TOPIC 18: Alternating Current

GUIDING QUESTIONS

- ❖ How is alternating current different from direct current?
- ❖ How can we describe alternating current, e.g., mathematically, and graphically?
- ❖ Why is alternating current used in the generation and transmission of electricity?

Content

- Characteristics of alternating currents
- The transformer
- Rectification with a diode

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage

(b) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin \omega t$

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = I_o / \sqrt{2}$ for the sinusoidal case

(e) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $N_s / N_p = V_s / V_p = I_p / I_s$ for an ideal transformer

(f) explain the use of a single diode for the half-wave rectification of an alternating current.

18.0 Introduction

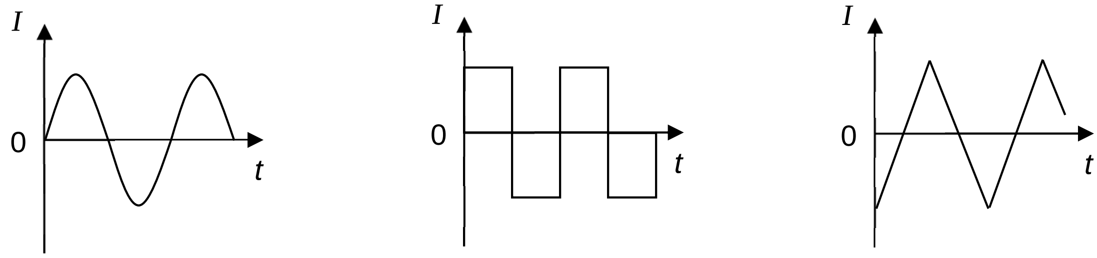
During the 1880s in the US there was a heated and acrimonious debate between two inventors over the best method of electric-power distribution. Thomas Edison (1847-1931) favoured direct current (d.c.) – i.e., steady current that does not vary with time. George Westinghouse (1846 - 1914) favoured alternating current (a.c.), with sinusoidally varying voltages and currents. He argued that transformers (which we will study in this chapter) can be used to step the voltage up and down with a.c. but not with d.c.; low voltages are safer for consumer use, but high voltages and correspondingly low currents are best for long-distance power transmission to minimize I^2R losses in the cables.

Eventually, Westinghouse prevailed, and most present-day household and industrial power-distribution systems operate with a.c.. Every time you turn on a light, a television set, a stereo, or any of a multitude of other electrical appliances in a home, you are using alternating currents to provide the power to operate them.

In this chapter we will learn how resistors behave in circuits with sinusoidally varying voltages and currents, why transformers are useful and how they work, and how a single diode converts the a.c. to d.c.

An alternating current is an electric current that periodically reverses its direction in a circuit with a frequency.

Below are some examples of a.c.:

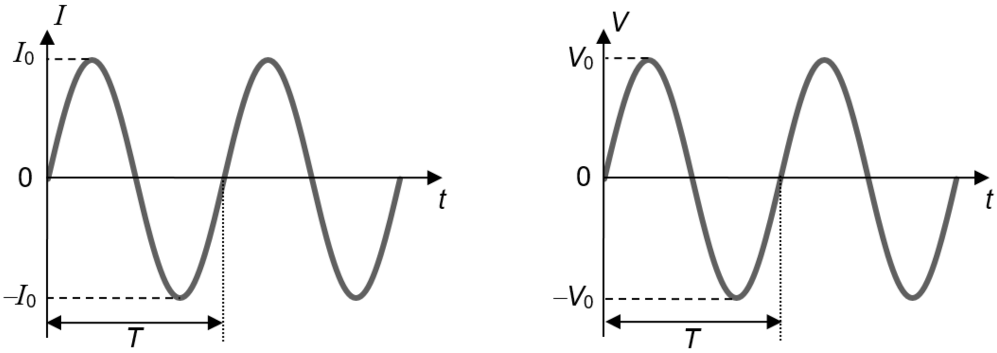


The principles of direct current in resistors learned in previous topics can be applied to resistors in a.c. circuits. However, major differences in circuit analysis arise when inductors and capacitors are to be considered.

18.1 Quantities characterising an alternating current or voltage

Quantity	***Symbol***	***Description***
period	T	The period of an A.C. source refers to the time taken to complete one cycle. The SI units for time is seconds (s).
frequency	f	The frequency of an A.C. source refers to the number of complete cycles per unit time. The SI units for frequency is Hertz (Hz) or s^{-1}. Frequency is related to period by the equation $f = \frac{1}{T}$. For a sinusoidal A.C. waveform, the concept of angular frequency $\omega = 2\pi f$ is useful.
peak value	I_0	Maximum absolute value of the alternating current or voltage in either direction of zero value in a periodic cycle. For a sinusoidal waveform, this corresponds to the *amplitude* for simple harmonic motion. If the waveform is not symmetrical, it makes more sense to distinguish peak positive value and peak negative value.
root mean square value	I_{rms}	Root-mean-square current of alternating current is the value of the steady direct current which would dissipate heat at the same average rate in a given resistor.
peak to peak value		Difference between the positive peak value and the negative peak value of the a.c. within a cycle.

The most commonly encountered form of a.c. is the <u>sinusoidal</u> form, that is, it varies with time according to a sine or cosine function.

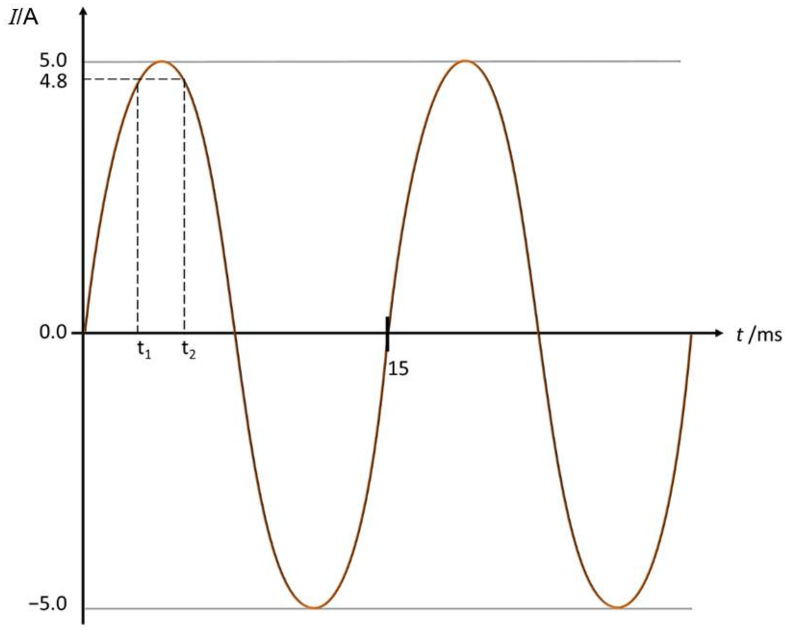


The sinusoidal alternating current can be represented by the equations:

$$I = I_0 \sin \omega t$$
$$V = V_0 \sin \omega t$$

In these expressions, I and V are the *instantaneous* current and voltage; I_0 and V_0 are the *peak* current and voltage (current and voltage amplitude); and ω is the *angular frequency*, equal to $2\pi f$ or $2\pi/T$.

<u>Self-assessment</u>:



1. For this AC source, the peak current is ……………..…… A.

2. The period is …………………….. s.

3. We can write the equation, with I measured in amperes and t measured in seconds, as

I = ………… sin …………. πt

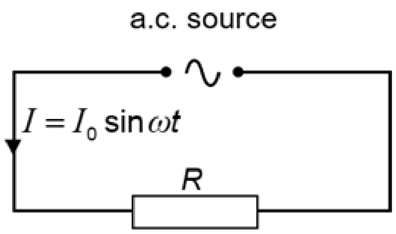
4. The time when the current first reaches 4.8 A is t_1 and the next time the current has the same value of 4.8 A is t_2. Using the equation to solve for these times, we have t_1 = …………………….. ms and t_2 = …………………….. ms.

18.1.1 Mean value of a.c.

For the case of <u>sinusoidal</u> current, any positive value of current, there will be a corresponding negative value within a complete cycle, thus the *mean value* of current I is <u>*zero*</u>.

However, heat is dissipated when it flows in a resistor, implying that the mean value of an a.c. does **not** represent the *effective value of the a.c*.

18.1.2 Mean power of sinusoidal alternating current in a resistive load



For a sinusoidal alternating current,

$$I = I_0 \sin \omega t$$

The instantaneous power dissipated in the resistor

$$P_{\text{instantaneous}} = I^2 R = I_0^2 R \sin^2 \omega t$$

Mean power,

$$\begin{aligned}\langle P \rangle &= \left(\text{mean value of } I^2\right) R \\ &= \left\langle I_0^2 \sin^2 \omega t \right\rangle R \\ &= I_0^2 \left\langle \sin^2 \omega t \right\rangle R \\ &= I_0^2 \left(\frac{1}{2}\right) R \\ &= \left(\frac{1}{2}\right) I_0^2 R \\ &= \frac{1}{2} P_{\max}\end{aligned}$$

So, the mean power dissipated in a resistive load is …………………………………………….

The mean power delivered by the source is converted to internal energy in the resistor, just as in the case of a d.c. circuit.

18.1.3 Root-mean-square value of a.c.

Although the current is not in one direction only, power is converted in the resistor. This is because the power/heating depends on I^2, so independent of current direction.

Recall that the electrical power P dissipated in a resistor is

$$P = I^2 R$$

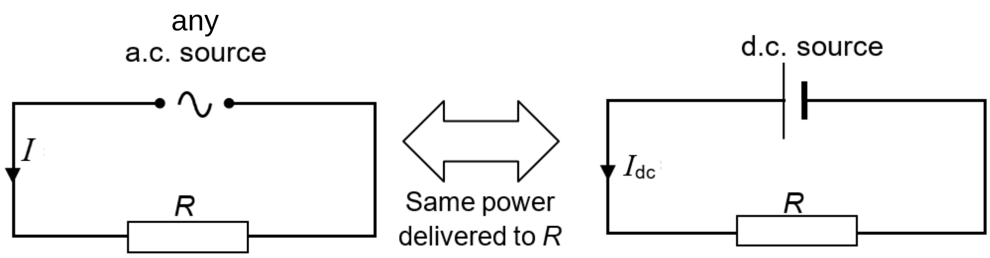
In an a.c., the instantaneous power dissipated in a resistor is given by

$$P_{\text{instantaneous}} = I^2 R$$

Mean power dissipated in a resistor over one cycle:

$$\begin{aligned}\langle P\rangle &= \text{mean value of } P_{\text{instantaneous}}\\ &= \langle I^2R\rangle\\ &= \langle I^2\rangle R\\ &= \left(\sqrt{\langle I^2\rangle}\right)^2 R\\ &= I_{\text{rms}}^2 R\end{aligned}$$

I_{rms} is the square root of the mean value of I^2 and hence is known as the root-mean-square (r.m.s.) current of the a.c. source.



To find what value of current by a d.c. source will dissipate the *same power* in a resistor as the ***mean power*** *dissipated by an* a.c. when it flows through the resistor, we equate:

$$\begin{aligned}P_{\text{dc}} &= \langle P\rangle\\ I_{\text{dc}}^2R &= \langle I^2R\rangle\\ &= \langle I^2\rangle R \quad \text{since } R \text{ is constant}\\ I_{\text{dc}}^2 &= \langle I^2\rangle\\ I_{\text{dc}} &= \sqrt{\langle I^2\rangle} = I_{\text{rms}}\end{aligned}$$

In other words, a d.c. of magnitude I_{rms} will produce the same heating effect as the a.c.

Thus the r.m.s. value can be considered as the *effective value of the a.c.*

The r.m.s. value of alternating current is the value of the steady direct current which would dissipate heat at the same average rate in a given resistor.

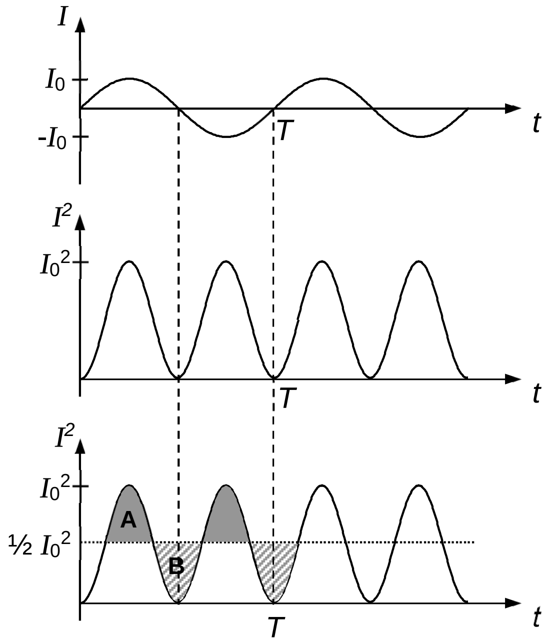
Graphically, it is the square root of the mean value of the square of the instantaneous current over one cycle.

$$I_{\text{rms}} = \sqrt{\langle I^2\rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2\text{-}t \text{ graph over one cycle}}{\text{period}}}$$

Procedure for calculating the r.m.s. current:

1. Square the current at each point in time for the entire repeating pattern.
2. Find the average (mean value) of these squared values, over the entire period.
3. Take the square root of this average.

In general, the r.m.s. current of an a.c. source is also known as the d.c. equivalent current.



step

square ①

mean = ½ I_0^2 (∵ area A = area B) ②

square **r**oot = $\dfrac{I_0}{\sqrt{2}}$ ③

For a sinusoidal a.c., $I_{rms} = \dfrac{I_0}{\sqrt{2}}$ (refer to Annex A)

Note that in a similar way, we can also find the root-mean-square (r.m.s.) voltage for an a.c. circuit.

So, $V_{rms} = \dfrac{V_0}{\sqrt{2}}$

Note that: (i) When we say that an alternating current is 10 A, we mean that the r.m.s. value is 10 A unless otherwise state. (ii) The waveform is assumed to be sinusoidal unless otherwise stated.

Recall that $\langle P \rangle = \dfrac{1}{2} P_{max}$

therefore $\langle P \rangle = \dfrac{1}{2} I_0 V_0 = \dfrac{I_0}{\sqrt{2}} \dfrac{V_0}{\sqrt{2}} = \underline{I_{rms} V_{rms}}$

For a resistance, $V_{rms} = I_{rms} R$

so $\langle P \rangle = \underline{I_{rms}^2 R = \dfrac{V_{rms}^2}{R}}$

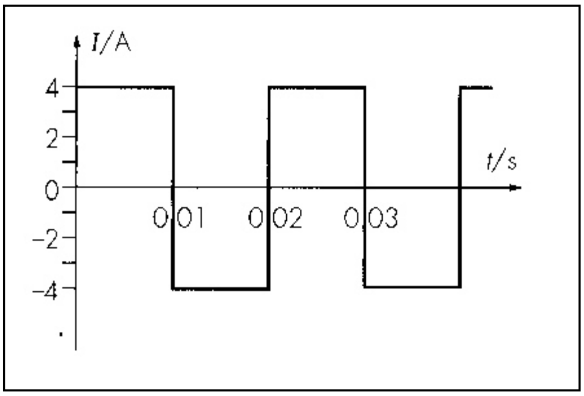
The mean power delivered to a resistor that carries an a.c. have the same form as the corresponding relationships for a d.c. circuit.

You may refer to Annex B for comparisons of I_{rms}, $\langle P \rangle$ and $\langle I \rangle$ for other variations of current.

Example 1

Calculate the mean value and r.m.s. value for each of the a.c. shown

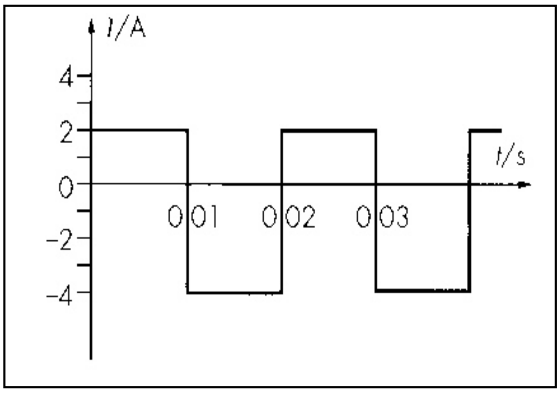
(a)



mean value:

r.m.s. value:

(b)



mean value:

r.m.s. value:

Example 2

A sinusoidal current described by the equation $I = 9.0 \sin \omega t$ flows through a 12.0 Ω resistor. Calculate

(a) the r.m.s. value of the current and voltage.

(b) the maximum instantaneous power in the resistor.

(c) the average power dissipated in the resistor.

Example 3

A tourist from U.S.A. brings an electric water kettle designed to operate with 110 V to Singapore and plugs it into a 230 V outlet. The heater breaks down due to overheating.

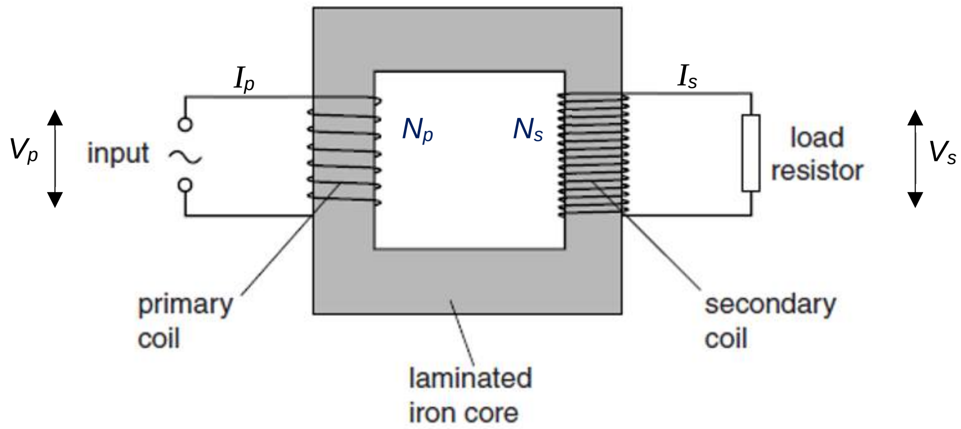
(a) If the heater normally consumes 500 W, what is the resistance of the heating coil?

(b) How much power does it consume using our 240 V electrical supply?

(c) What is the r.m.s. current that flows when the potential difference is 230 V?

18.2 The Transformer

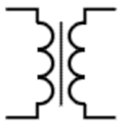
A simple iron-core transformer is illustrated below:



Two insulated copper coils, called the primary and secondary coils, not electrically connected to one another, are wound on separate limbs of a soft iron core (which is laminated to reduce power loss due to eddy currents).

The iron core keeps the magnetic flux due to a current in the primary coil almost completely within the core and hence almost all the magnetic flux pass through the secondary coil.

Circuit symbol for a transformer (with an iron core):



Principle of operation:

A transformer works by electromagnetic induction.

The a.c. source causes an alternating current in the primary, which sets up an alternating magnetic flux (almost completely contained) in the iron core.

This induces an alternating e.m.f. in each coil, in accordance with Faraday's law of induction.

The induced e.m.f. in the secondary give rise to an alternating current in the secondary, and this delivers energy to the device to which the secondary is connected.

All currents and e.m.f.s have the same frequency as the a.c. source.

18.2.1 The Ideal Transformer

An ideal transformer is one in which there is no power loss (i.e. input power = output power). In addition, the same magnetic flux passes through each turn of both the primary and secondary coils, that is, there is no flux leakage.

Using the Laws of Electromagnetic Induction, it can be shown (in Annex C) that for an ideal transformer, turn ratio of a transformer is given by

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

where, N_p = number of turns on the primary coil,
N_s = number of turns on the secondary coil,
V_p = input voltage to the primary coil,
V_s = output voltage across the load resistor connected to the secondary coil

For an ideal transformer, the power input to the primary coil is <u>equal to</u> the power output at the secondary circuit. Hence:

$$I_p V_p = I_s V_s$$

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Thus, for an <u>ideal</u> transformer:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

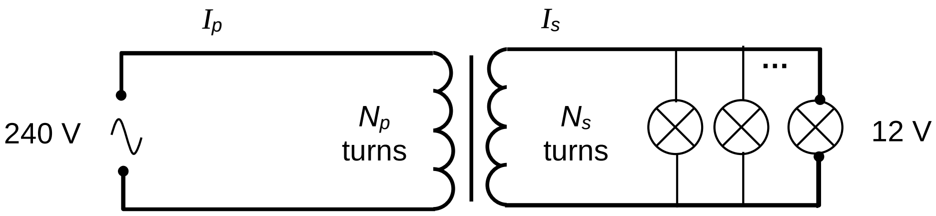
If $N_s > N_p$, the transformer is called a ***<u>step-up</u>*** transformer since $V_s > V_p$. A ***<u>step-down</u>*** transformer has $N_s < N_p$.

The above equation tells us that an *ideal transformer* that steps up the voltage *<u>simultaneously</u>* steps down the current and a transformer that steps down the voltage steps up the current. However, the power is neither stepped up nor stepped down.

Practical transformers always have some power losses. The causes of power loss are given in Annex D.

Example 4

Ten lamps each rated 24 W, 12 V are connected in parallel with the mains supply of 240 V as shown.

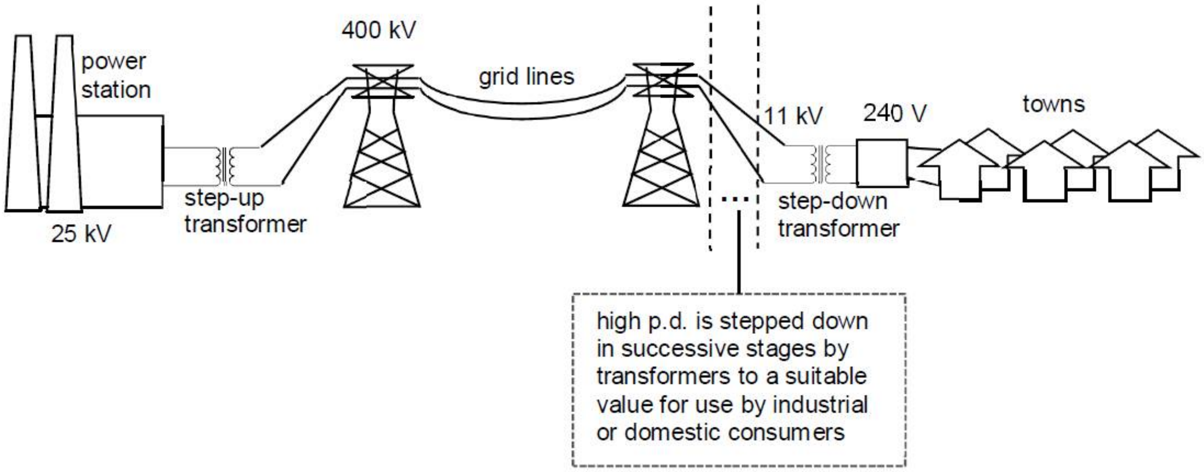


(a) Calculate the turns-ratio of the transformer needed.

(b) What would be the current drawn from the mains supply if the transformer is 100% efficient?

(c) What would be the current drawn from the mains supply if the transformer is 91% efficient?

18.2.2 Transmission of electrical power



When it comes to heating and lighting applications, a.c. has no practical advantage over d.c. because the heating effect of a current is independent of its direction. In fact, many practical systems like large motors and railway systems can only run on d.c.

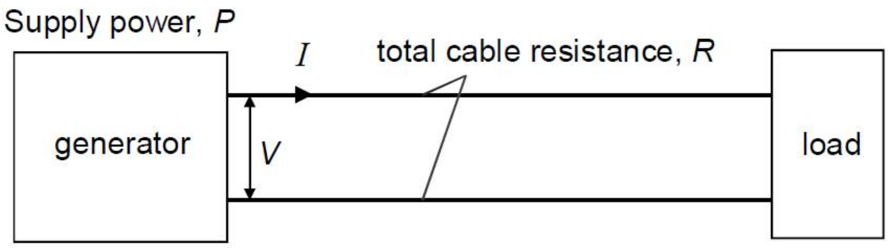
The main consideration influencing the use of high alternating, rather than d.c., power supplies in long-distance transmission is that it is more effective in minimising power loss in the transmission lines with the use of transformers to efficiently step up the voltage and transmit the electrical power at high voltages (low currents), and, subsequently, to step down the voltage for use by homes as household appliances do not need high voltages.

Transformers, works only with a.c. and not d.c., meet these requirements at both ends of the transmission line.

Example 5

A generator produces power at a p.d. of about 10 kV to 30 kV. It is then stepped up by transformers to a high p.d. (e.g. 400 kV) for transmission. Why is the power transmitted at a high p.d.?

Consider a transmission line delivering power P from a generator to a load as shown. Suppose the total resistance of the cables between the generator and the load is R, and that the load is such that the effective value of the current is I and the p.d. at the supply end is V.



(a) What is the current in the transmission line?

(b) What is the power loss P_{loss} in the transmission line?

(c) For a given value of P, how can this power loss be minimized?

(d) How does the power loss for a 400 kV system compare to that a 132 kV system?

Example 6

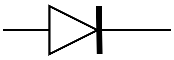
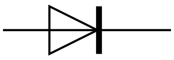
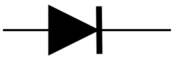
An electricity-generating station delivers energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(a) If the resistance of the wires is 2.0 Ω and energy costs $0.20 per kWh, estimate the cost for the energy converted to internal energy in the wires in one day.

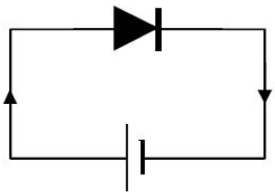
(b) Find the cost in (a) if the energy is delivered at its original voltage of 22 kV.

18.3 Rectification with a diode

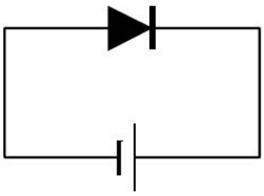
Rectification is the conversion of a.c. to d.c. by a rectifier (or diode).

Circuit symbol for a a diode:  *or*  *or* 

Diodes are used for rectification. A diode is a device that conducts current better in one direction than in the other; an ideal diode has zero resistance for one direction of current and infinite resistance for the other. When connection is made to a supply so that a diode conducts it is said to be *forward biased*; in the non-conducting state it is *reversed biased*. The arrow head on the symbol for a diode indicates the forward direction of conventional current flow. \

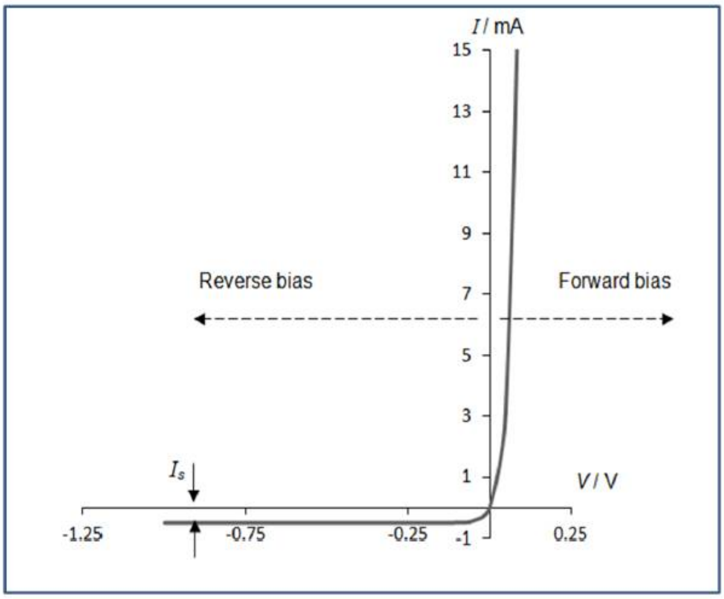


Forward biased; current flows



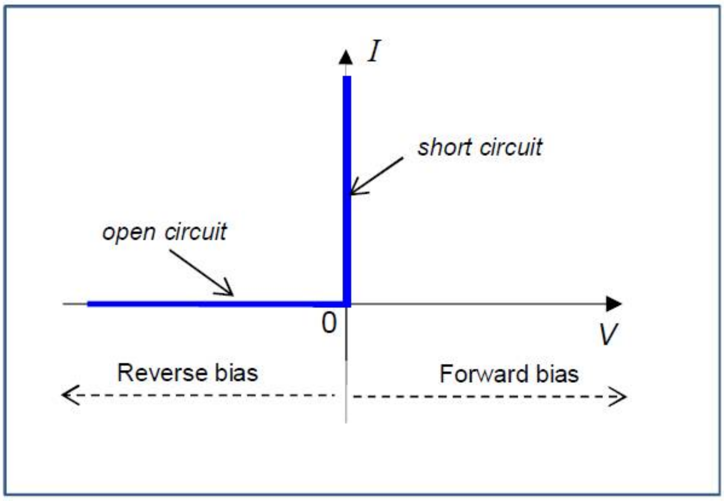
Reversed biased; negligible current

I-*V* characteristics of a typical diode:

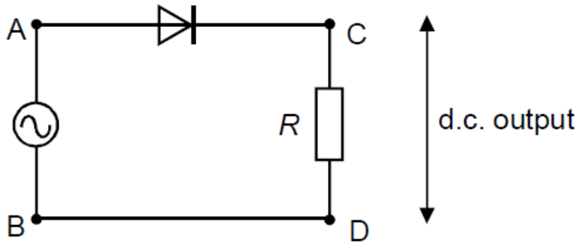


I-*V* characteristic of an ideal diode:

When a reverse bias voltage is applied the current through the diode is zero. When the current becomes greater than zero the voltage drop across the diode is zero.



The circuit consists of one diode in series with an a.c. input to be rectified and a ‘load’ requiring d.c. output. For simplicity the ‘load’ is represented by a resistor R but it might be some piece of electronic equipment.

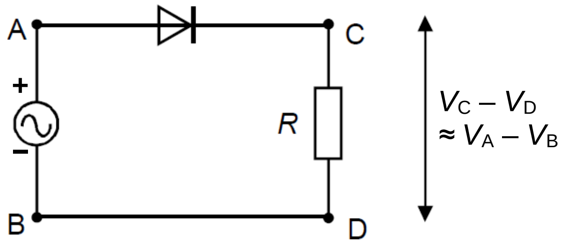
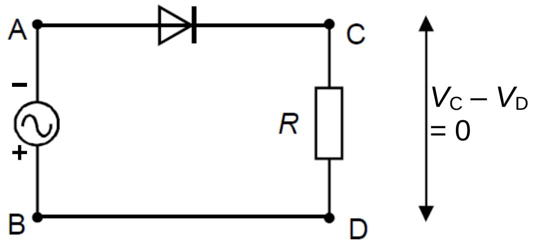


Half-wave rectifier

In a half wave rectifier, current flows through the resistor only during <u>one half</u> of every a.c. cycle.

Let us consider the case of a sinusoidal a.c.

If the <u>first half</u> cycle acts in the <u>forward-biased</u> direction of the diode (i), current flows round the circuit, creating a p.d. $V_C - V_D$ across R which will have almost the same value as the input p.d. $V_A - V_B$ if the forward resistance of the diode is small compared with R. During the <u>second half</u> cycle, the diode is <u>reverse-biased</u> (ii). Little or no current flows in the circuit and the p.d. across R is zero. This is repeated for each cycle of a.c. input. The current is unidirectional and so the p.d. across R is direct, for although it fluctuates it never changes direction.

<table>
<tr><th>(i) Forward-biased</th><th>(ii) Reverse-biased</th></tr>
<tr><td>
</td><td>
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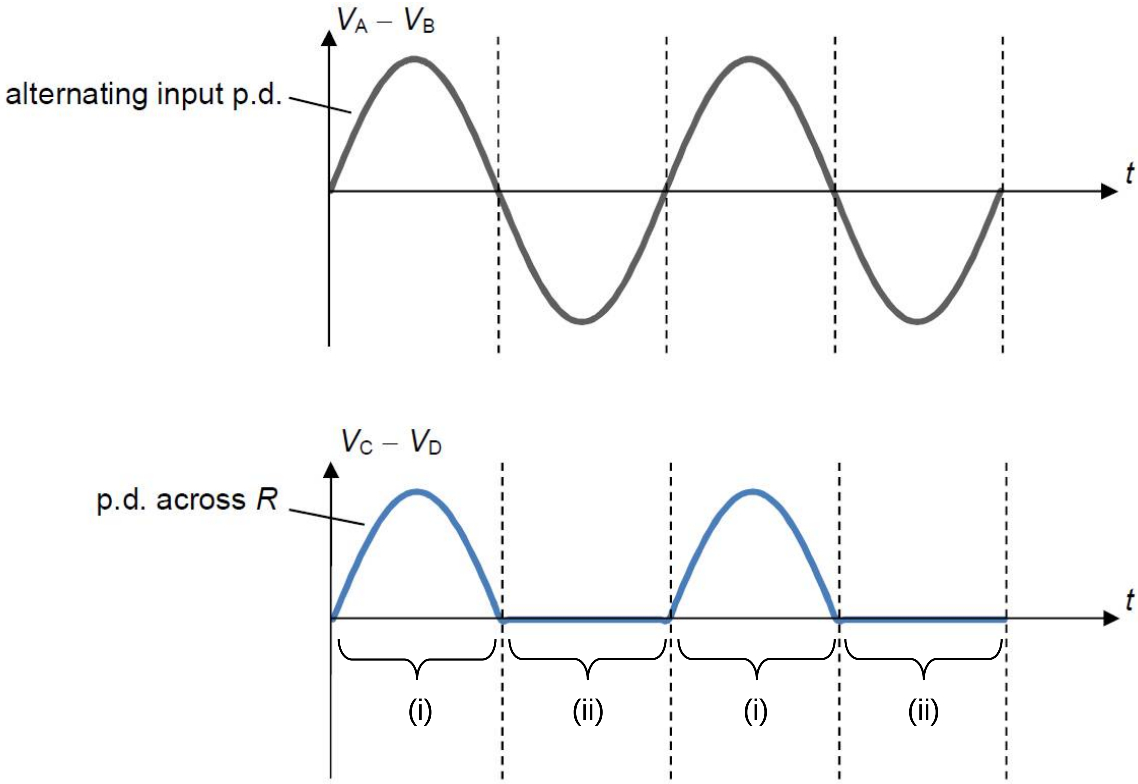
Hence, the output p.d. across R is a pulsating d.c.

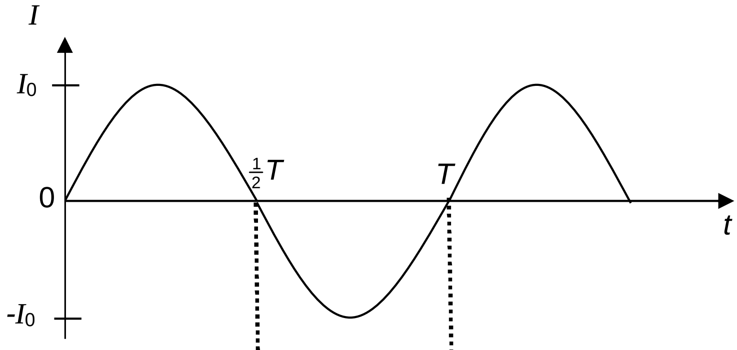


Question: If the sinusoidal alternating supply in a half-wave rectifier circuit provides a peak current of I_0, what is the value of the mean current in R? Assuming the diode is ideal with a zero forward resistance and an infinite reverse resistance.

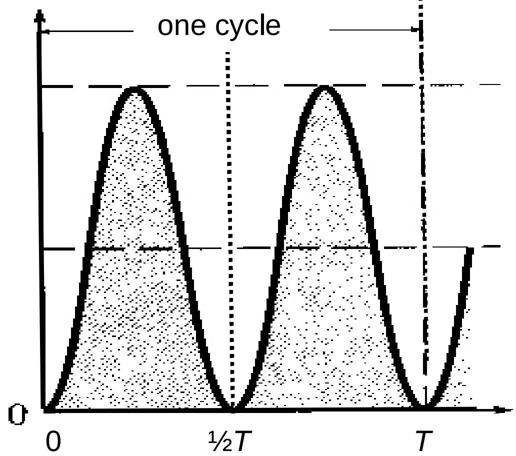
Full-wave rectification of a sinusoidal a.c. can be achieved using four diodes in a circuit shown in Annex E.

Annex A *To obtain the root-mean-square value of a sinusoidal a.c.*

For a sinusoidal a.c. $I = I_0 \sin \omega t$



mean current, $\langle I \rangle = \dfrac{\int_0^T I\,dt}{T} = 0$



mean-square current, $\langle I^2 \rangle = \langle I_0^2 \sin^2 \omega t \rangle = I_0^2 \langle \sin^2 \omega t \rangle$

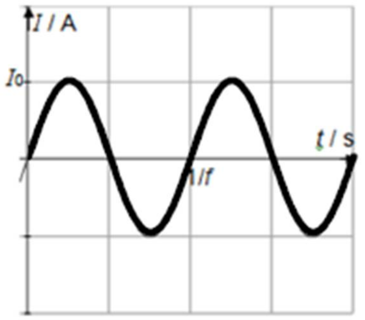
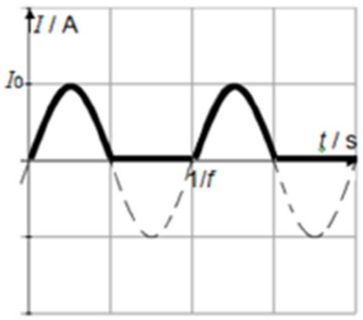
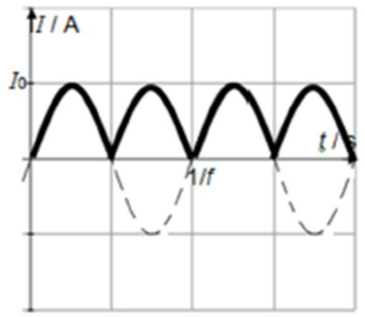
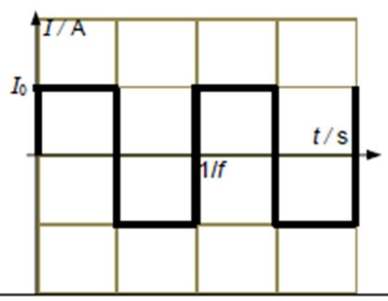
but $\langle \sin^2 \omega t \rangle = \dfrac{\int_0^T \sin^2 \omega t\,dt}{T} = \dfrac{\int_0^T (1 - \cos 2\omega t)dt}{2T} = \dfrac{1}{2T}\left[t - \dfrac{1}{2\omega}\sin 2\omega t\right]_0^T = \dfrac{1}{2}$

so $\langle I^2 \rangle = \dfrac{I_0^2}{2}$

or $I_{rms} = \sqrt{\langle I^2 \rangle} = \dfrac{I_0}{\sqrt{2}}$

Annex B

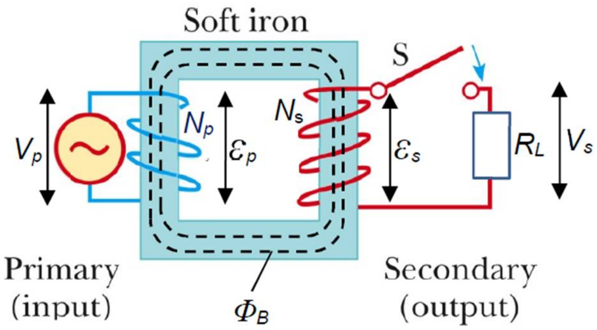
Comparison of $I_{r.m.s.}$, < P > and < I >

Variation of current I with time t		r.m.s. current	**mean power** for a resistive load of resistance R	whole cycle **average** current
	*Sinusoidal*	$\frac{I_0}{\sqrt{2}}$	$\frac{I_0^{\ 2}}{2}R$	0
	*Half-wave rectified*	$\frac{I_0}{2}$	$\frac{I_0^{\ 2}}{4}R$	$\frac{I_0}{\pi}$
	*Full-wave rectified*	$\frac{I_0}{\sqrt{2}}$	$\frac{I_0^{\ 2}}{2}R$	$\frac{2I_0}{\pi}$
	*Square*	I_0	$I_0^{\ 2}R$	0

<u>Annex C</u> *Derivation of* $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ *for an ideal transformer*

Assuming an ideal transformer, one in which the energy losses in the coils and core are zero, and there is no leakage flux (all the magnetic field lines are confined to the iron core). So at any instant the magnetic flux Φ_B through each turn is the same in the primary and secondary coils.

The primary coil has N_p turns and the secondary coil has N_s turns.



When the magnetic flux changes because of changing current in the primary, the resulting induced e.m.f.s in the two coils are

$$\varepsilon_p = -N_p \frac{d\Phi_B}{dt} \quad \text{and} \quad \varepsilon_s = -N_s \frac{d\Phi_B}{dt}$$

The flux per turn Φ_B is the same in both the primary and the secondary, so these equations show that the induced e.m.f. per turn is the same in each, i.e. $\frac{\varepsilon_p}{N_p} = \frac{\varepsilon_s}{N_s}$

Therefore,

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$$

Since ε_p and ε_s both oscillate with the same frequency as the a.c. source, this also gives the ratio of the peaks or of the r.m.s. values of the induced e.m.f.s. If the coils have zero resistance, the induced e.m.f.s ε_p and ε_s are equal to the terminal voltages across the primary and the secondary, respectively; hence

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

where V_p and V_s are either the peaks or the r.m.s. values of the terminal voltages. By choosing the appropriate turns ratio N_s/N_p, we may obtain any desired secondary voltage from a given primary voltage. If $N_s > N_p$, then $V_p > V_p$ and we have a step-up transformer; if $N_s < N_p$, then $V_p < V_p$ and we have a step-down transformer.

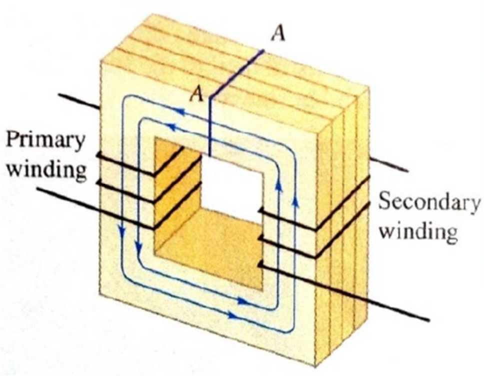
<u>Annex D</u> *Causes of power loss in practical transformers*

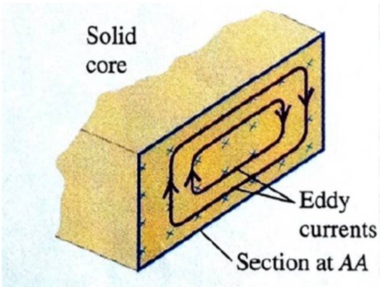
Real transformers always have some power losses. That is why an a.c. adapter feels warm to the touch after it's been in use for a while; the transformer is heated by the dissipated energy.

Causes of power loss:

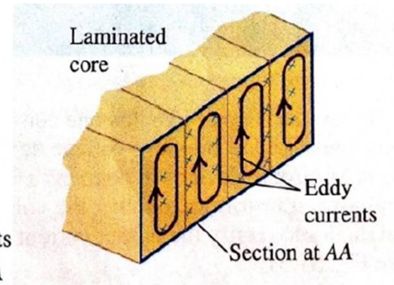
(i) *Resistance of the windings (coils):*
The wires used for the windings of the coils have <u>resistance</u> and so heating occurs, resulting in power loss (I^2R). This form of power loss is also known as <u>Joule heating</u>. Thicker wires made of material with low resistivity (i.e. high purity copper) are used to reduce this power loss.

(ii) *Eddy Currents:*
The alternating magnetic flux induces eddy currents in the iron core and causes heating.
The effect is minimised by the use of a laminated core, i.e., one built up of thin sheets or laminae. The large electrical surface resistance of each lamina, due to an insulating varnish, effectively confines the eddy currents to individual laminae. The possible eddy-current paths are narrower (the effective resistance of eddy-current paths is considerably increased), and the eddy-currents and their heating effect are greatly reduced.





(large eddy currents in solid core)



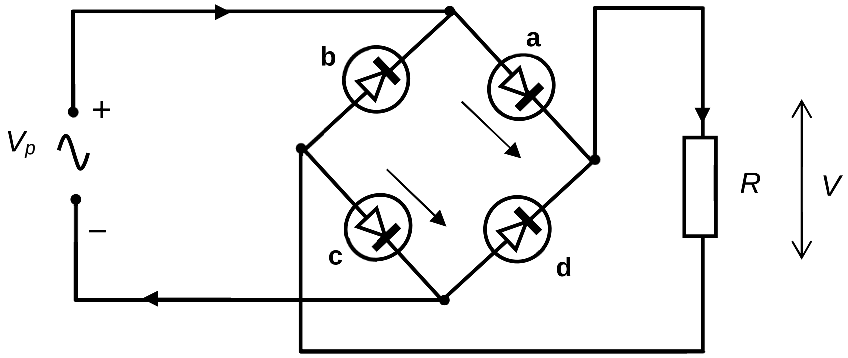
(smaller eddy currents In laminated core)

(iii) *Hysteresis:*
There are hysteresis losses in the core. The magnetization of the core is repeatedly reversed by the alternating magnetic field. The energy required to magnetise the core (while the current is increasing) is <u>not entirely recovered</u> during demagnetisation. The difference in energy is lost as heat in the core. The energy loss is kept to a minimum by using a magnetic material with low hysteresis – can be easily magnetised and demagnetised by the magnetic field caused by alternating current in the primary winding.

(iv) *Flux Leakage:*
The flux due to the primary <u>*may not all link*</u> to the secondary coil if the core is badly designed or has <u>air gaps</u> in it. This causes the secondary voltage not to be directly proportional to the primary voltage, not all the power from the primary coil can be transferred to the secondary coil.

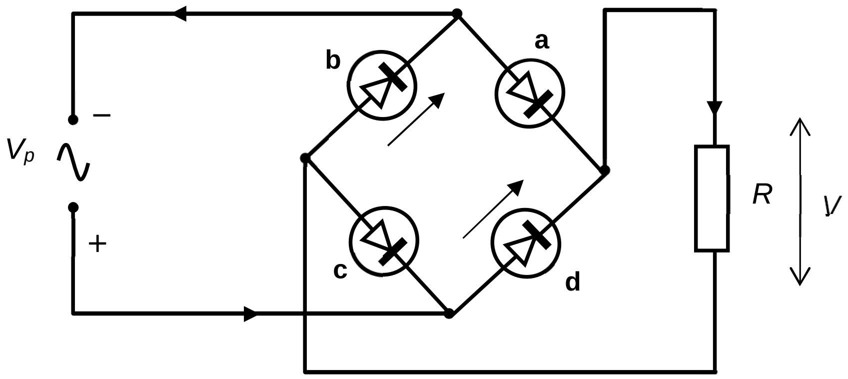
Annex E *Full-wave rectification of a sinusoidal a.c.*

Full-wave rectification is achieved with a four diode bridge connection as shown.

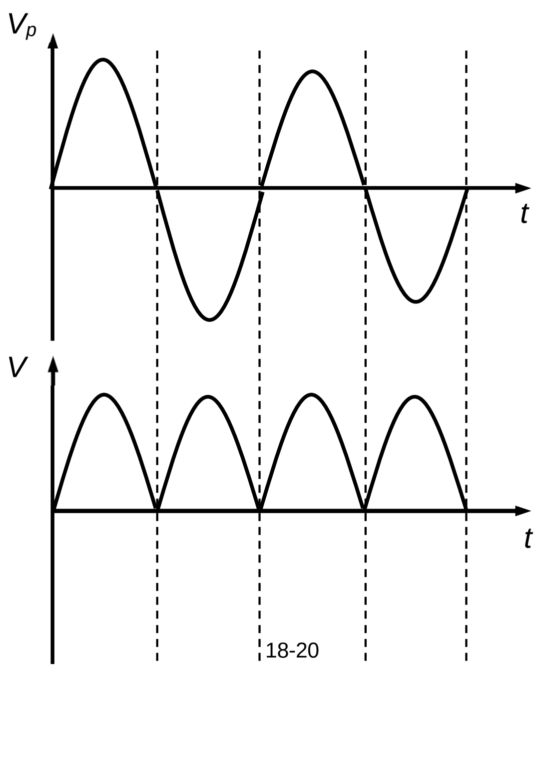


For the first half cycle, only two of the four diodes (**a** and **c**) are forward-biased and the current flows through R as shown.

For the second half cycle (when the polarity of the a.c. changes), the other two diodes (**b** and **d**) are forward-biased and the current flows through R as shown.



Notice that the direction of the *current through R remains the same* during both half cycles of the a.c. The output p.d. across R is a pulsating d.c.

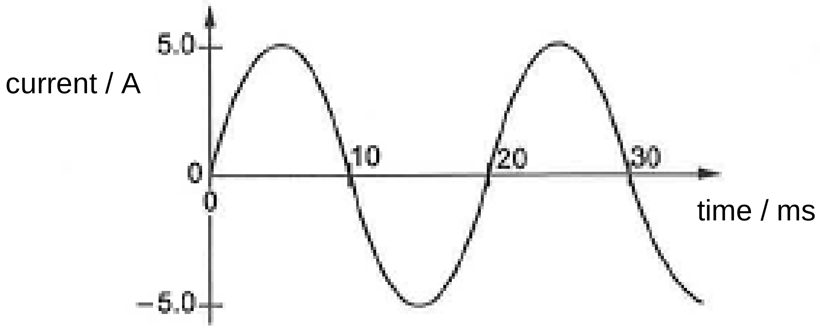


Tutorial Questions

1 If the peak sinusoidal alternating voltage across a pure resistor is doubled, how does the r.m.s. current and mean power in the resistor change?

	r.m.s. current	mean power
A	doubles	doubles
B	doubles	increases by a factor of 4
C	increases by $\sqrt{2}$	doubles
D	increases by $\sqrt{2}$	increases by a factor of 4

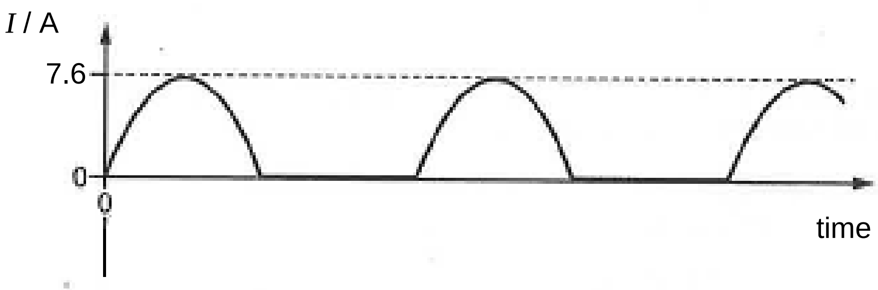
2 The graph represents an alternating current in a 10 Ω resistor.



What is the mean power generated in the resistor?

A 25 W **B** 50 W **C** 125 W **D** 250 W

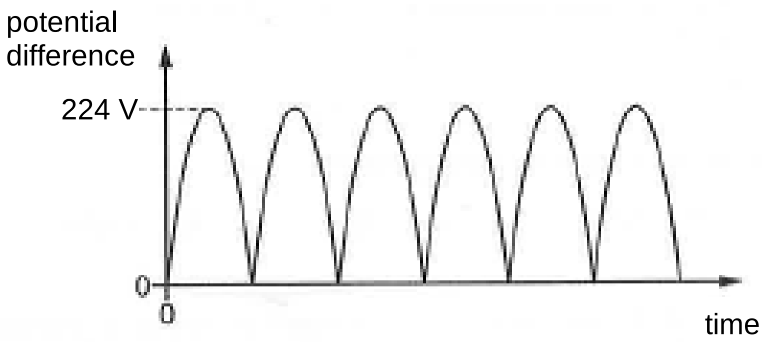
3 An alternating voltage is connected in series to a diode and a resistor of resistance 9.4 Ω. The current in the circuit varies as shown.



What is the mean power dissipated in the resistor?

A 34 W **B** 68 W **C** 140 W **D** 270 W

4 An alternating current is supplied to a rectifier. The output from the rectifier has the shape shown in the diagram, with a maximum of 224 V.



This output is used as the supply to a resistor of resistance 70.0 Ω.

What are the r.m.s. values of the current and the power to the resistor?

	r.m.s. current / A	power / W
A	2.26	358
B	2.26	507
C	3.20	507
D	3.20	717

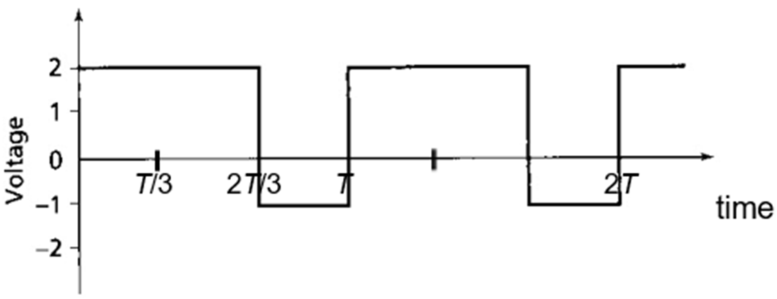
5 A sinusoidal current I through a 10.0 Ω resistor varies with time t according to the equation $I = 1.2 \sin 100\pi t$. Determine the current and power dissipated at $t = 0.018$ s.

[–0.705 A, 4.98 W]

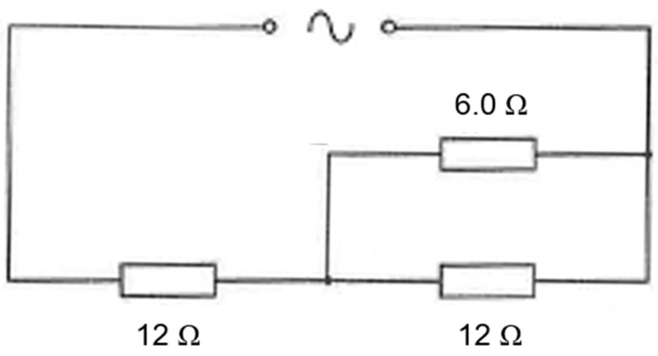
6 A steady current I dissipates a certain power in a variable resistor. The resistance has to be halved to obtain the same power when a sinusoidal alternating current is used.
What is the r.m.s. value of the alternating current? [$\sqrt{2}I$]

7 Determine the r.m.s. current in each case.

(a) A sinusoidal current of peak value 2.0 A.
(b) A full-wave rectified sinusoidal current of peak value 3.0 A.
(c) A square wave current with a frequency of 1.0 Hz and has a value of 0.10 A for one half cycle and –0.10 A for the next half cycle.
(d) An uneven square wave voltage as shown below.



8 An a.c. power supply is connected to three resistors as shown.



The variation with time t of the voltage V of the power suply is given by the expression

$$V = 9.0 \sin 120\pi t$$

(a) For the power supply, determine

(i) the frequency f,

(ii) the root-mean-square (r.m.s.) voltage [60 Hz, 6.4 V]

(b) Calculate the peak current from the power supply. [0.56 A]

(c) Calculate the mean power dissipated in the resistor of resistance 6.0 Ω. [0.42 W]

9 **(a)** By reference to heating effect, state what is meant by the *root-mean-square* (r.m.s.) value of an alternating current.

(b) An alternating power supply is connected to a resistor of resistance 58 Ω, as shown in Fig. 9.1.

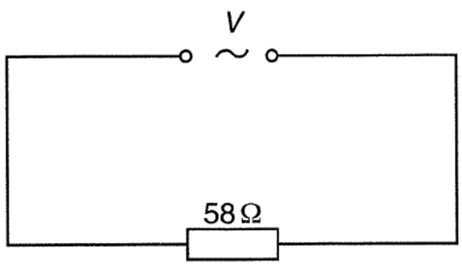


Fig. 9.1

The variation with time t of the potential difference V across the supply is given by the expression

$$V = 170 \sin (377t).$$

where V is measured in volts and t is in seconds.

For this alternating voltage, determine

(i) the root-mean-square (r.m.s.) potential difference V_{rms} of the supply

(ii) the frequency f of the supply

(iii) the mean power $< P >$ transferred in the resistor.

(c) Use your answers in (b) to sketch, on the axes of Fig. 9.2, the variation with time *t* of the power *P* transferred in the resistor. Include on your graph a time equal to two periods of the alternating potential difference.

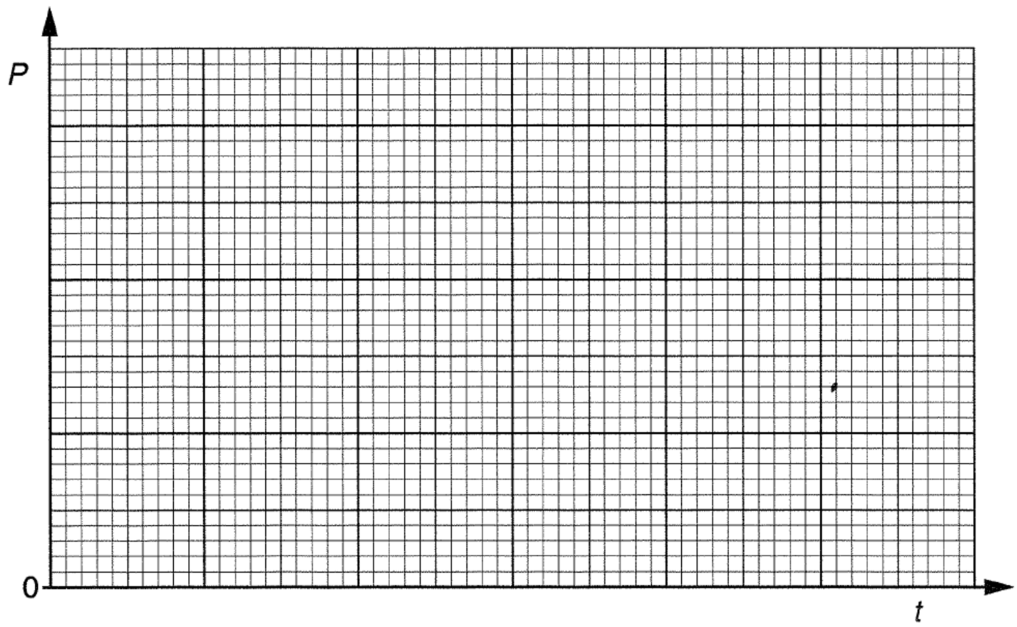
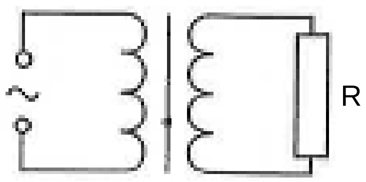


Fig. 9.2

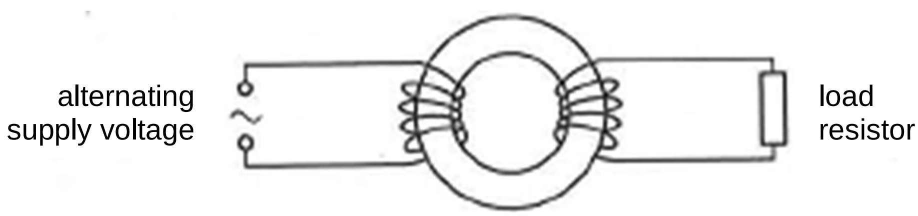
10 A 16:1 step down transformer is connected to a load resistor R. The resistor dissipates power at a rate of 32 W when the peak alternating voltage of the supply is 180 V.



What is the value of R?

A 1.0 Ω **B** 2.0 Ω **C** 4.0 Ω **D** 32 Ω

11 An alternating supply voltage is connected across the primary coil of an ideal iron-cored transformer. The secondary coil has 600 turns and is connected across a load resistor output. The r.m.s. potential difference of the output is 9.0 V.



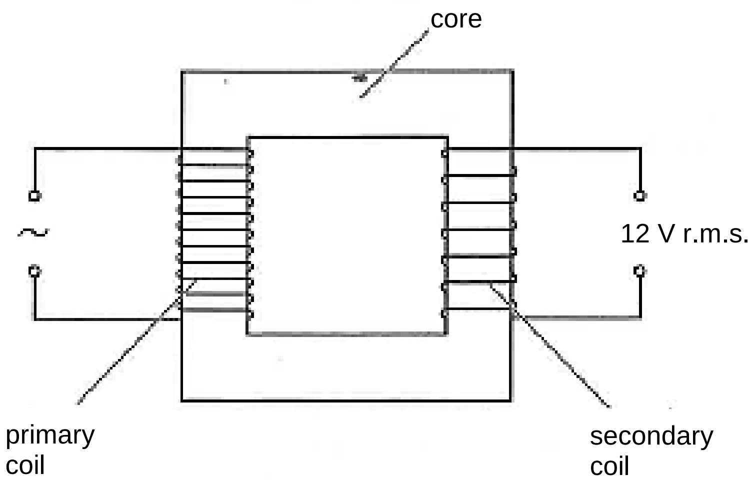
The r.m.s. current is 250 mA in the primary coil and 750 mA in the secondary coil. What are the number of turns on the primary coil and the r.m.s. voltage of the supply?

	number of turns on the primary coil	r.m.s. voltage of the supply / V
A	200	3.0
B	200	27
C	1800	3.0
D	1800	27

12 The number of turns on the secondary coil of an ideal transformer is doubled and the number on the primary coil stays the same. How do the ratios of the current in the primary coil to the current in the secondary $\frac{I_p}{I_s}$ and the voltage across the primary to the voltage across the secondary $\frac{V_p}{V_s}$ change?

	$\frac{I_p}{I_s}$	$\frac{V_p}{V_s}$
A	doubles	doubles
B	doubles	halves
C	halves	doubles
D	halves	halves

13 An alternating voltage in the primary coil can be stepped down to 12 V r.m.s. using a transformer as shown.



(a) Use Faraday's law to explain why an alternating current in the primary coil gives rise to an alternating e.m.f. in the secondary coil. [4]

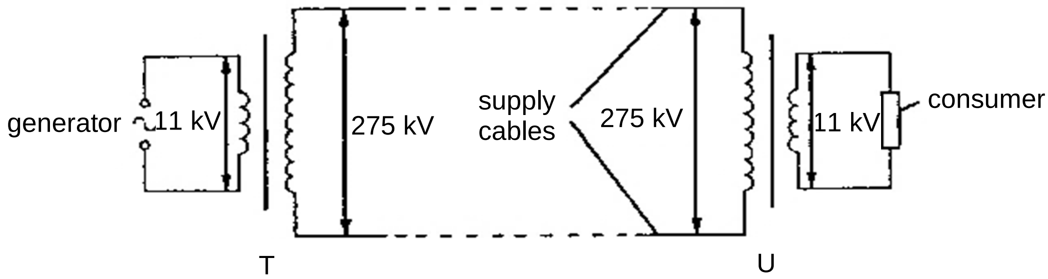
(b) In practice, the core of some transformer is made of laminated soft iron.

(i) State two reasons why soft iron, which is easily magnetized and demagnetized, is used as the core. [2]

(ii) Explain how the lamination of the core reduces energy losses. [2]

14 Jun98/II/5

Electrical power of 4400 kW is supplied to an industrial consumer at a considerable distance from a generating station. This is represented below.



In order to do this, the electricity supply company makes use of a circuit containing two transformers T and U. The transformers can be considered to be ideal and the supply cables are said to have negligible resistance.

(a) The power is generated at 11 kV r.m.s. and is supplied to the consumer at 11 kV r.m.s. Calculate the r.m.s. current supplied to the consumer.

(b) There is a potential difference of 275 kV r.m.s. between the supply cables. Calculate

1. the ratio $\frac{N_s}{N_p}$ for each transformer

2. the current in the supply cables

(c) Explain why, when the resistance of the supply cables cannot be neglected, this arrangement is preferable to a system which generates and transmits the power at the same voltage of 11 kV r.m.s.

(d) Find the peak value of the current in (a).

- END -