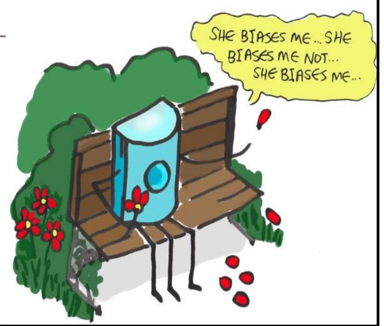
SECTION V ELECTRICITY AND MAGNETISM Topic 18

ALTERNATING CURRENTS



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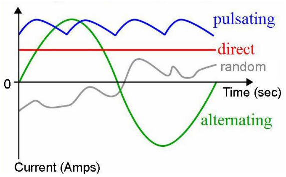
ALTERNATING CURRENTS

- Characteristics of alternating currents
- The transformer
- Rectification with a diode



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GUIDING QUESTIONS

- How is alternating current <u>different</u> from direct current?
- How can we <u>describe</u> alternating current, e.g., mathematically and graphically?
- Why is alternating current used in the <u>generation and transmission of electricity</u>?

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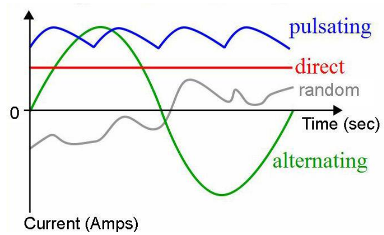
Content

- Characteristics of alternating currents
- The transformer
- Rectification with a diode



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Learning Outcomes

- understand and use
  - period (T)
  - frequency (f)
  - peak value (I_0, V_0)
  - root-mean-square value (I_{rms}, V_{rms})
- deduce <P> = P_0 / 2 for sinusoidal I
- represent a.c. by $I = I_0 \sin \omega t$
- solve problems using $I_{rms} = I_0/\sqrt{2}$ for sinusoidal case

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Learning Outcomes (continue)

- understand principle of operation of a simple iron-cored transformer
- for an ideal transformer, recall and solve problems using $N_s/N_p = V_s/V_p = I_p/I_s$
- explain the use of a single diode for the half-wave rectification of an a.c.

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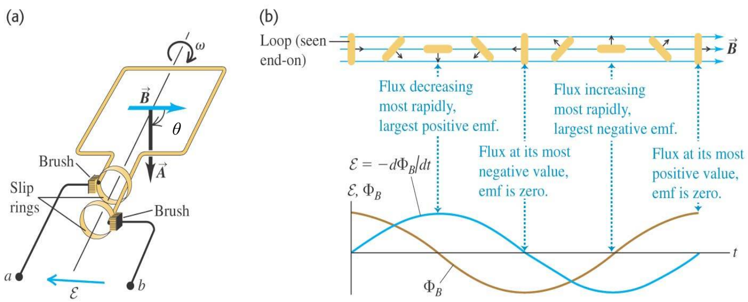
Recall: A simple alternator

sinusoidal e.m.f. is induced in a coil rotating with constant speed in a uniform B field



$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\varepsilon = -\frac{dN\Phi_B}{dt} = \omega NBA\sin\omega t$$

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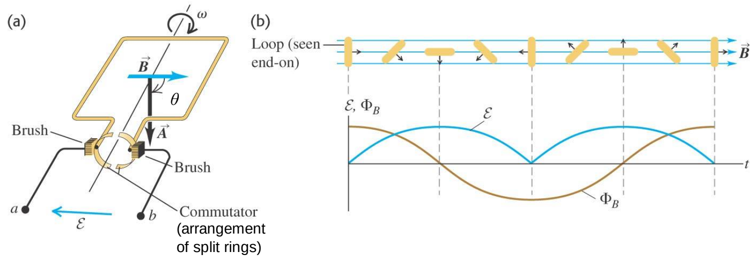
Recall: A d.c. generator

the ring halves are attached to the loop and rotate with it; the commutator reverses the connections to the external circuit at regular positions where the e.m.f. reverses



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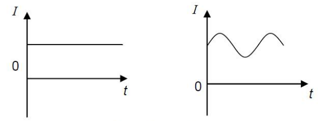
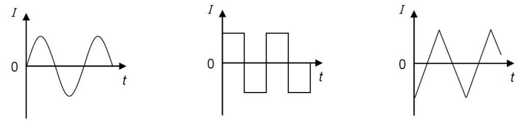
Difference between AC & DC

Alternating current (A.C.) : an electric current which periodically reverses direction

Direct current (D.C.) : an electric current which only flows in a single direction around a circuit



Examples of Direct Current (d.c.)



Examples of Alternating Current (a.c.)

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Characteristics of A.C.

period
The period of an A.C. source refers to the time taken to complete one cycle. The SI units for time is seconds (s).

frequency
The frequency of an A.C. source refers to the number of complete cycles per unit time. The SI units for frequency is Hertz (Hz) or s^{-1}.

peak value
Maximum absolute value of the alternating current or voltage in either direction of zero value in a periodic cycle.

peak to peak value
Difference between the positive peak value and the negative peak value of the a.c. within a cycle.

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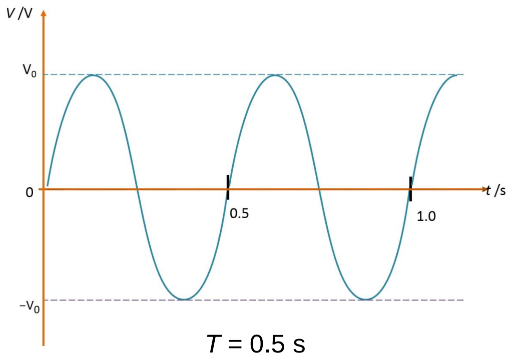
Characteristics of A.C.



T = 0.5 s

f = 2 Hz

$\omega = 2\pi f$ = 13 rad s^{-1}

peak value = V_0

peak to peak value = $2V_0$

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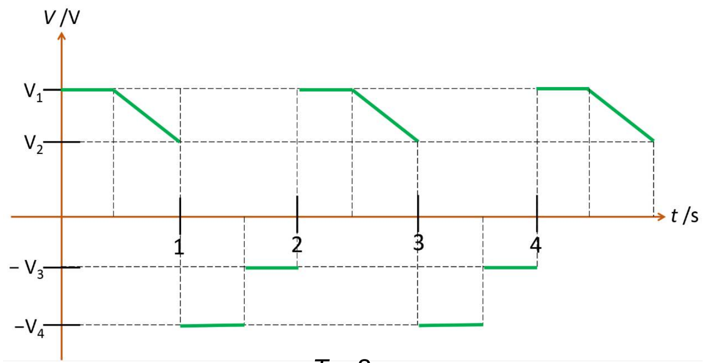
Characteristics of A.C.



T = 2 s

f = 0.5 Hz

positive peak value = V_1

negative peak value = V_4

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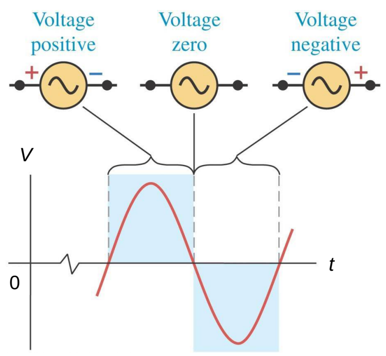
Voltage across a sinusoidal source



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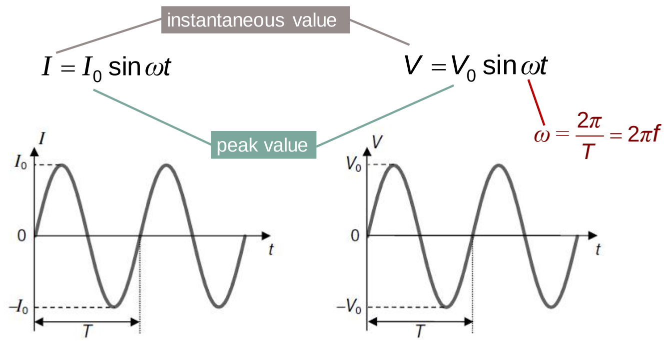
Equations for sinusoidal A.C.



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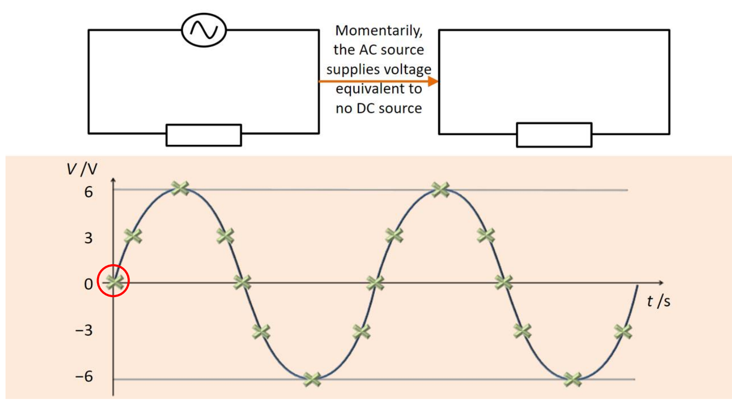
mathematical symbols ↔ physical situation



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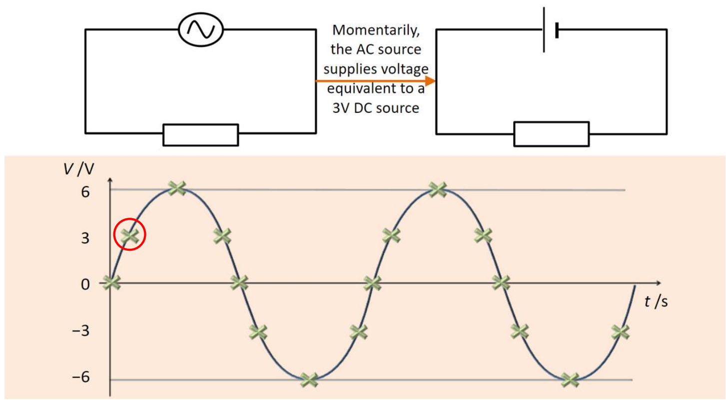
mathematical symbols ↔ physical situation



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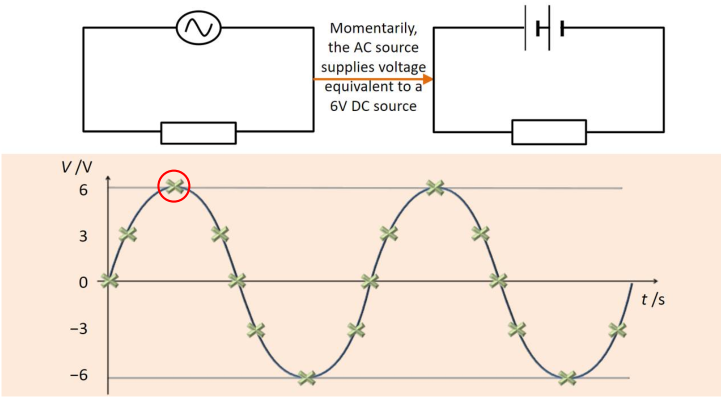
mathematical symbols ↔ physical situation



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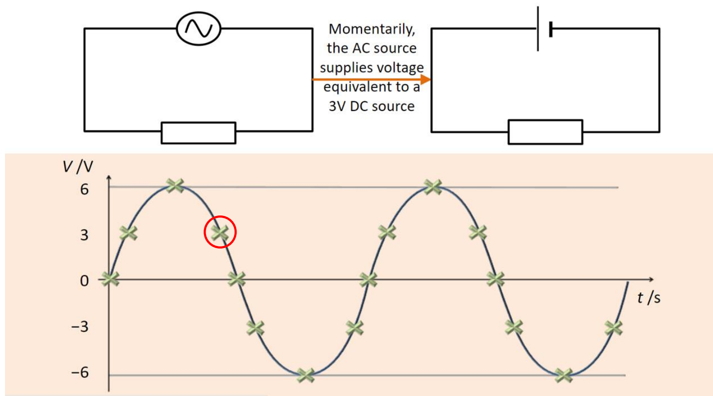
mathematical symbols ↔ physical situation



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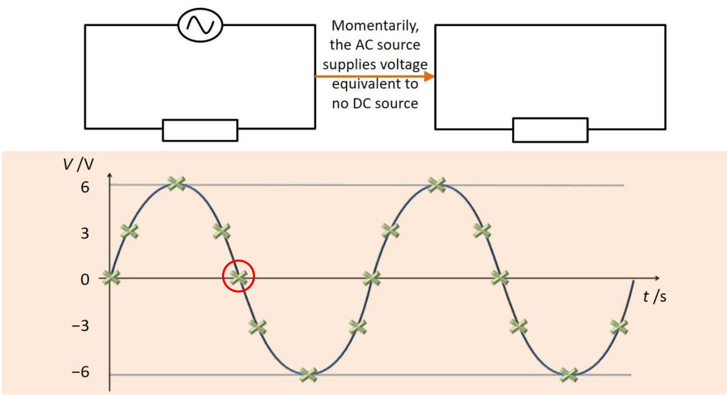
mathematical symbols ↔ physical situation



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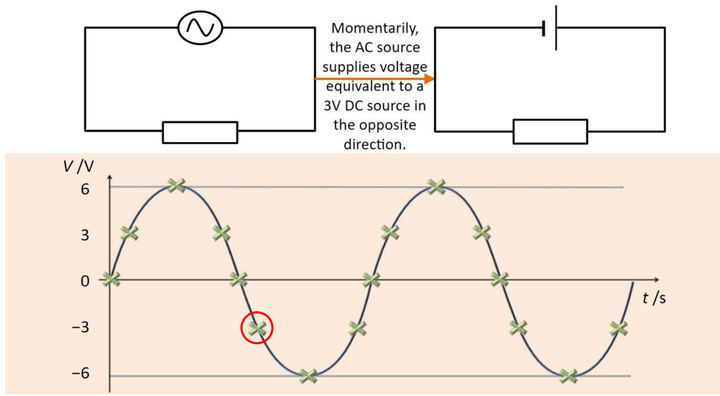
mathematical symbols ↔ physical situation



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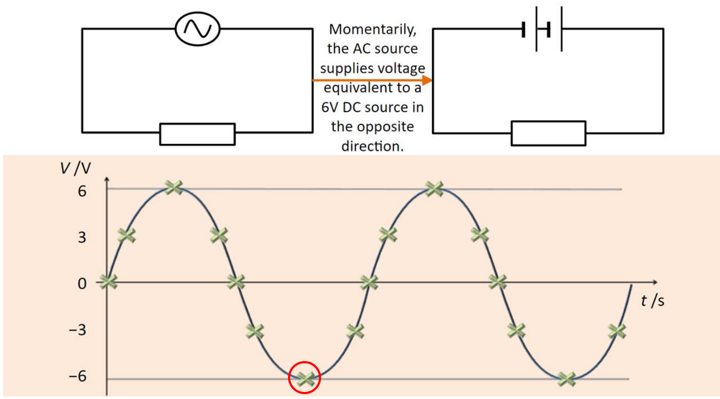
mathematical symbols ↔ physical situation



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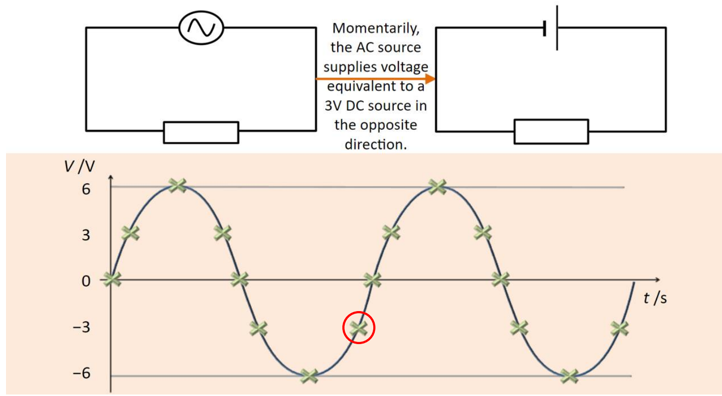
mathematical symbols ↔ physical situation



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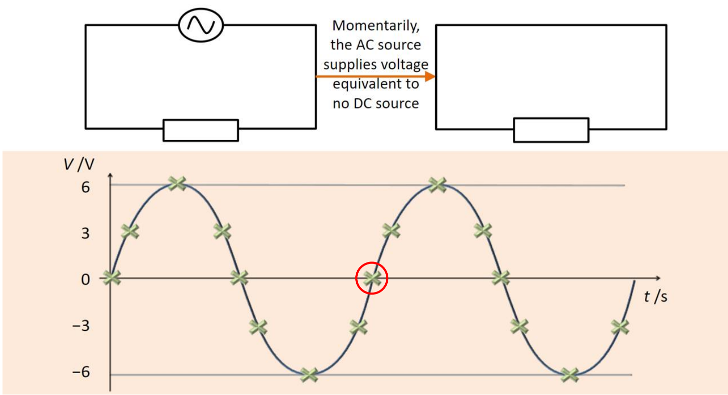
mathematical symbols ↔ physical situation



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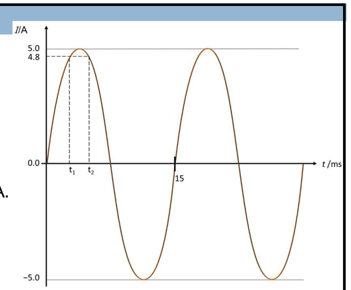
Self-assessment (2 min)



1. For this AC source, the peak current is ……. A.

2. The period is ……….. s.

3. We can write the equation, with I measured in amperes and t measured in seconds, as

I = ………… sin …………. πt

4. The time when the current first reaches 4.8 A is t_1 and the next time the current has the same value of 4.8 A is t_2.
Using the equation to solve for these times, we have t_1 = …………………….. ms and t_2 = …………………….. ms.

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Self-assessment (Answer)

1. For this AC source, the peak current is 5.0 A.

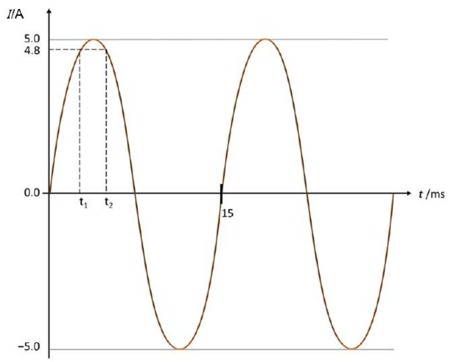
2. The period is 15×10^{-3} s.

3. We can write the equation, with I measured in amperes and t measured in seconds, as

$$I\,(\text{A}) = 5.0\,\sin\left(\frac{400}{3}\right)\pi t$$

4. The time when the current first reaches 4.8 A is t_1 and the next time the current has the same value of 4.8 A is t_2.

Using the equation to solve for these times, we have t_1 = 3.07 ms and t_2 = 4.43 ms.



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Self-assessment (working)

3. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{15 \times 10^{-3}\ \text{s}} = \dfrac{400}{3}\pi\ \text{rad s}^{-1}$

$\Rightarrow I = I_0 \sin \omega t = 5.0 \sin\left(\dfrac{400}{3}\pi\right)t$

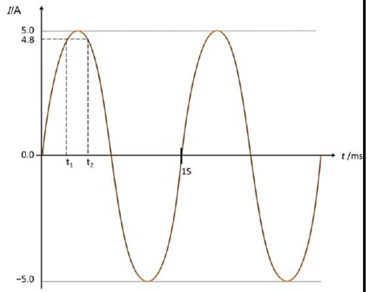
4. $4.8 = 5.0 \sin\left(\dfrac{400}{3}\pi\right)t$

$\Rightarrow \sin\left(\dfrac{400}{3}\pi\right)t = \dfrac{4.8}{5.0} = 0.960$

$\Rightarrow \left(\dfrac{400}{3}\pi\right)t = 73.74^{\circ}\ \text{ or }\ 106.26^{\circ}$

$\Rightarrow \left(\dfrac{400}{3}\pi\right)t = 0.4097\pi\ \text{ or }\ 0.5903\pi\ \text{ rad}$

$\Rightarrow t = 3.07\ \text{s}\ \text{ or }\ 4.43\ \text{s}$



Note: If $\theta \leq 180°$, sin (θ) = sin (180° - θ)

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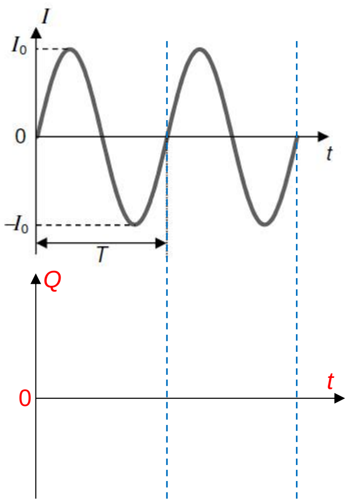
Quiz

The graph shows the variation with time t of the current I in a wire.

How would you represent the variation with time t of the net amount of charge Q which has flowed past a point in the wire?



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Mean value of A.C.

For the case of <u>sinusoidal</u> current, any positive value of current, there will be a corresponding negative value within a complete cycle, thus the *mean value* of current I is *<u>zero</u>*.

However, heat is dissipated when it flows in a resistor, implying that the <u>mean</u> value of an a.c. does **not** represent the *effective value of the a.c.*

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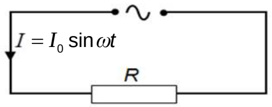
Mean power in a resistive load for a sinusoidal a.c.

$$\langle P\rangle = (\text{mean value of } I^2)R$$
$$= \langle I_0^2 \sin^2 \omega t\rangle R$$
$$= I_0^2 \langle \sin^2 \omega t\rangle R$$
$$= I_0^2 \left(\frac{1}{2}\right) R \qquad \because \langle \sin^2 \omega t\rangle = \left\langle \frac{1}{2}(1-\cos 2\omega t)\right\rangle = \frac{1}{2}$$
$$= \left(\frac{1}{2}\right) I_0^2 R$$
$$= \frac{1}{2} P_{max}$$

mean power in a resistive load is half the maximum power for a sinusoidal a.c.



The mean power delivered by the source is converted to internal energy in the resistor, just as in the case of a d.c. circuit.

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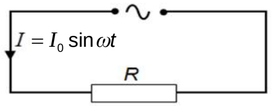
Mean power in a resistive load for a sinusoidal a.c. (derived in terms of V)

$$\langle P\rangle = \frac{(\text{mean value of } V^2)}{R}$$
$$= \frac{\langle V_0^2 \sin^2 \omega t\rangle}{R}$$
$$= \frac{V_0^2 \langle \sin^2 \omega t\rangle}{R}$$
$$= \frac{V_0^2 (1/2)}{R} \qquad \because \langle \sin^2 \omega t\rangle = \left\langle \frac{1}{2}(1-\cos 2\omega t)\right\rangle = \frac{1}{2}$$
$$= \left(\frac{V_0^2}{R}\right)(1/2)$$



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Mean power in a resistive load for a sinusoidal a.c. (derived in terms of V)

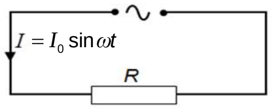
$$\langle P\rangle = \frac{(\text{mean value of } V^2)}{R}$$
$$= \frac{\langle V_0^2 \sin^2 \omega t\rangle}{R}$$

. . .

. . .

. . .

$$= \frac{1}{2} P_{max}$$



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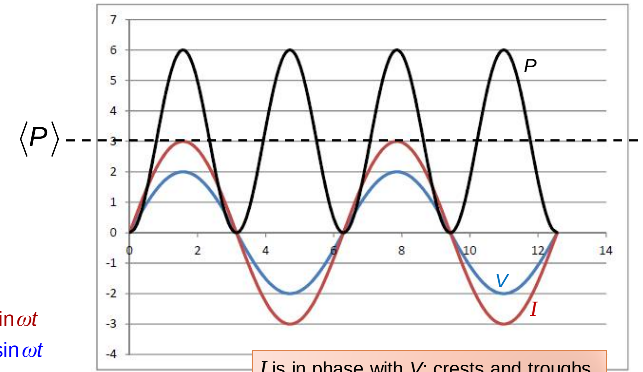
Power in a resistive load for a sinusoidal a.c.



E.g.

$I = 3.0 \sin \omega t$

$V = 2.0 \sin \omega t$

$\Rightarrow P = 6.0 \sin^2 \omega t$

$\therefore P_0 = 6.0$ W, $\langle P\rangle = \frac{6.0}{2} = 3.0$ W

I is in phase with V: crests and troughs occur together

Peaks are related as for a d.c. circuit: $V_0 = I_0 R$

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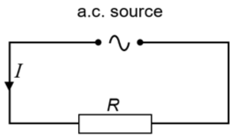
Root-mean-square (r.m.s.) value



Although the current is not in one direction only, power is converted in the resistor. This is because the power/heating depends on I^2, so independent of current direction.

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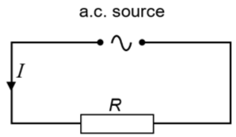
Root-mean-square (r.m.s.) value



Recall that the electrical power P dissipated in a resistor is

$$P = I^2R$$

In an a.c., the instantaneous power dissipated in a resistor is given by

$$P_{\text{instantaneous}} = I^2R$$

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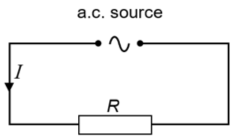
Root-mean-square (r.m.s.) value



Since <*P*> delivered by source is converted to internal energy in the resistor, can we define an equivalent constant current that would produce the same heating effect?

How can we express this current in a simple, single value?

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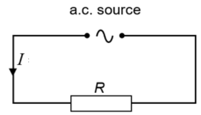
Root-mean-square (r.m.s.) value



Mean power dissipated in a resistor over one cycle:

$$\begin{aligned}\langle P\rangle &= \text{mean value of } P_{\text{instantaneous}}\\ &= \langle I^2R\rangle\\ &= \langle I^2\rangle R\\ &= \left(\sqrt{\langle I^2\rangle}\right)^2 R\\ &= I_{\text{rms}}^2R\end{aligned}$$

I_{rms} is the square root of the mean value of I^2 and hence is known as the <u>root-mean-square (r.m.s.) current</u> of the a.c. source.

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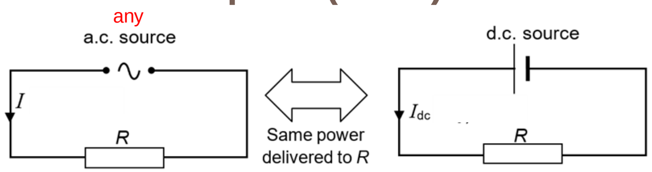
Root-mean-square (r.m.s.) value



To find what value of current by a d.c. source will dissipate the *__same__ power* in a resistor as the ***__mean power__*** *dissipated by an* a.c. when it flows through the resistor, we equate:

$$P_{dc} = \langle P \rangle$$
$$I_{dc}^2 R = \langle I^2 R \rangle$$
$$= \langle I^2 \rangle R \quad \text{since } R \text{ is constant}$$
$$I_{dc}^2 = \langle I^2 \rangle$$
$$I_{dc} = \sqrt{\langle I^2 \rangle} = I_{rms}$$

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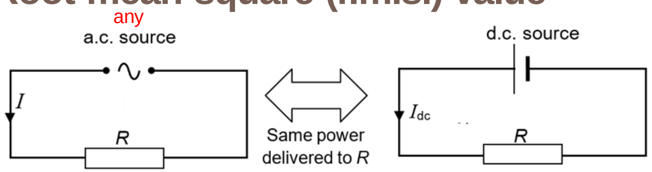
Root-mean-square (r.m.s.) value



To find what value of voltage by a d.c. source will dissipate the *__same__ power* in a resistor as the ***__mean power__*** *dissipated by an* a.c. source applying across the resistor, we equate:

$$P_{dc} = \langle P \rangle$$
$$\frac{V_{dc}^2}{R} = \left\langle \frac{V^2}{R} \right\rangle$$
$$= \frac{\langle V^2 \rangle}{R} \quad \text{since } R \text{ is constant}$$
$$V_{dc}^2 = \langle V^2 \rangle$$
$$V_{dc} = \sqrt{\langle V^2 \rangle} = V_{rms}$$

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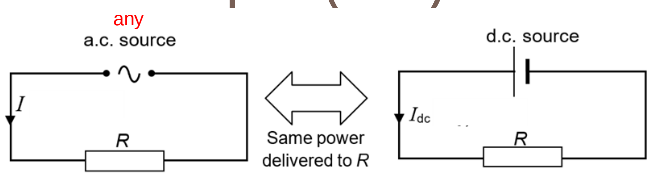
Root-mean-square (r.m.s.) value



In other words, a d.c. of magnitude I_{rms} (or V_{rms}) will produce the same heating effect as the a.c.

Thus the __r.m.s. value__ can be considered as the *__effective value of the a.c.__*

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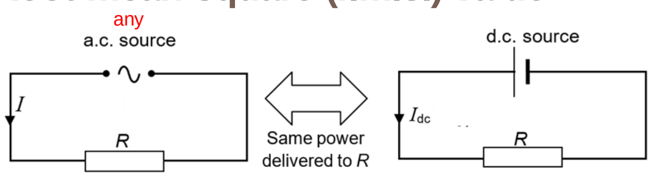
Root-mean-square (r.m.s.) value



The r.m.s. value of alternating current is the *__value of the steady direct current__ which would dissipate heat at the __same average rate__ in a given resistor.*

Graphically, it is the square root of the mean value of the square of the instantaneous current over one cycle.

$$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2\text{-}t \text{ graph over one cycle}}{\text{period}}}$$

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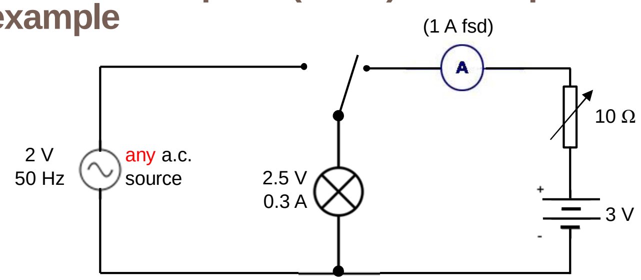
Root-mean-square (r.m.s.) value – practical example



If the lamp is lit first from an a.c. and the brightness noted, then if 0.3 A d.c. produces the same brightness, the r.m.s. value of the a.c. is 0.3 A.

the *mean* power delivered by the a.c. (I) to the lamp equals the *steady* power delivered by the d.c. ($I_{d.c.}$)

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Procedure for calculating the r.m.s. current:

- Square the current at each point in time for the entire repeating pattern.
- Find the average (mean value) of these squared values, over the entire period.
- Take the square root of this average.

In general, the r.m.s. current of an a.c. source is also known as the d.c. equivalent current.

$$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{\int_0^T I^2 dt}{T}} = \sqrt{\frac{\text{area under } I^2\text{-}t \text{ graph over one cycle}}{\text{period}}}$$

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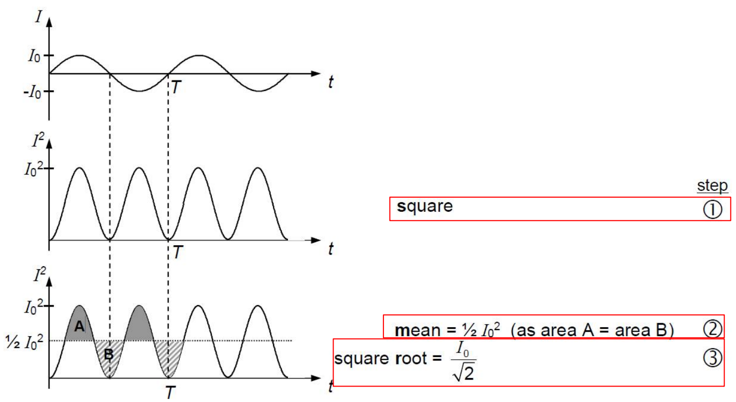
Find I_{rms} in terms of I_0 for a sinusoidal current:



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Find I_{rms} in terms of I_0 for a sinusoidal current:

For a sinusoidal a.c. (not valid for other types of a.c.):

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

Note: Recall that $\langle P \rangle = \frac{1}{2} P_{max}$ for a sinusoidal a.c.

therefore $\langle P \rangle = \frac{1}{2} I_0 V_0 = \frac{I_0}{\sqrt{2}} \frac{V_0}{\sqrt{2}} = \underline{I_{rms} V_{rms}}$

For a resistance, $V_{rms} = I_{rms} R$

so $\langle P \rangle = \underline{I_{rms}^2 R = \frac{V_{rms}^2}{R}}$

The mean power delivered to a resistor that carries an a.c. have the same form as the corresponding relationships for a d.c. circuit.

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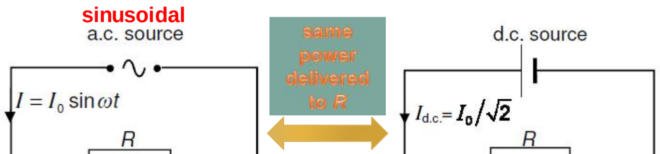
Root-mean-square (r.m.s.) value of sinusoidal a.c. source



An sinusoidal a.c. whose peak value is I_0 delivers to a resistor the same power as a direct current that has a value of $I_0/\sqrt{2}$.

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Note:

(i) When we say that an alternating current is 10 A, we mean that the r.m.s. value is 10 A unless otherwise state.

(ii) The waveform is assumed to be sinusoidal unless otherwise stated.

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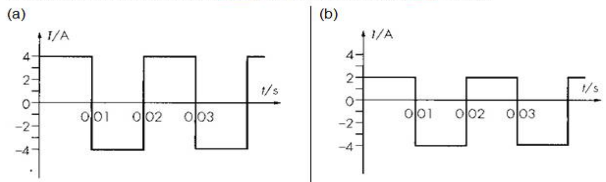
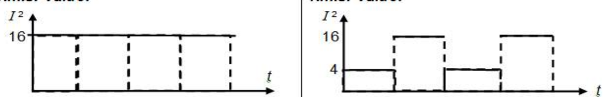
Example 1



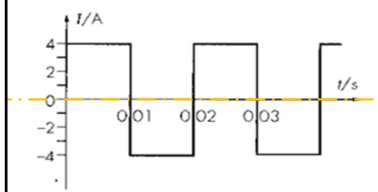
Mean value:

r.m.s. value:



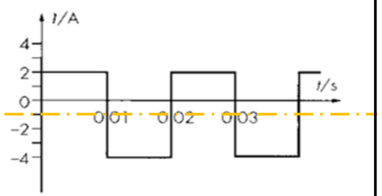
23

Example 1 (a)



$$\langle I \rangle = \frac{\int_0^T I\,dt}{T} = \frac{4(0.01) - 4(0.01)}{0.02} = 0 \text{ A}$$

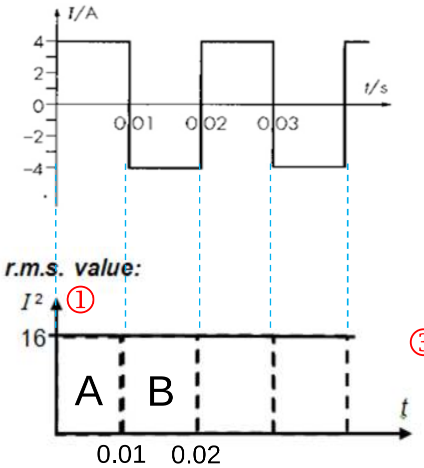
Example 1 (b)



$$\langle I \rangle = \frac{\int_0^T I\,dt}{T} = \frac{2(0.01) - 4(0.01)}{0.02} = -1 \text{ A}$$

24

Example 1 (a)



②

$$\langle I^2 \rangle = \frac{\int_0^T I^2 dt}{T}$$

$$= \frac{\text{Area A+Area B}}{\textit{Period}}$$

$$= \frac{16(0.01)+16(0.01)}{0.02}$$

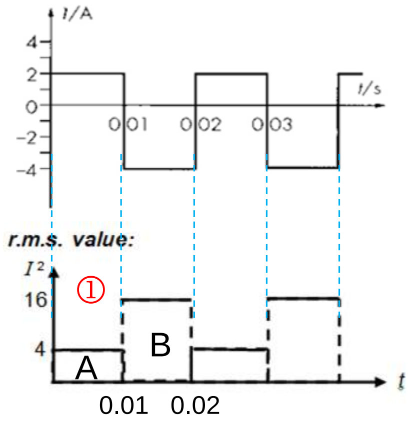
$$= 16 \text{ A}^2$$

③

$$I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{16} = 4 \text{ A}$$

25

Example 1 (b)



②

$$\langle I^2 \rangle = \frac{\int_0^T I^2 dt}{T}$$

$$= \frac{\text{Area A+Area B}}{\textit{Period}}$$

$$= \frac{4(0.01)+16(0.01)}{0.02}$$

$$= 10 \text{ A}^2$$

③

$$I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{10} = 3.16 \text{ A}$$

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Example 2

A sinusoidal current described by the equation I = 9.0 sin ωt flows through a 12.0 Ω resistor. Calculate

(a) the r.m.s. value of the current and voltage.

For a sinusoidal current, $I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}} = \dfrac{9.0}{\sqrt{2}} = 6.36$ A

$V_{\text{rms}} = I_{\text{rms}}R = I_{\text{rms}}(12) = 76.4$ V

(b) the maximum instantaneous power in the resistor.

$P_0 = I_0^2 R = (9.0)^2(12.0) = 972$ W

(c) the average power dissipated in the resistor.

$\langle P \rangle = \dfrac{P_0}{2} = 486$ W or $\langle P \rangle = I_{\text{rms}}^2 R = \left(\dfrac{9.0}{\sqrt{2}}\right)^2 (12.0) = 486$ W

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Example 3

A tourist from U.S.A. brings an electric water kettle designed to operate with 110 V to Singapore and plugs it into a 230 V outlet. The heater breaks down due to overheating.

(a) If the heater normally consumes 500 W, what is the resistance of the heating coil?

$\langle P \rangle = \dfrac{V_{\text{rms}}^2}{R} \Rightarrow 500 = \dfrac{110^2}{R} \Rightarrow R = 24.2\ \Omega$

(b) How much power does it consume using our 230 V electrical supply?

Assuming that R remains constant.

$\langle P \rangle = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{230^2}{24.2} = 2190$ W.

The heating coil develops $\left(\dfrac{230}{110}\right)^2 \approx 4$ times the designed power & burns out fast.

(c) What is the r.m.s. current that flows when the potential difference is 230 V?

$I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{230}{24.2} = 9.50$ A

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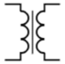
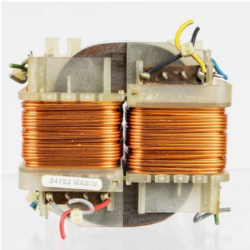
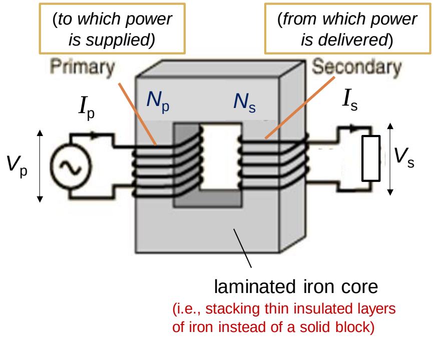
The Transformer

- changes an alternating voltage from one value to another
- two insulated coils of wire wound around laminated iron core
- iron core keeps the magnetic flux almost completely within the core
- symbol: 





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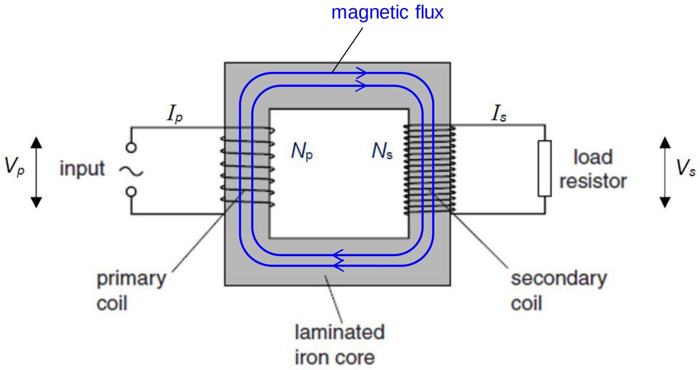
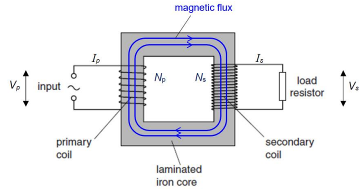
The Transformer



1

The Transformer



Principle of operation

- works by electromagnetic induction
- a.c. source causes an alternating current in the primary, which sets up an alternating magnetic flux (almost completely contained) in the iron core (which couples the flux around through the secondary coil)
- this induces an alternating e.m.f. in each coil, in accordance with Faraday's law of induction
- the induced e.m.f. in the secondary give rise to an alternating current in the secondary, & this delivers energy to the device to which the secondary is connected
- all currents and e.m.f.s have the same frequency as the a.c. source

2

The Ideal Transformer

- no power loss (i.e. input power = output power)
- the *same* flux passes through *each turn* of both the primary and secondary coils (i.e. no flux leakage)

Using the Laws of Electromagnetic Induction, it can be shown that for an ideal transformer, turn ratio of a transformer is

$$\frac{N_s}{N_p} = \frac{V_s}{V_p}$$

either ratio of peaks or r.m.s. values

N_p = number of turns on the primary coil,
N_s = number of turns on the secondary coil,
V_p = input voltage to the primary coil,
V_s = output voltage across the load resistor connected to the secondary coil

Thus $$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$ (ideal transformer)

3

The Ideal Transformer

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

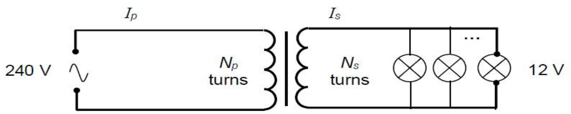
- If $N_s > N_p$, then $V_s > V_p$ → a ***step-up*** transformer.
- If $N_s < N_p$, then $V_s < V_p$ → a ***step-down*** transformer.
- Steps up the voltage *simultaneously* steps down the current and steps down the voltage steps up the current. However, the power is neither stepped up nor stepped down.

Practical transformers always have some power losses. (Refer to Annex D of the notes for causes of power loss.)

4

Example 4

Ten lamps each rated 24 W, 12 V are connected in parallel with the mains supply of 240 V as shown.



(a) Calculate the turns-ratio of the transformer needed.

$$\frac{N_s}{N_p}=\frac{V_s}{V_p}=\frac{12}{240}=\frac{1}{20}$$

(b) What would be the current drawn from the mains supply if the transformer is 100% efficient?

Current in each lamp, $I_{rms}=\langle P\rangle/V_{rms}=24/12=2.0$ A
The lamps are connected in parallel, so total current drawn in the secondary coil = 20 A.
Since transformer is 100 % efficient, $I_pV_p=I_sV_s \Rightarrow I_p(240)=(20)(12)\Rightarrow I_p=1.0$ A

(c) What would be the current drawn from the mains supply if the transformer is 91% efficient?

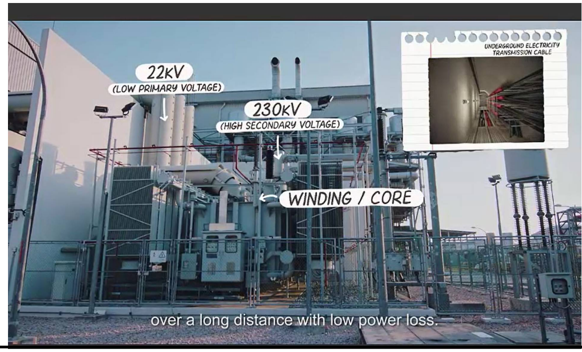
$$(0.91)(I_pV_p)=I_sV_s \Rightarrow (0.91)I_p(240)=(20)(12)\Rightarrow I_p=1.1 \text{ A}$$

5

Use of transformers

At a power generating station

Power source — **Step-up transformers** — Transmission lines

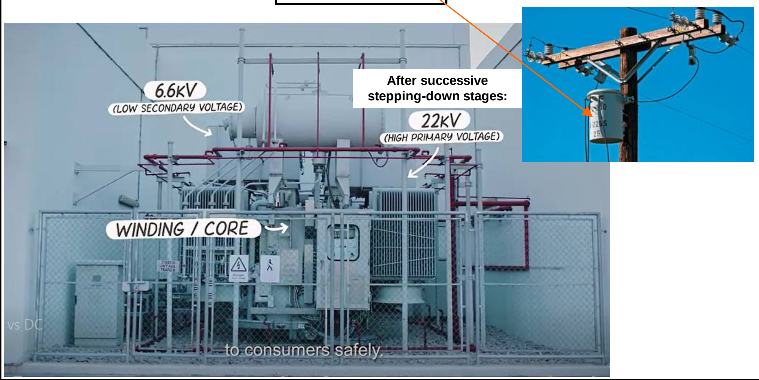


6

Use of transformers

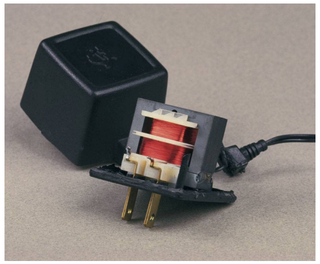
Near the consumer

Transmission lines — **Step-down transformers** — Industries or homes (in successive stages)



7

a.c. adapter (power adapter)



- contains a step-down transformer that converts household line voltage to a lower value, typically 3 to 20 V, as well as diodes to convert a.c. to the d.c. that many small electronic devices required

rectification (to be discussed later)

8

Transmission of electrical power

For heating and lighting applications, a.c. has no practical advantage over d.c. because the heating effect of a current is independent of its direction.

So why do we use a.c. power supplies?

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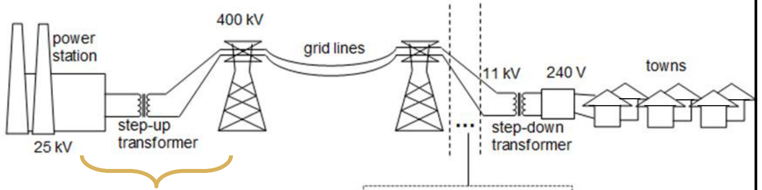
Transmission of electrical power



A generator produces power at a p.d. of about 10 kV to 30 kV. It is then stepped up by transformers to a high p.d. for transmission.

high p.d. is stepped down in successive stages by transformers to a suitable value for use by industrial or domestic consumers

10

Transmission of electrical power

Use high alternating, rather than d.c., power supplies in long-distance transmission because

more effective in minimising power loss in the transmission lines with the use of transformers

to efficiently step up the voltage & transmit the electrical power at high voltages (low currents)

&, subsequently,

to step down the voltage for use by homes as household appliances do not need high voltages

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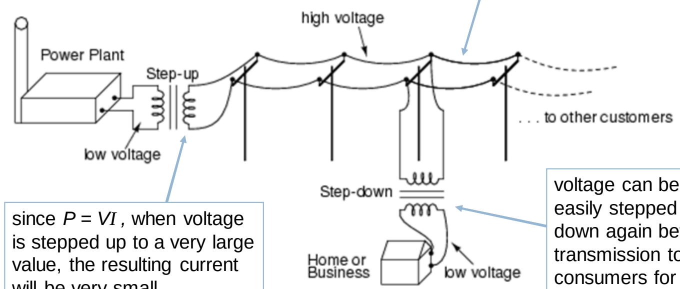
Transmission of electrical power

a small current flowing through the transmission wires will result in a small power loss, where, $P_{Loss} = I^2R_{cables}$



since $P = VI$, when voltage is stepped up to a very large value, the resulting current will be very small

voltage can be easily stepped down again before transmission to consumers for safety reasons.

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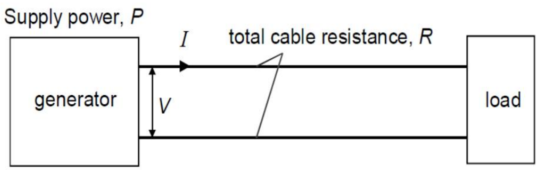
Example 5

A generator produces power at a p.d. of about 10 kV to 30 kV. It is then stepped up by transformers to a high p.d. (e.g. 400 kV) for transmission. Why is the power transmitted at a high p.d.?

Consider a transmission line delivering power P from a generator to a load as shown. Suppose the total resistance of the cables between the generator and the load is R, and that the load is such that the effective value of the current is I and the p.d. at the supply end is V.



To **reduce power losses** during transmission, more power reaches the consumers efficiently.

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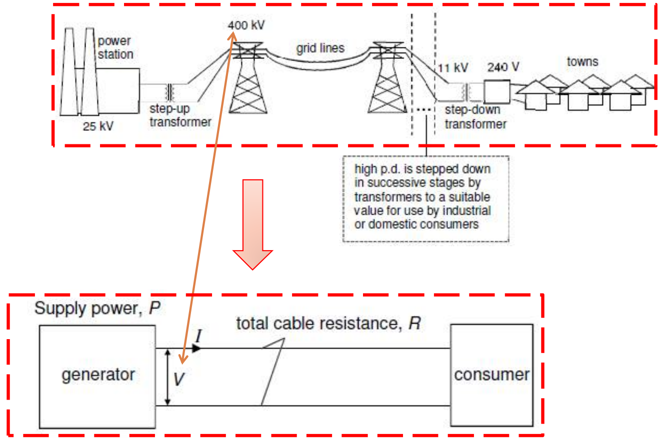
Example 5



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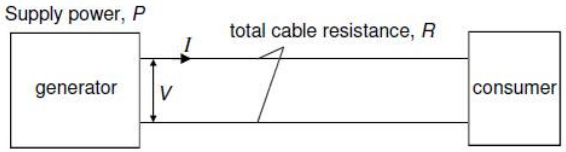
Example 5



(a) What is the current in the transmission line?

$$I = \frac{P}{V}$$

(b) What is the power loss P_{loss} in the transmission line?

$$P_{loss} = I^2R = \left(\frac{P}{V}\right)^2 R$$

(c) For a given value of P, how can this power loss be minimized?

Reduce I (i.e. increase V); reduce R.

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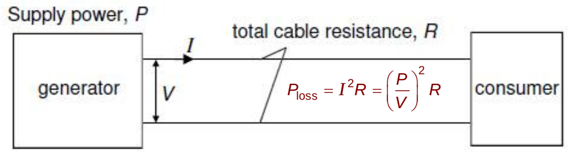
Example 5



(d) How does the power loss for a 400 kV system compare to that a 132 kV system?

Since P_{loss} is proportional to $\frac{1}{V^2}$, the loss in the 400 kV system is about $\frac{1}{9}$ of the loss in the 132 kV system.

$$P_{loss} \propto \frac{1}{V^2}$$

if P and R are the same

$$\Rightarrow \frac{(P_{loss})_{400\ kV}}{(P_{loss})_{132\ kV}} = \left(\frac{132}{400}\right)^2 = \left(\frac{33}{100}\right)^2 \approx \left(\frac{1}{3}\right)^2$$

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Example 6

An electricity-generating station delivers energy at a rate of 20 MW to a city 1.0 km away. A common voltage for commercial power generators is 22 kV, but a step-up transformer is used to boost the voltage to 230 kV before transmission.

(a) If the resistance of the wires is 2.0 Ω and energy costs $0.20 per kWh, estimate the cost for the energy converted to internal energy in the wires in one day.

(b) Find the cost in (a) if the energy is delivered at its original voltage of 22 kV.

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Example 6

(a) If the resistance of the wires is 2.0 Ω and energy costs $0.20 per kWh, estimate the cost for the energy converted to internal energy in the wires in one day.

(a) $I_{rms} = \dfrac{P}{V_{rms}}$

$= \dfrac{20\times10^6}{230\times10^3}$

$= 87.0$ A

$P_{loss} = I_{rms}{}^2 R$

$= (87.0)^2(2.0)$

$= 15.1$ kW

Energy converted to internal energy in one day $= 15.1\times 24$

$= 362$ kWh

Cost $= 362 \times \$0.20$

$= \$72.40$

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Example 6

(b) Find the cost in (a) if the energy is delivered at its original voltage of 22 kV.

(b) $I_{rms} = \dfrac{P}{V_{rms}}$

$= \dfrac{20\times10^6}{22\times10^3}$

$= 909$ A

$P_{loss} = I_{rms}{}^2 R$

$= (909)^2(2.0)$

$= 1652.9$ kW

Energy converted to internal energy in one day $= 1652.9\times 24$

$= 39670$ kWh

Cost $= 39670 \times \$0.20$

$= \$7930$

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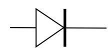
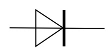
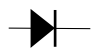
Rectification

Many household devices run on constant voltage but the e.m.f obtained from the mains supply is alternating in nature.

Thus, a means of deriving D.C from A.C. is desired.

This process of converting A.C. to D.C. is known as rectification and is achieved by using rectifiers that conduct current in one direction only, such as a **diode**.

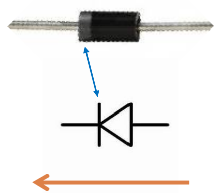
symbol:  or  or 

20

Diode

Most made with semiconductor materials such as silicon or germanium

(Silver ring denotes the negative terminal of the diode.)



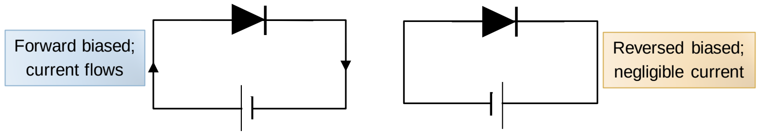
forward direction of conventional current flow

21

Rectification

A **diode** is forward-biased when connected to a supply such that current flows through it.

It is reverse-biased if connected such that little or no current flows through it.



The arrow head indicates the forward direction of conventional current flow.

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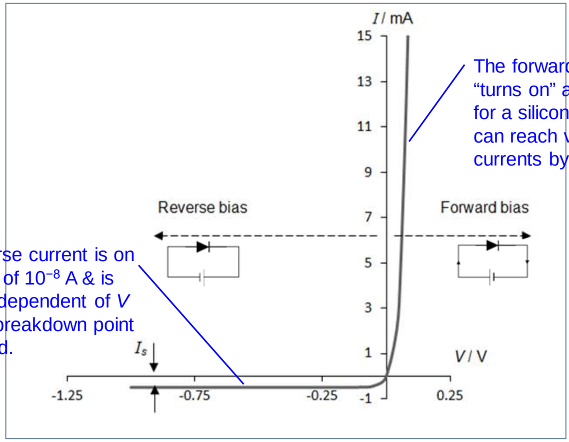
I-*V* characteristics of a typical diode



The forward current "turns on" at $V \approx 0.5$ V for a silicon diode & can reach very high currents by 0.7 V.

The reverse current is on the order of 10^{-8} A & is almost independent of V until the breakdown point is reached.

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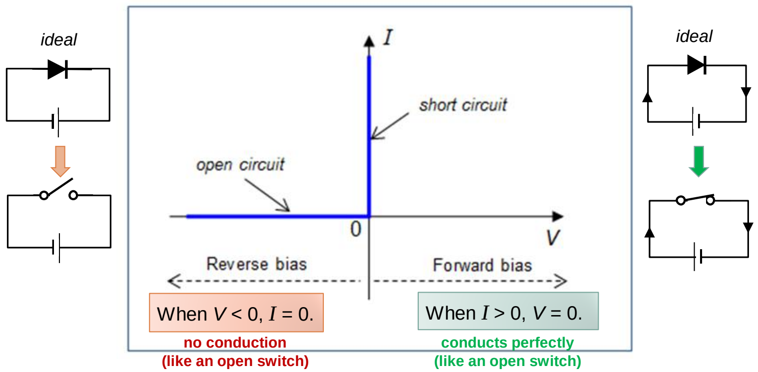
I-*V* characteristic of an ideal diode

- an *ideal* diode has **zero** resistance for one direction of current and **infinite** resistance for the other



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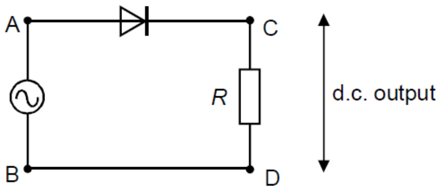
Rectifier circuit

Consists of one diode in series with an a.c. input to be rectified and a ‘load’ requiring d.c. output.



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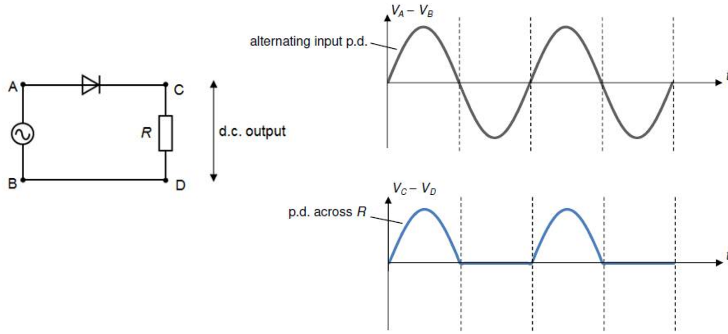
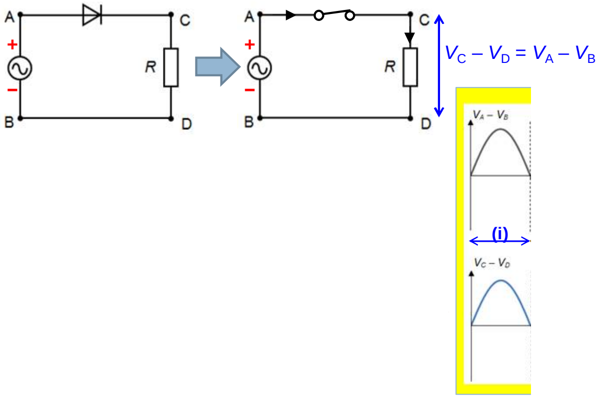
Half-wave rectification



26

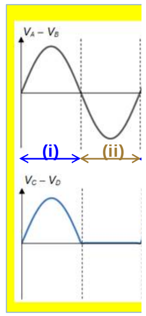
Half-wave rectification

(i) If $V_A - V_B > 0$, diode is **FB**, current flows

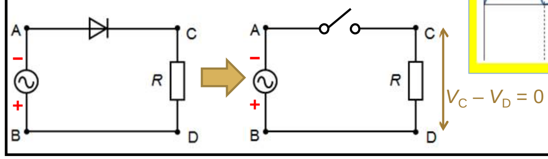


27

Half-wave rectification



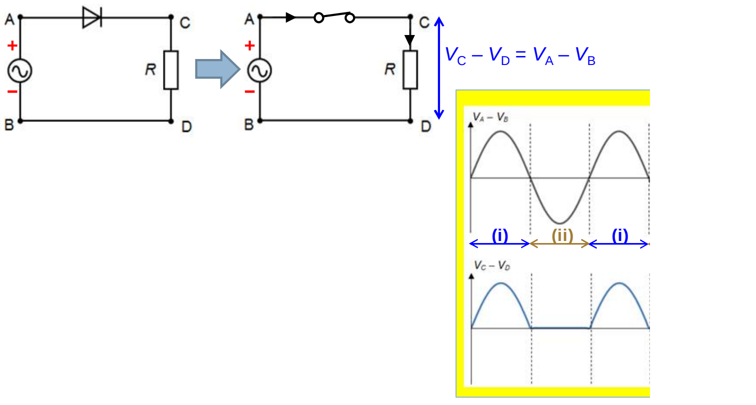
(ii) If $V_A - V_B < 0$, diode is **RB**, no current flows



28

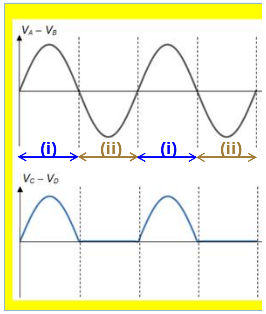
Half-wave rectification

(i) If $V_A - V_B > 0$, diode is **FB**, current flows

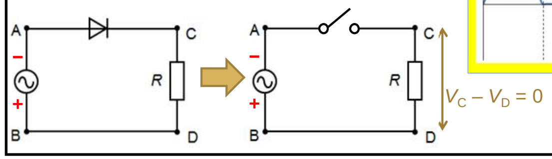


29

Half-wave rectification



(ii) If $V_A - V_B < 0$, diode is **RB**, no current flows



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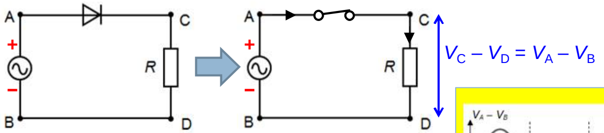
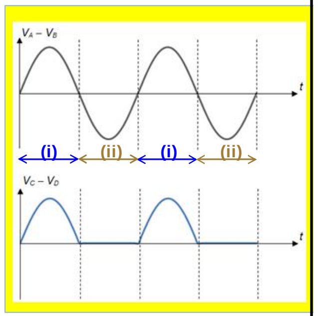
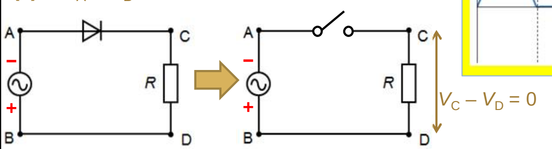
Half-wave rectification

(i) If $V_A - V_B > 0$, diode is **FB**, current flows





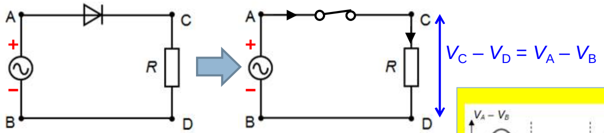
(ii) If $V_A - V_B < 0$, diode is **RB**, no current flows

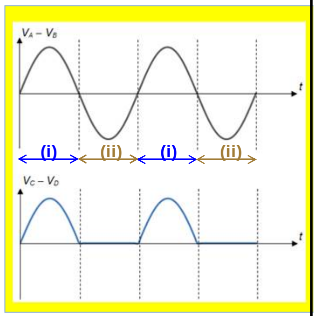


31

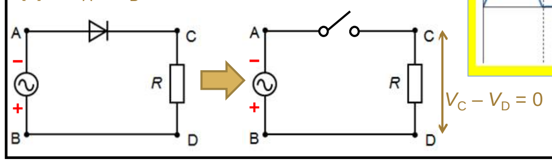
Half-wave rectification

(i) If $V_A - V_B > 0$, diode is **FB**, current flows





(ii) If $V_A - V_B < 0$, diode is **RB**, no current flows



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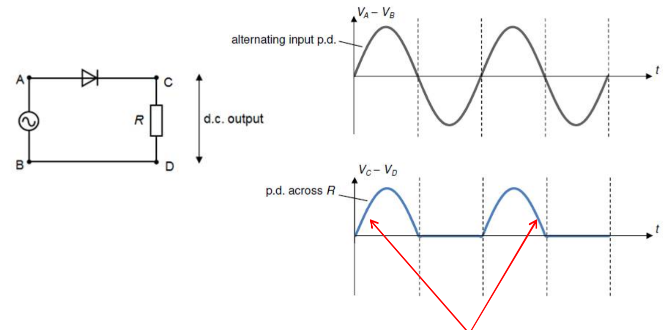
Half-wave rectification

The above process is repeated for each cycle of a.c. input.



The current is unidirectional and so the p.d. across R is direct, for although it fluctuates it never changes direction.

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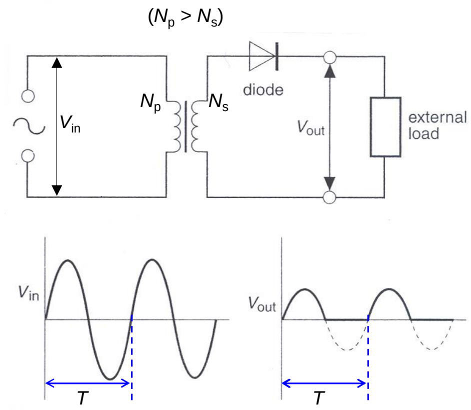
Half-wave Rectification (after stepping down of voltage)



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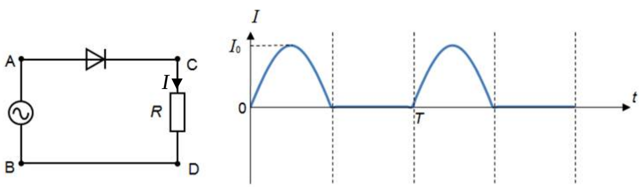
Rectified Alternating Current

Question: If the sinusoidal alternating supply in a half-wave rectifier circuit provides a peak current of I_0, what is the value of the mean current in R? Assuming the diode is ideal.



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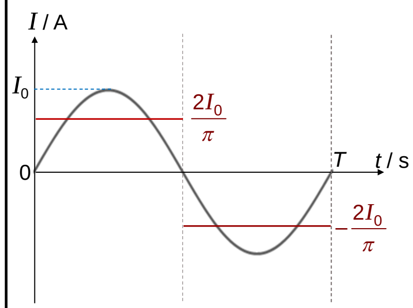
Half-cycle mean current

Not in syllabus



$$\langle I \rangle_{\text{half-cycle}} = \left(\frac{1}{T/2}\right)\int_0^{\frac{T}{2}} (I_0 \sin \omega t)dt$$

$$= \left(\frac{1}{T/2}\right)\left(\frac{I_0}{\omega}\right)\int_0^{\frac{T}{2}} (\sin \omega t) d(\omega t)$$

$$= \frac{I_0}{\pi}\left[-\cos \omega t\right]_0^{T/2}$$

$$= \frac{I_0}{\pi}\left[\left(-\cos \omega \left(\frac{T}{2}\right)\right) - (-\cos 0)\right]$$

$$= \frac{I_0}{\pi}\left[(-\cos \pi) - (-\cos 0)\right]$$

$$= \frac{I_0}{\pi}\left[1 - (-1)\right]$$

$$= \frac{2I_0}{\pi}$$

Back

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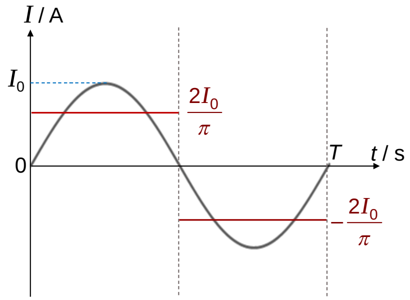
Not in syllabus

Half-cycle mean current



It can be shown mathematically that for sinusoidal current the mean current during a half-cycle is equal to $2I_0/\pi$, and over the next half-cycle it is – $2I_0/\pi$.

Half-cycle mean value ≈ 0.637I_0

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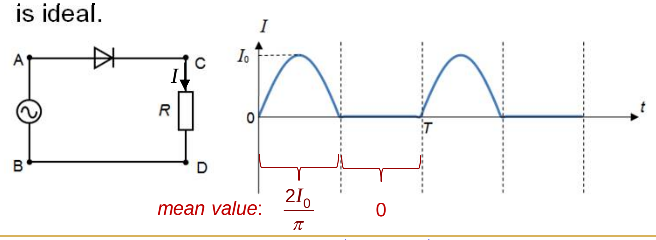
Not in syllabus

Rectified Alternating Current

Question: If the sinusoidal alternating supply in a half-wave rectifier circuit provides a peak current of I_0, what is the value of the mean current in R? Assuming the diode is ideal.



$$\therefore \text{mean current in } R = \frac{\left(\frac{2I_0}{\pi}+0\right)}{2} = \frac{I_0}{\pi} \approx 0.32I_0$$

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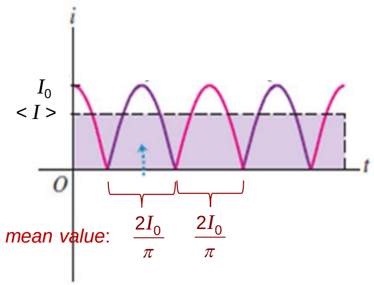
Full-wave rectified current

Not in syllabus



$$\langle I \rangle = \frac{2I_0}{\pi} \approx 0.64I_0$$

Full-wave rectified mean current

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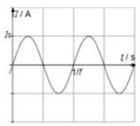
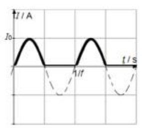
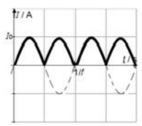
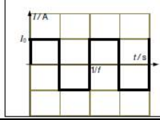
Comparison of $I_{r.m.s.}$ and <I>

Variation of current I with time t		r.m.s. current	whole cycle average current
	*Sinusoidal*	$\frac{I_0}{\sqrt{2}}$	0
	*Half-wave rectified*	$\frac{I_0}{2}$	$\frac{I_0}{\pi}$
	*Full-wave rectified*	$\frac{I_0}{\sqrt{2}}$	$\frac{2I_0}{\pi}$
	*Square*	I_0	0

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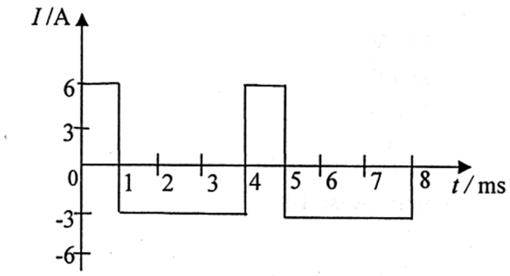
Additional Example 1

Calculate the root-mean-square value of the current.



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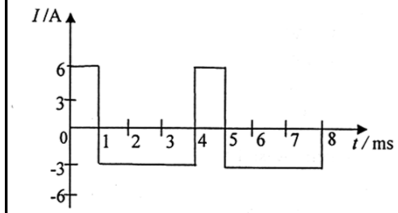
Additional Example 1



Solution:

$$\langle I^2 \rangle = \frac{(36\times 1)+(9\times 3)}{1+3}$$

$$\therefore\ I_{\text{rms}} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{(36\times 1)+(9\times 3)}{1+3}} = 3.97\text{ A}$$

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Additional Example 2

When an alternating current flows through a resistor of 5.0 Ω, heat is dissipated at a rate of 20 W.

What is the peak value of this alternating current?

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Additional Example 2

Question:

When an alternating current flows through a resistor of 5.0 Ω, heat is dissipated at a rate of 20 W.

What is the peak value of this alternating current?

Solution:

$$\langle P \rangle = I_{\text{rms}}^2 R$$

$$I_{\text{rms}}^2 = \frac{\langle P \rangle}{R} = \frac{20}{5.0} = 4.0\text{ A}^2$$

$$I_{\text{rms}} = \sqrt{4.0} = 2.0\text{ A}$$

$$I_0 = \left(\sqrt{2}\right) I_{\text{rms}} = 2.8\text{ A}$$

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Additional Example 3

A 100% efficient transformer is connected to a a.c. supply. There are 200 turns on the primary coil and 1000 turns in the secondary coil. If the output voltage is 80 V and the secondary coil is connected to a 100 Ω resistor, what is the current in the primary coil?

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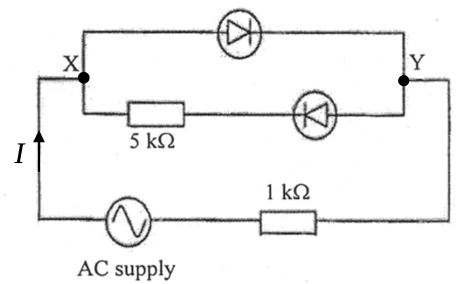
Additional Example 4

Sketch the I vs t graph for the circuit below.



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END

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