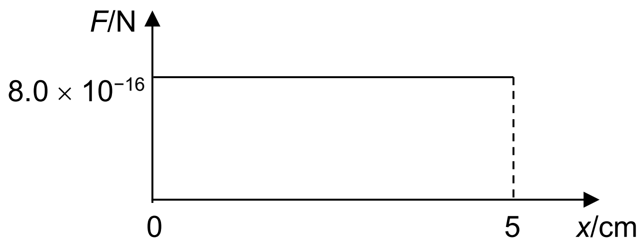
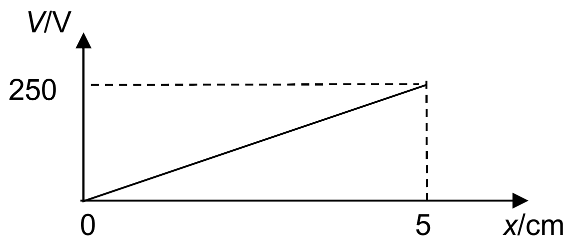
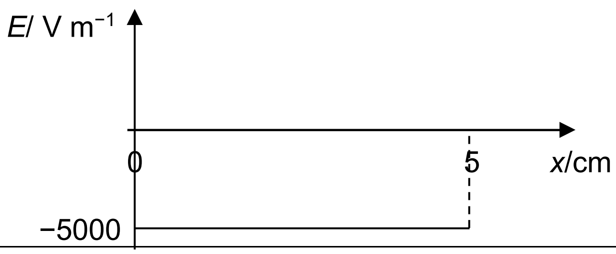
Magnitude of force = |gradient of graph| = $\left|\dfrac{1.5\times10^{-16}-0}{0-0.020}\right|$

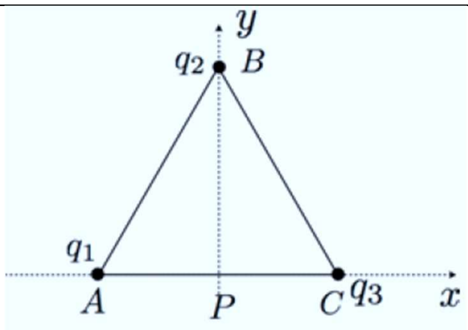
$= 7.5 \times 10^{-15}$ N

</td></tr>
<tr><td></td><td colspan="2"></td></tr>
<tr><td>11</td><td colspan="2">

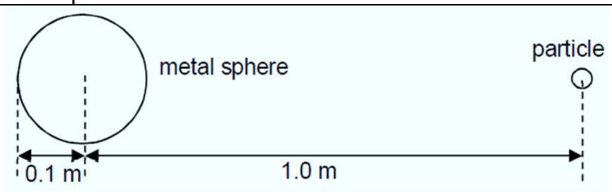


</td></tr>
</table>

	$F = qE = q\dfrac{\lvert\Delta V\rvert}{d} = (5.0 \times 10^{-6})\left(\dfrac{100-40}{2.0\times10^{-2}}\right)$ $= 1.5 \times 10^{-2}$ N (in direction of the electric field)		
12	(a)	Horizontally, the electron moves with uniform velocity u_x; using $s_x = u_x t$, time interval that the electron is inside the electric field is $t = \dfrac{s_x}{u_x} = \dfrac{0.10}{2\times10^{7}} = 5.0 \times 10^{-9}$ s	
	(b)	Vertically, the electron moves with uniform acceleration a: $ma = qE = e\dfrac{\lvert\Delta V\rvert}{d}$ $\Rightarrow \quad a = \dfrac{eV}{md} = \dfrac{(1.60\times10^{-19})(250)}{(9.11\times10^{-31})(5.0\times10^{-2})}$ $= 8.78 \times 10^{14}$ m s^{-2} (vertically upwards)	
	(c)	Using $s_y = u_y t + \frac{1}{2}a_y t^2$, $s_y = \frac{1}{2}a_y t^2 = \dfrac{eV}{2md}\left(\dfrac{s_x}{u_x}\right)^2$ $= \dfrac{(1.60\times10^{-19})(250)}{2(9.11\times10^{-31})(5.0\times10^{-2})}\left(\dfrac{0.10}{2.0\times10^{7}}\right)^2$ $= 0.011$ m	
	(d)	The direction of the electric field strength (high V to low V): **downwards**. The direction of the electric force acting on an electron (–ve): **upwards**.	
	(e)	(i)	
		(ii)	
		(iii)	

13	(a)	$E = \dfrac{45}{0.006} = 7500$ N C^{-1} Newton's 2nd law: $F = ma \Rightarrow qE = ma$ $7500 \times 1.60 \times 10^{-19} = 9.11 \times 10^{-31}\, a$ $a = 1.317 \times 10^{15}$ m s^{-1}, downwards The electric field lines should be straight and equally spaced parallel lines directed upwards. The path of the beam is a parabolic curve.
	(b)	Let the speed of electrons emerging from B be u and their time of travel between A and B be T. $s_x = u_x t$: $0.025 = (u \cos 15^\circ)\, T$ $T = \dfrac{0.025}{u \cos 15^\circ}$ ——— (1) When the beam is at the highest point, $v_y = u_y + a_y t$: $0 = u \sin 15^\circ - (1.317 \times 10^{15})\left(\dfrac{T}{2}\right)$ $u \sin 15^\circ = (1.317 \times 10^{15})\left(\dfrac{T}{2}\right)$ ——— (2) Substitute (1) into (2): $u \sin 15^\circ = (1.317 \times 10^{15})\left(\dfrac{0.025}{2u \cos 15^\circ}\right)$ $u = 8.11 \times 10^{6}$ m s^{-1}
	Electric potential energy	
14	 	
	$EPE_{\text{system}} = \left(\dfrac{1}{4\pi\varepsilon_0 r}\right)(q_1q_2 + q_2q_3 + q_1q_3)$ $= 0.798$ J	
	in general, total $EPE_{\text{system}} = \left(\dfrac{1}{4\pi\varepsilon_0}\right)\sum_{i<j}\dfrac{q_iq_j}{r_{ij}}$	

15



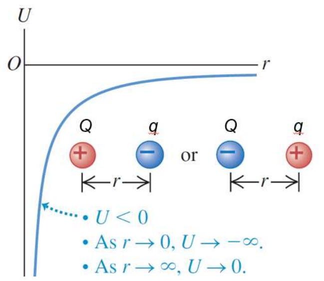
Conservation of energy:

$$\Rightarrow \Delta K + \Delta U_{electric} = 0$$

$$\Rightarrow \left(\frac{1}{2}mv^2 - 0\right) + q\Delta V = 0$$

$$\Rightarrow \frac{1}{2}mv^2 + q\left(V_f - V_i\right) = 0$$

$$\Rightarrow \frac{1}{2}mv^2 + q\left[\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)\right] = 0$$



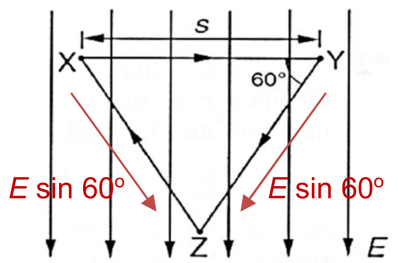
$$\Rightarrow \frac{1}{2}\left(2.0\times10^{-5}\right)v^2 + \left(-1.5\times10^{-10}\right)\left[\frac{1.0\times10^{-4}}{4\pi\varepsilon_0}\left(\frac{1}{0.1} - \frac{1}{1.0}\right)\right] = 0$$

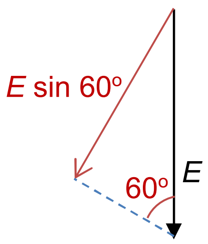
$$\Rightarrow \frac{1}{2}\left(2.0\times10^{-5}\right)v^2 = \left(1.5\times10^{-10}\right)\left[\frac{1.0\times10^{-4}}{4\pi\varepsilon_0}(9)\right]$$ {i.e. increase in KE = decrease in EPE}

$$\Rightarrow v = 11 \text{ m s}^{-1}$$

16

Along XY, the electric force is perpendicular to the path XY, work done by electric force is zero. Hence work done (by an external force) **against** electric force is zero.





The component of E along YZ is E sin 60°, pointing from Y to Z.

The component of electric force along YZ on the **unit** positive test charge
= (1 C) (E sin 60°) = E sin 60°, pointing from Y to Z.

Work done by electric force in moving the charge along YZ
= (E sin 60°) (s) [cos 0°] = Es sin 60°

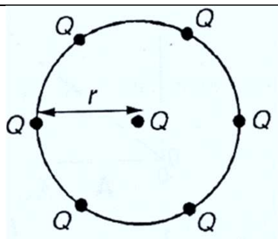
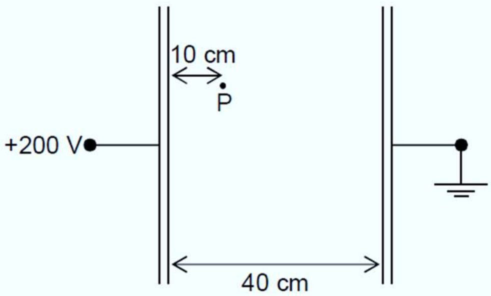
Work done (by an external force) **against** electric force in moving the charge along YZ is −Es sin 60°

The component of E along ZX is E sin 60°, pointing from X to Z.

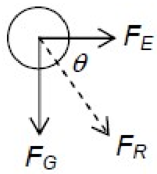
The component of electric force along ZX on the **unit** positive test charge
= (1 C) (E sin 60°) = E sin 60°, pointing from X to Z.

Work done by electric force in moving the charge along ZX
= (E sin 60°) (s) [cos 180°] = −Es sin 60°

Work done (by an external force) **against** electric force in moving charge along YZ is +Es sin 60°

17	 potential when the charge Q is at centre of circle, V_C = sum of potential due to individual charge $= 6\left(\frac{Q}{4\pi\varepsilon_0 r}\right)$ potential when the charge Q is at infinity, $V_\infty = 0$ alternative & equivalent viewpoint work done by repulsive electric-field forces to move Q from centre of circle to infinity = work done by an external force (equal and opposite to the electric-field force) to move charge Q from infinity to centre of circle $= Q(\Delta V_{\infty \text{ to C}}) = Q(V_C - V_\infty) = Q\left(\frac{6Q}{4\pi\varepsilon_o r} - 0\right) = \frac{6Q^2}{4\pi\varepsilon_o r}$
	Sample Examination Question(s)
18	**2012/RJC/P3/Q3** **(a)** A student states that "*the electrostatic force acting on a charged particle placed in an electric field always acts in the direction of the field*". Comment on the statement. [2] **(b)** Two parallel, infinitely-long vertical plates are placed 40 cm apart in a vacuum, with a potential difference of 200 V applied across them, as shown in Fig. 3. An oil drop of mass 1.4×10^{-15} kg is released from rest at P, 10 cm away from the plate of higher potential. The oil drop carries a charge of $+1.5 \times 10^{-17}$ C. **Fig. 3** (i) Calculate the angle to the horizontal at which the oil drop will accelerate from P. [61.4°] [3] (ii) The vertical distance travelled by the oil drop when it hits the plate is 55 cm. By considering the change in potential energies, calculate the kinetic energy of the oil drop when it hits the plate. [9.80×10^{-15} J] [3]

18	(a)	The **electrostatic force on a positively charged particle is in the same direction** as the field lines as a positively charged particle is attracted to the lower electric potential region.

		However, the **electrostatic force on a negatively charged particle is in the opposite direction** to the direction of field lines as a negatively charged particle is attracted to the higher electric potential region. That is, **direction of electrostatic force depends on the sign of the charge.** The statement is **untrue / false**.	
	(b)	(i)	$F_G = mg = (1.4\times10^{-15})(9.81)$ $F_E = qE = \frac{q\lvert\Delta V\rvert}{d} = \left(\frac{(1.5\times10^{-17})(200)}{0.4}\right)$ $\tan\theta = \frac{F_G}{F_E}$ $\theta = \tan^{-1}\left(\frac{F_G}{F_E}\right) = 61.4^\circ$ 
		(ii)	The electric potential decreases uniformly from +200 V to 0 V in the uniform field between the plates, so the electric potential at point P is +(200/40) ×30 = +150 V. $\Delta EPE = q\Delta V' = (1.5\times10^{-17}\text{ C})(0-150\text{ V})$ $\Delta GPE = mg\Delta h = (1.4\times10^{-15}\text{ kg})(9.81\text{ m s}^{-2})(-0.55\text{ m})$ Conservation of energy: $\Delta EPE + \Delta GPE + \Delta KE = 0$ $\Rightarrow \Delta KE = -(\Delta EPE + \Delta GPE) = 9.80\times10^{-15}\text{ J}$

19 **N2011/P2/4**

A small metal sphere S is supported by an insulating string that is fixed at point P, as shown.

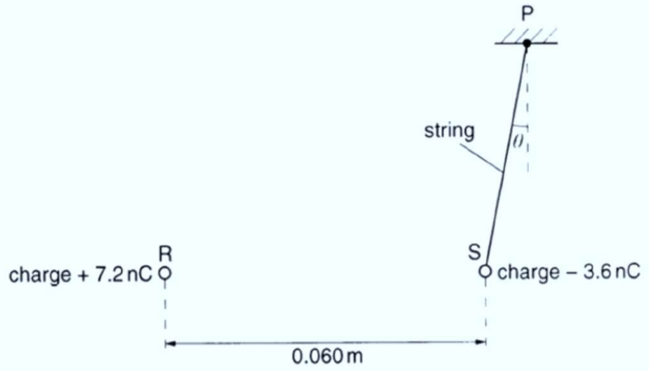


Fig. 4

Sphere S has a weight of 0.49×10^{-3} N and charge −3.6 nC. Another metal sphere R with a charge of +7.2 nC is placed 0.060 m from S. The diameter of the spheres is negligible compared to the distance between them.
A line joining R and S is horizontal. The string makes an angle θ with the vertical.

(a) **(i)** On Fig 4, draw electric field lines to represent the electric field in the region between R and S. [3]

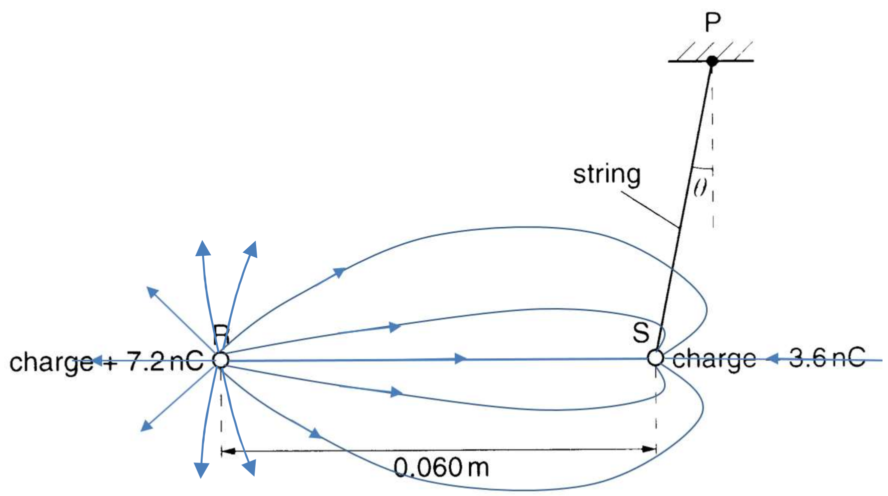
(ii) Calculate the magnitude of the electric field strength at the mid-point between R and S. [1.1×10^5 N C^{-1}] [2]

(iii) Show that the electric force acting on S is 6.5×10^{-5} N. [2]

(b) Using resolution of forces or a vector triangle, determine the angle θ and the tension in the string. [7.5°, 4.9×10^{-4} N] [4]

(c) The horizontal distance from R towards S is x. Sketch the variation with x of the potential V. [2]

19 (a) (i)



Notes:
RS is a line of symmetry.
The field lines in the region close to R is denser than the field lines in the region close to S.
Field lines point away from R and point towards S.

(a) (ii)

$$|E_{\text{resultant}}| = |E_R| + |E_S|$$

$$= \left|\frac{Q_R}{4\pi\varepsilon_o r^2}\right| + \left|\frac{Q_S}{4\pi\varepsilon_o r^2}\right|$$

$$= \frac{1}{4\pi\left(8.85\times10^{-12}\right)}\left[\frac{1}{0.030^2}(7.2+3.6)\times10^{-9}\right]$$

$$= 107900 \text{ N C}^{-1}$$

$$\approx 1.1\times10^5 \text{ N C}^{-1}$$

Notes: Should add instead of subtract the magnitudes of the electric fields due to the two charges.

(a) (iii)

Using Coulomb's law,

$$F = \frac{Q_R Q_S}{4\pi\varepsilon_o r^2}$$

$$= \frac{1}{4\pi\left(8.85\times10^{-12}\right)}\left[\frac{\left(7.2\times10^{-9}\right)\left(3.6\times10^{-9}\right)}{0.060^2}\right]$$

$$\approx 6.5\times10^{-5} \text{ N}$$ (Shown)

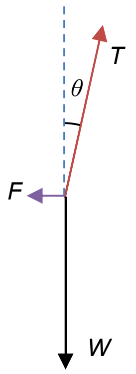
(b)

Using resolution of forces,

$$F / W = \tan\theta$$

$$\frac{6.5\times10^{-5}}{0.49\times10^{-3}} = \tan\theta$$

$$\theta = 7.5^\circ$$



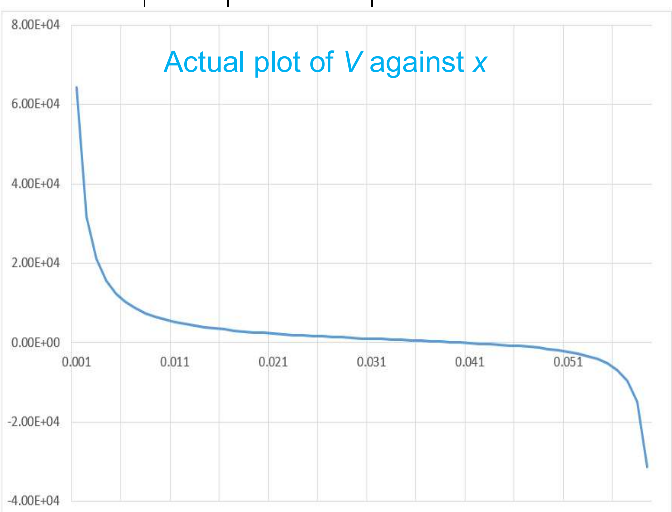
By Pythagora's theorem,

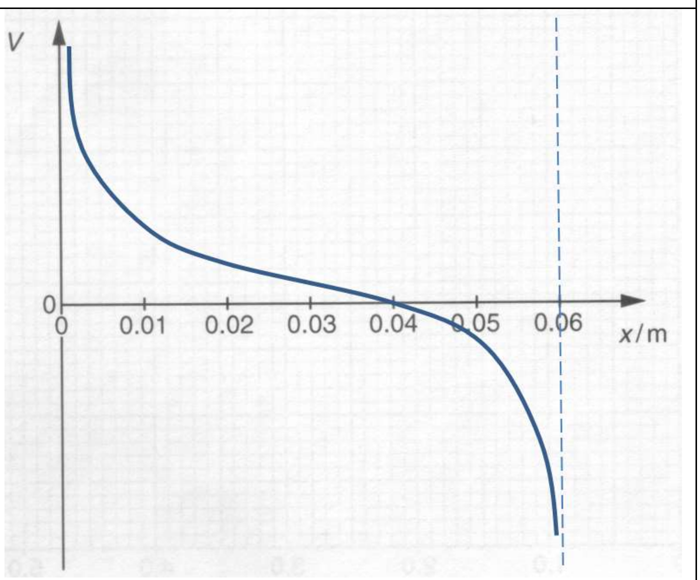
$$T^2 = (F^2 + W^2)^{1/2}$$

$$T = [(6.5\times10^{-5})^2 + (0.49\times10^{-3})^2]^{1/2}$$

$$= 4.9\times10^{-4} \text{ N}$$

(c)





when $\frac{1}{4\pi\varepsilon_o}\left[\frac{2Q}{r} - \frac{Q}{0.060 - r}\right] = 0$

we have $2(0.06 - r) - r = 0$

$\Rightarrow r = 0.040 \text{ m}$

20 (2014 P3 Q6)
Two charged solid metal spheres A and B are situated in a vacuum. Their centres are separated by a distance of 30.0 cm, as illustrated in Fig. 20.1. The diagram is not to scale.

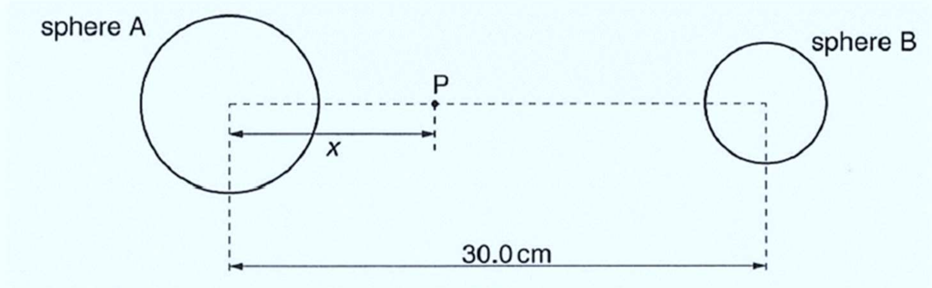


Fig. 20.1

Point P is a point on the line joining the centres of the two spheres. Point P is a distance *x* from the centre of sphere A.

The variation with distance *x* of the electric field strength *E* at point P is shown in Fig. 20.2.

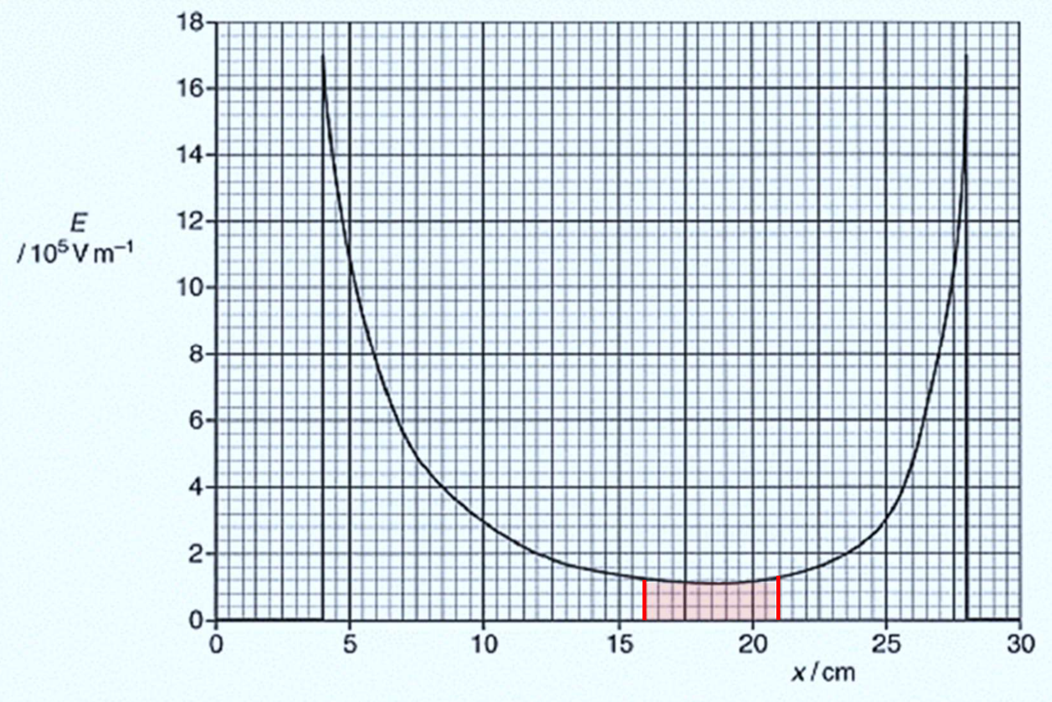


Fig. 20.2

(a) Suggest why the electric field strength is zero for two regions of *x*. [1]

(b) Use Fig. 20.2 to

(i) determine the radius of each sphere, [2]

(ii) state and explain whether the spheres have charges of the same, or opposite, sign. [2]

(c) A lithium-7 $\left({}^{7}_{3}\text{Li}\right)$ nucleus moves along the line joining the centres of the two spheres.

(i) Estimate the energy gained by this nucleus as it moves from point P where x = 16.0 cm to the point P where x = 21.0 cm. [2.88×10^{-15} J][5]

(ii) Calculate the acceleration of the nucleus at point P where x = 25.0 cm. [1.24×10^{13} m s^{-2}] [2]

(iii) The nucleus is at rest at point P where x = 4.0 cm. Describe qualitatively the variation with x of the acceleration of the nucleus for x = 4.0 cm to x = 28.0 cm. [3]

20	(a)	The two regions are within the metal spheres A and B. A conductor[1] forms an equipotential volume and has an equipotential surface. All points in the conductor are at the same potential (i.e., potential gradient is zero), so the net electric field must be zero[2] inside a conductor. Mathematically, V is constant, $-\dfrac{dV}{dx} = 0$ and $E = 0$.
	(b) (i)	Radius of sphere A = 4.00 cm Radius of sphere B = 2.00 cm
	(b) (ii)	The spheres have charges of opposite sign. Because the resultant electric field strength is **never zero along the line joining the centres of the two spheres**, implying that the individual electric field strength, which is a vector quantity, due to each sphere at a point is always acting in the same direction. OR Their resultant field strength did not fall to zero in the region of space between them. If their charges have the same sign, their fields would have been in opposite directions and be able to cancel out vectorially at some point in between them.

	(c) (i)	A $^{7}_{3}\text{Li}$ nucleus has a mass of 7*u* and an elementary charge of +3*e*. (1 *u* = 1.66 × 10^{-27} kg) change in electric potential energy of nucleus, $= \Delta U = q\Delta V = q\left(-\int_{x_1}^{x_2} E dx\right)$ $\left\{E = -\frac{dV}{dx} \Rightarrow \Delta V = -\int_{x_1}^{x_2} E dx\right\}$ = − *q* × (area under the graph from *x* = 16.0 cm to *x* = 21.0 cm, which can be approximated as a rectangle) $\approx -\left(3 \times 1.60 \times 10^{-19}\right)\left[\left(\frac{21.0 - 16.0}{100}\right) \times \left(1.2 \times 10^{5}\right)\right]$ = −2.88 × 10^{-15} J Conservation of energy: $\Delta K + \Delta U = 0 \Rightarrow \Delta K = -\Delta U = +2.88 \times 10^{-15}$ J Thus, kinetic energy gained by nucleus = 2.88 × 10^{-15} J
	(c) (ii)	From Newton's 2nd Law, $\sum F = ma = qE$ $(7)\left(1.66 \times 10^{-27}\right)a = (3)\left(1.60 \times 10^{-19}\right)\left(3.0 \times 10^{5}\right)$ $a \approx 1.24 \times 10^{13}$ m s^{-2}
	(c) (iii)	According to Newton's second law of motion, the acceleration *a* of the nucleus is directly proportional to its resultant force *F* and thus also directly proportional to the electric field strength: $a = \frac{F}{m} = \frac{qE}{m} \Rightarrow a \propto E$ Its acceleration *a* thus decreases in a similar manner as *E* from a maximum value at *x* = 4.0 cm to a minimum value at around *x* = 18.5 cm before increasing again to the initial maximum value at *x* = 28.0 cm.

\- End -

[1] Every point inside a perfect conductor must have exactly the same potential. This is because the charge carriers are free to move about inside the material. If there were points of different potential the charge carriers would move in order to balance them out. Conductors may be charged and yet still be an equipotential surface.
[2] Electric fields connect points of different potential. If all points in the conductor are at the same potential, then the net electric field inside must be zero.