



## Content

- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle

## Learning Outcomes

Candidates should be able to:

- show an appreciation of the particulate nature of electromagnetic radiation
- recall and use the equation  $E = hf$  for the energy of a photon
- show an understanding that the photoelectric effect provides evidence for the particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for the wave nature
- recall the significance of threshold frequency
- recall and use the equation  $\frac{1}{2}mv_{\max}^2 = eV_s$ , where  $V_s$  is the stopping potential
- explain photoelectric phenomena in terms of photon energy and work function energy
- explain why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency
- recall, use and explain the significance of the equation  $hf = \Phi + \frac{1}{2}mv_{\max}^2$
- describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles
- recall and use the relation for the de Broglie wavelength  $\lambda = h/p$
- show an understanding of the existence of discrete electronic energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to the observation of spectral lines
- distinguish between emission and absorption line spectra
- recall and solve problems using the relation  $hf = E_2 - E_1$
- explain the origins of the features of a typical X-ray spectrum
- show an understanding of and apply  $\Delta p \Delta x \gtrsim h$  as a form of the Heisenberg position-momentum uncertainty principle to new situations or to solve related problems.

## Guiding Questions

- What roles do experiments play in constructing knowledge about the photoelectric effect, the discrete energy levels in atoms, and wave-particle duality?
- How does the wave model of light fail to explain the photoelectric effect? What does the existence of atomic line spectra suggest about light?
- How does quantum physics modify our ideas about matter?
- How can we use conservation laws to analyse the photoelectric effect?
- What is the significance of Heisenberg's uncertainty principle?

## 19.0 Introduction

### Links between sections and topics

Modern physics, developed from 1900 onwards, focuses on quantum and nuclear physics. It provides the foundation for much of chemistry and explains the behaviour of matter at the atomic scale, which differs significantly from classical Newtonian physics.

Electromagnetic radiation exhibits both wave-like properties (interference, diffraction, polarisation) and particle-like behaviour (photoelectric effect, atomic spectra). This led to the concept of wave-particle duality, where electromagnetic radiation is composed of discrete 'photons' with energy proportional to frequency.

Quantum physics introduced revolutionary ideas such as:

- Quantisation of energy in atoms
- Wave properties of particles (de Broglie wavelength)
- Probabilistic nature of sub-atomic phenomena

These concepts explained various experimental observations, including X-ray production and scattering, and led to technological advancements like lasers. While quantum mechanics challenged the deterministic view of the universe, it ultimately strengthened physics by providing a unified framework that encompasses both the microscopic and macroscopic realms.

### Applications and relevance to daily life

Quantum mechanics plays a crucial role in our understanding of the world around us, particularly at the atomic and molecular level. It provides the foundation for explaining the structure of atoms and molecules, their emission and absorption spectra, chemical behaviours, and various material properties. This deep understanding has led to significant advancements in modern chemistry and materials science.

Beyond its theoretical importance, quantum mechanics is paving the way for revolutionary technologies. One of the most exciting areas of development is quantum computing. Unlike traditional computers that use bits (0s and 1s), quantum computers utilise qubits, which can exist in a superposition of both states simultaneously, allowing quantum computers to process multiple possibilities at once. This field brings together experts from physics, computer science, and mathematics, promising to solve complex problems far more efficiently than classical computers.

### 19.1 Photon

Instead of treating electromagnetic radiation as a continuous wave, both Max Planck and Albert Einstein independently postulated electromagnetic radiation as discrete packets of energy which behave like particles.

A photon is a discrete packet (or quantum) of energy of electromagnetic radiation.

Energy of one photon is directly proportional to the frequency ( $f$ ) of electromagnetic radiation,

$$E = hf$$

where  $h$  is the Planck constant ( $= 6.63 \times 10^{-34} \text{ J s}$ )

Q1: Determine the energy in joule of a high-energy gamma photon of frequency  $10^{26} \text{ Hz}$ .

The example shows that the energy for a 'high-energy' photon is far less than 1.0 J. Hence the joule is not a convenient unit for measuring photon energies. The **electronvolt (eV)** is often used for such purposes.

*1 eV is the energy gained by an electron when it is accelerated through a potential difference of 1 volt.*

$$\text{Energy gain} = Q\Delta V = (1 \text{ e}) (1 \text{ V}) = (1.60 \times 10^{-19} \text{ C}) (1 \text{ V}) = 1.60 \times 10^{-19} \text{ J}$$

Therefore,  $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ .

To convert from eV to J, multiply by  $1.60 \times 10^{-19}$

To convert from J to eV, divide by  $1.60 \times 10^{-19}$

□

Q2: Visible light has wavelength spanning from 400 nm (violet) to 700 nm (red). Find, in eV, the maximum energy of a photon of visible light.

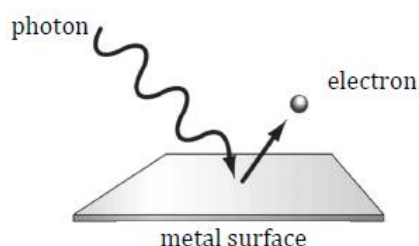
$$E_{\text{max}} =$$

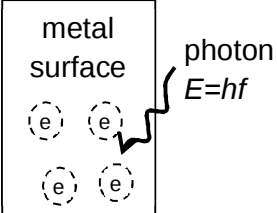
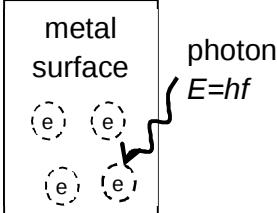
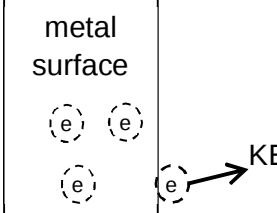
Q3: Determine the number of photons emitted per second by a 60 W violet light source. Wavelength of the violet light source is 400 nm.

$$\begin{aligned}\text{Power of the light source} &= \text{Total energy of light source per second} \\ &= \text{number of photons per second} \times \text{energy of a violet photon}\end{aligned}$$

## 19.2 Photoelectric effect

The photoelectric effect refers to the emission of electrons (also known as photoelectrons) from a cold metal surface when electromagnetic radiation of a sufficiently high frequency falls on it. In 1905, Albert Einstein came up with an explanation for this phenomenon based on the idea of photons.



|                                                                                                                                                                 |                                                                                                                                                   |                                                                                                                                                                                                                    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  <p>A photon of energy <math>hf</math> is incident on the metal surface.</p> |  <p>Photon meets an electron. Electron absorbs the energy.</p> |  <p>Some energy is used to overcome the forces holding the electron in the metal, the rest is KE of the emitted electron.</p> |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

**19.2.1 Einstein's Photoelectric Equation**

$$\begin{array}{c} \text{energy provided} \\ \text{by a single photon} \end{array} = \begin{array}{c} \text{work function energy} \\ \text{of a metal} \end{array} + \begin{array}{c} \text{max KE of} \\ \text{emitted electrons} \end{array}$$

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

The work function energy  $\Phi$  of a metal is the minimum energy of photon to cause emission of electron from surface of a metal. Work function energy depends on the nature of the material of the metal and its surface condition.

The threshold frequency  $f_0$  is the lowest frequency of electromagnetic radiation that gives rise to the ejection of electrons from the metal surface.

The threshold wavelength  $\lambda_0$  is the highest wavelength of electromagnetic radiation that gives rise to the ejection of electrons from the metal surface.

From the above definitions,

$$hf_0 = \frac{hc}{\lambda_0} = \Phi$$

Q4: The maximum KE of the electrons emitted from a metallic surface is 1.0 eV when the frequency of the incident radiation is  $7.5 \times 10^{14}$  Hz. Calculate

- (a) the work function energy of the metal in joules,
- (b) the threshold wavelength for emission of electrons from the metal.

- (a) energy of photon = work function energy + maximum kinetic energy of electron

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

(b) 
$$\Phi = \frac{hc}{\lambda_0}$$

Q5: Suggest why the emitted electrons are likely to have a range of kinetic energy (instead of just the maximum KE) for any one frequency of the electromagnetic radiation.

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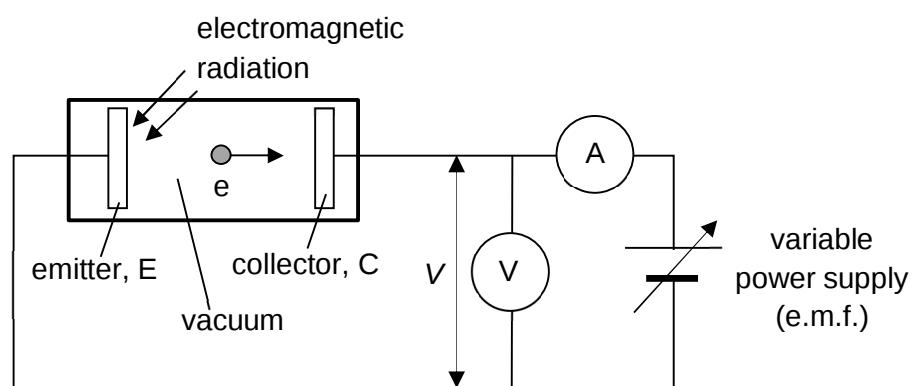
Some characteristics of photoelectric effect:

- A single photon can only interact, and hence exchange its energy with a single electron (one-to-one interaction). It is extremely unlikely for any electron to simultaneously receive energy from multiple photons.
- Not all photons (of sufficient energy) get to interact with electrons.
- The photoelectrons are emitted in all random directions with varying speeds.

*What happens when the incident radiation has frequency less than the threshold frequency?*

A single photon can still give up its energy to a single electron, but this electron cannot escape from the metal. The energy absorbed from the photons appears as kinetic energy of the electrons. These electrons lose their kinetic energy to the metal ions when they collide with them. This warms up the metal. This is why a metal plate placed in the vicinity of a table lamp gets hot.

### 19.2.2 Experimental Observations and Wave vs Photon (Particle) theory of Light

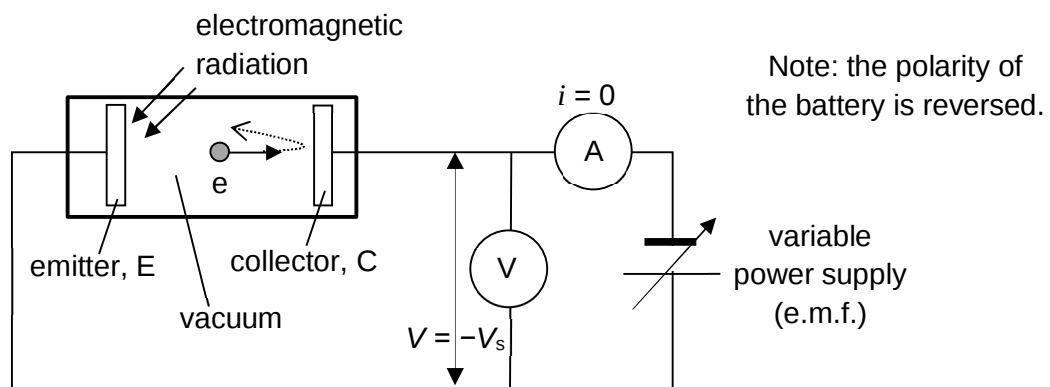


The above figure shows an experimental setup in which photoelectric effect can be observed. An evacuated glass contains a metal electrode (emitter, E), connected to the negative terminal of an e.m.f. source. Another metal electrode (collector, C) is connected to the positive terminal instead.  $V$  is the potential of the collector relative to the emitter. When the apparatus is kept in the dark, the ammeter registers no current. However, when monochromatic light, of frequency above a certain

threshold, shines on plate E, a current is detected by the ammeter, indicating a flow of electrons (called photoelectrons) across the plates E and C.

The illuminated electrode emits photoelectrons with various kinetic energies. If the electric field points toward the collector, as shown in the above figure, all the electrons are accelerated toward the collector and contribute to the photocurrent.

But by reversing the electric field (by reversing the polarities of the e.m.f. source) and adjusting its strength as in the figure below, we can prevent the less energetic electrons from reaching the collector. In fact, we can determine the maximum kinetic energy of the emitted electrons by making  $V$  just negative enough so that the current  $I$  stops. This occurs for  $V = -V_s$ , where  $V_s$  is called the stopping potential.



As an electron moves from the emitter to the collector, the potential decreases by  $V_s$  and negative work is done by electric force on the (negatively charged) electron. The most energetic electron leaves the emitter with kinetic energy  $KE_{\max} = \frac{1}{2}mv_{\max}^2$  and has zero kinetic energy at the collector. From the conservation of energy, we have

$$\begin{aligned}\Delta KE + \Delta EPE &= 0 \\ \Rightarrow (0 - KE_{\max}) + (-e)(-V_s) &= 0 \\ \Rightarrow KE_{\max} &= \frac{1}{2}mv_{\max}^2 = eV_s\end{aligned}$$

Hence by measuring the stopping potential  $V_s$ , we can determine the maximum kinetic energy with which electrons leave the emitter.

This minimum potential,  $V_s$ , that is applied to stop the most energetic electrons is the **stopping potential**.

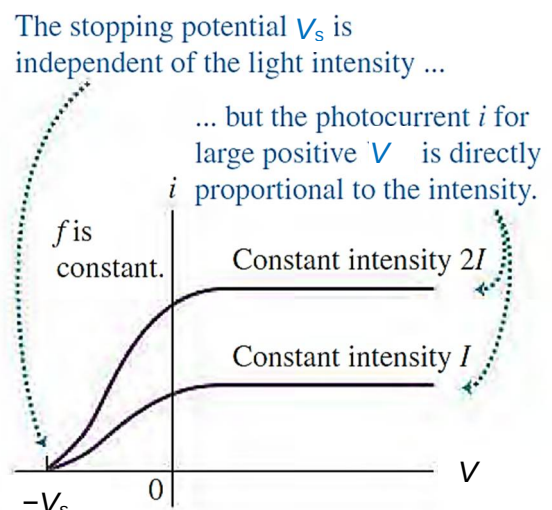
In this experiment, how does the photocurrent  $i$  depend on the voltage  $V$  across the electrodes and on the frequency  $f$  and intensity  $I$  of the light?

| Wave-Model Prediction (Based on Maxwell's picture of light as an electromagnetic wave)                                                                                                                                                                                                                                                    | Experimental Result                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. The intensity of an electromagnetic wave depends on its amplitude but not on its frequency. So, <i>the photoelectric effect should occur for light of any frequency, and the magnitude of the photocurrent should not depend on the frequency of the light.</i>                                                                        | 1. <i>The photocurrent depends on the light frequency.</i> For a given material, monochromatic light with a frequency below a minimum <b>threshold frequency</b> produces <b>no</b> photocurrent, regardless of intensity.                                                                                                                                                                                                                                                                                                               |
| 2. If the light falling on the surface is very faint, some time may elapse before the total energy absorbed by the surface equals the work function energy. Hence, for faint illumination, <i>we expect a time delay</i> between when we switch on the light and when photoelectrons appear.                                              | 2. <i>There is no measurable time delay</i> between when the light is turned on and when the cathode emits photoelectrons (assuming the frequency of the light exceeds the threshold frequency). This is true no matter how faint the light is.                                                                                                                                                                                                                                                                                          |
| 3. Because the energy delivered to the emitter surface depends on the intensity of illumination, <i>we expect the stopping potential to increase with increasing light intensity.</i> Since intensity does not depend on frequency, we further expect that <i>the stopping potential should not depend on the frequency of the light.</i> | 3. <i>The stopping potential does not depend on intensity but does depend on frequency.</i> Figure below shows $i$ as a function $V$ for light of a given $f$ & two different intensities $I$ . The only effect of increasing the intensity is to increase the number of electrons per second and hence the photocurrent. If the intensity is held constant but the frequency is increased, the stopping potential also increases. In other words, the greater the light frequency, the higher the energy of the ejected photoelectrons. |

The classical wave model of light predicted behavior that significantly differed from what was observed in photoelectric effect experiments.

According to the wave theory, light was considered a continuous electromagnetic wave. This model suggested that electrons in a metal should be able to gradually accumulate energy from the incident light, regardless of its frequency. Given enough time, electrons should eventually gain sufficient energy to be ejected from the metal surface, even at low light frequencies.

However, the experimental results painted a very different picture. Scientists observed that electron emission occurred only above a certain threshold frequency, which varied for different metals. Below this frequency, no electrons were emitted, no matter how intense the light or how long the metal was exposed. Furthermore, when emission did occur, it happened almost instantaneously, with no detectable time lag.



### 19.2.3 Einstein's Photon Explanation

Einstein made the radical postulate that a beam of light consists of small packages of energy called photons or quanta. This postulate was an extension of an idea developed five years earlier by Max Planck to explain the properties of blackbody radiation.

In Einstein's picture, the energy  $E$  of an individual photon is equal to  $hf$ . An individual photon arriving at the surface is absorbed by a single electron. This energy transfer is an all-or-nothing process, in contrast to the continuous transfer of energy in the wave theory of light; the electron gets all of the photon's energy or none at all. The electron can escape from the surface only if the energy it acquires is greater than the work function energy  $\Phi$ . Thus, photoelectrons will be ejected only if  $hf > \Phi$  or  $f > \Phi/h$ . Einstein's postulate therefore explains why the photoelectric effect occurs only for frequencies greater than a minimum threshold frequency  $f_0$ . This postulate is also consistent with the observation that greater intensity causes a greater photocurrent  $i$  (Figure on page 19-8). Greater intensity  $I$  at a particular frequency means a greater number of photons per second absorbed, and thus a greater number of electrons emitted per second and a greater photocurrent.

Einstein's postulate also explains why there is no delay between illumination and the emission of photoelectrons. As soon as photons of sufficient energy strike the surface, electrons can absorb them and be ejected.

Finally, Einstein's postulate explains why the stopping potential  $V_s$  for a given surface depends only on the light frequency. Recall that  $\Phi$  is the minimum energy needed to remove an electron from the surface. Einstein applied conservation of energy to find that the maximum kinetic energy  $KE_{\max}$  for an emitted electron:

$$KE_{\max} = \frac{1}{2}mv_{\max}^2 = hf - \Phi$$

which is Einstein's photoelectric equation.

Substitute  $KE_{\max} = eV_s$ , we find, for photoelectric effect:

$$eV_s = hf - \Phi$$

The intensity doesn't appear in this equation, so  $V_s$  is independent of intensity.

Q6: UV light of wavelength 350 nm falls onto Cesium metal of work function 2.1 eV. Find the stopping potential (in eV) required for the ammeter to register zero current.

$$hf = \Phi + \frac{1}{2}mv_{\max}^2$$

maximum KE in eV:

stopping potential:  $KE_{\max} = \frac{1}{2}mv_{\max}^2 = eV_s$

Q7: A metal surface is illuminated with a beam of monochromatic electromagnetic radiation. What determines the maximum speed of the photoelectrons that are emitted from the surface?

- (A) The intensity of the radiation only.
- (B) The frequency of the radiation only.
- (C) The intensity and the frequency of the radiation.
- (D) The frequency of the radiation and the work function energy of the metal.

#### 19.2.4 Graphical representations of the Photoelectric effect results

We can measure the stopping potential  $V_s$  for each of several values of frequency  $f$  for a given cathode material. A graph of  $V_s$  against  $f$  turns out to be a straight line, verifying  $eV_s = hf - \Phi$ , and from such a graph we can determine both the work function energy  $\Phi$  for the material and the value of the quantity  $h/e$ .

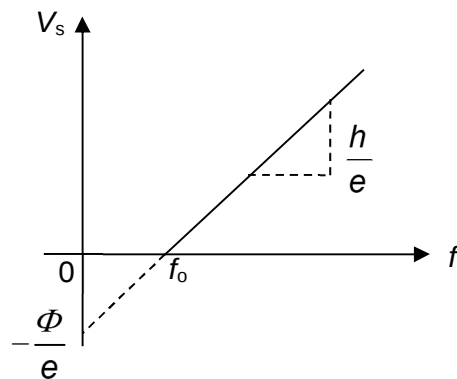
$$eV_s = hf - \Phi$$

$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

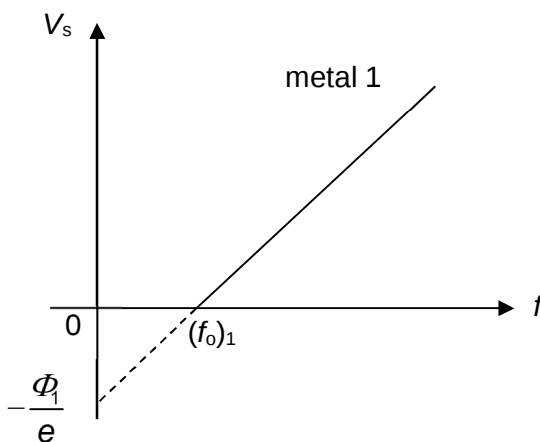
Gradient of the line =  $\frac{h}{e}$

Vertical intercept of the line =  $-\frac{\Phi}{e}$

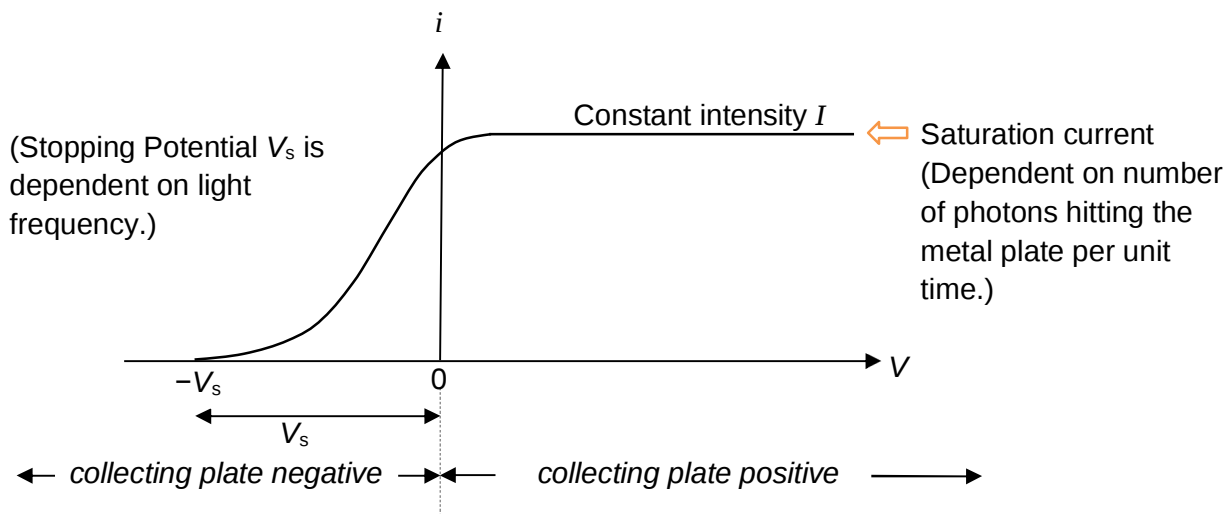
Horizontal Intercept of the line =  $f_0$



Q8: In the figure below, draw the graph when the experiment is repeated using a metal of greater work function energy.

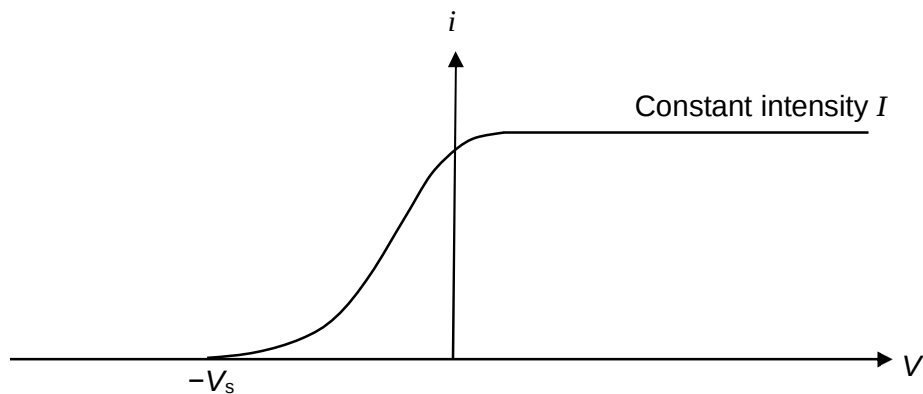


The graph of  $i$  against  $V$  for the circuit in the photoelectric experiment for light of a given  $f$  is shown below.



1. Negative  $V$  Region ( $V < 0$ ):
  - o As  $V$  increases from  $-V_s$  towards 0 V, more photoelectrons reach the collector.
  - o This results in an increase in the circuit current.
  - o The current-voltage relationship in this region is approximately linear.
2. Zero Voltage ( $V = 0$ ):
  - o At 0 V, there's no accelerating or retarding potential.
  - o Only photoelectrons with sufficient initial kinetic energy reach the collector.
3. Positive  $V$  Region ( $V > 0$ ):
  - o As  $V$  becomes positive, it accelerates all emitted photoelectrons towards the collector.
  - o The current rapidly increases until it reaches a saturation value.
4. Saturation Region:
  - o Beyond a certain positive voltage, the current reaches a maximum (saturation) value.
  - o Further increases in voltage do not increase the current.
  - o The saturation current is determined by: (a) The intensity of the incident light (number of photons per second), (b) The quantum efficiency of the emitter (electrons emitted per incident photon)
5. Intensity Dependence:
  - o The saturation current is directly proportional to the light intensity.
  - o Higher intensity means more photons per second, resulting in more emitted electrons per second.

Q9: Plot a new graph if the frequency of the light is increased but keeping the number of photons reaching the plate per second the same.



Hints:

1. Consider the photoelectric equation:  $hf = \Phi + eV_s$
2. Effect on stopping potential ( $V_s$ ): As  $f$  increases,  $V_s$  will increase.
3. Effect on maximum photoelectric current:
  - o Maximum current depends on the number of electrons emitted per second.
  - o This is determined by the number of photons hitting the metal surface per second.
  - o Since the number of photons per second remains constant, the maximum current will not change.
4. Shape of the new graph:
  - o The overall shape will be like the original graph.
  - o The saturation current (maximum current) will remain the same.
  - o The curve will shift to the **left** due to the increased stopping potential.
5. Consider the energy of individual photons:
  - o Higher frequency means each photon carries more energy ( $E = hf$ ).
  - o This results in photoelectrons being emitted with higher maximum kinetic energy.

### 19.3 Wave-particle duality

Light and other electromagnetic radiation act sometimes like waves and sometimes like particles. Interference and diffraction demonstrate wave behaviour, while emission and absorption of photons demonstrate particle behaviour.

If light waves can behave like particles, can the particles of matter behave like waves? The answer is a resounding yes. Electrons can interfere and diffract just like other kinds of waves.

If a particle acts like a wave, it should have a wavelength and a frequency. In 1923, De Broglie postulated that a free particle with rest mass  $m$ , moving with nonrelativistic speed  $v$ , should have a wavelength  $\lambda$  related to its momentum  $p = mv$ , as expressed by:  $\lambda = \frac{h}{p} = \frac{h}{mv}$ .

The **de Broglie** wavelength,  $\lambda$ , is the wavelength associated with a particle that is moving, is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where  $h$  is the Planck constant.

The frequency  $f$ , according to de Broglie, is also related to the particle's energy  $E$  in exactly the same way as for a photon:

$$E = hf$$

Similarly, for electromagnetic radiation of wavelength,  $\lambda$ , which exhibits particle behaviour, it will have an associated momentum,  $p$ , given by

$$p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c} \quad \left[ \text{from } c = f\lambda \Rightarrow \frac{1}{\lambda} = \frac{f}{c} \right]$$

[Caution] **Not all photon equations apply to particles with mass.**

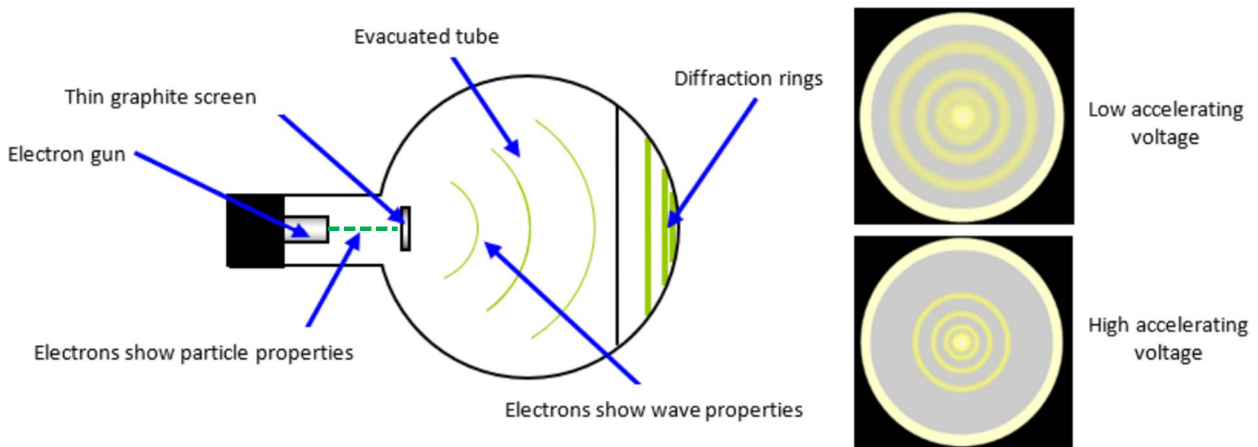
Be careful when applying  $E = hf$  to particles with nonzero rest mass, such as electrons. Unlike a photon, they do not travel at speed  $c$ , so the equations  $f = c/\lambda$  and  $E = pc$  do not apply to them!

**Wave-particle duality** refers to the fact that matter behave like waves in some situations and like particles in others.

Photons and atomic and sub-atomic particles must be treated as particles or waves, according to circumstances.

### 19.3.1 Electron-diffraction experiment

#### Experiment Setup



1. Electron Gun: produces a beam of electrons with controlled energy
  - (a) Heater: Heats the cathode to high temperatures,
  - (b) Cathode: Heated metal (usually tungsten) that emits electrons,
  - (c) Anode: Positively charged plate with a small hole, accelerates electrons towards it, and allows a narrow beam of electrons to pass through the hole
2. Target Graphite: acts as a diffraction medium for the electrons
  - o Thin film of crystalline graphite
  - o Placed in the path of the electron beam
3. Display: shows the diffraction pattern as glowing rings
  - o Fluorescent screen
  - o Located beyond the graphite target

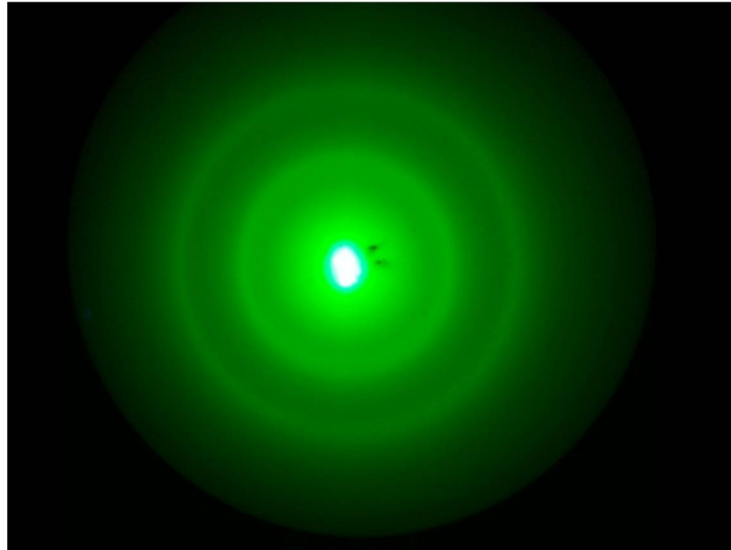
The entire apparatus, including the electron gun, is housed in a vacuum chamber. This prevents electron scattering by air molecules.

#### Formation of Circular Rings

1. Electron wavelength:
  - o Electrons have a de Broglie wavelength:  $\lambda = h/p = h/mv = h/\sqrt{2meV}$  where  $V$  is the accelerating potential in the electron gun
2. Diffraction by crystalline graphite:
  - o Graphite acts similar to a diffraction grating for electrons
  - o The spacing between carbon atom layers is analogous to grating spacing ( $d$ )
3. Diffraction pattern:
  - o Follows a principle similar to the grating equation:  $d \sin \theta = n\lambda$
  - o  $\theta$  is the diffraction angle,  $n$  is the order of diffraction
4. Circular rings:
  - o Formed due to the random orientation of graphite crystallites
  - o Each ring corresponds to a specific diffraction order ( $n$ )
5. Effect of accelerating potential ( $V$ ):

- o Higher  $V$ : Shorter  $\lambda$ , smaller  $\theta$ , more closely spaced rings
  - o Lower  $V$ : Longer  $\lambda$ , larger  $\theta$ , more widely spaced rings
6. Comparison of two patterns:
- o Pattern with closer rings: Higher accelerating potential
  - o Pattern with wider rings: Lower accelerating potential

Photograph of the electron diffraction pattern for a particular  $V$ :



The unobstructed electron beam is much brighter than the rings outside. Note the two distinct rings corresponding to two separate distances between carbon atoms in the graphite structure.

- Q10: Estimate the de Broglie wavelength associated with
- (a) an electron (mass  $9.11 \times 10^{-31}$  kg) accelerated through a p.d. of 1000 V from rest,
  - (b) a teacher (mass 100 kg) running at  $10 \text{ m s}^{-1}$  (chasing after students who owe him homework).
- (a)
- conservation of energy:
- $$\Delta KE + \Delta EPE = 0$$

The wavelength of the electron is in the X-ray region, which is in the same order as that of inter-atomic spacing in a graphic / carbon crystal. Therefore, a noticeable diffraction pattern can be observed by sending the electrons towards a thin film of carbon.

(b)

matter wavelength of the teacher

=

=

=

Ordinary objects of much bigger masses have extremely small wavelengths. The wave-like behaviour is not observable.

Q11: Estimate the minimum change in momentum of a photon of visible light when it is reflected normally off a plane mirror.

$$\begin{aligned}\text{minimum momentum of visible light } p &= \frac{h}{\lambda_{\text{red}}} \\ &= 6.63 \times 10^{-34} / (7 \times 10^{-7}) \\ &= 9.47 \times 10^{-28} \text{ kg m s}^{-1}\end{aligned}$$

minimum change in momentum =

direction of the change in momentum:

Q12: Calculate the number of photons of the light in Q11 needed per second to accelerate a mass of 1.0 kg by  $1.0 \text{ m s}^{-2}$ .

force = number of photons per second  $\times$  change in momentum of one photon

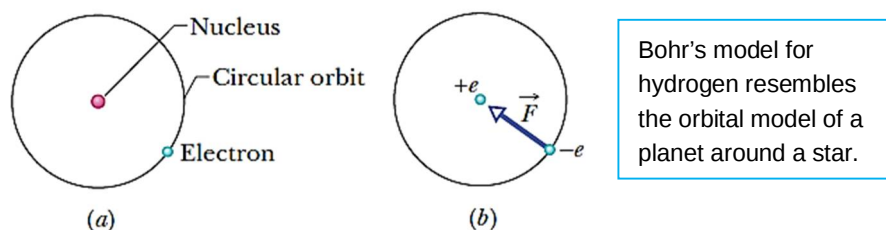
A futuristic application of the momentum of light could be the propulsion of space travel using solar sails. Advantages: no onboard fuel required; potentially unlimited range. Challenges: large sail area needed for significant thrust; limited effectiveness far from the Sun.

## 19.4 Energy levels in isolated atoms

### Atomic Structure and Discrete Energy Levels

Atoms, the building blocks of matter, consist of a nucleus surrounded by electrons. In isolated atoms, these electrons occupy specific energy levels or orbitals, characterized by principal quantum numbers ( $n$ ). The principal quantum number is a positive integer that primarily determines the energy of an electron in an atom. In quantum mechanics, we find that electrons in atoms can only exist in certain allowed energy states, which are **quantized**, meaning they have **discrete** values. The lowest energy state, corresponding to  $n = 1$ , is called the **ground state**. Any states above this ( $n > 1$ ) are referred to as **excited states**.

The Bohr model of the hydrogen atom provides the simplest illustration of these discrete energy levels. In this model, electrons can only occupy specific circular orbits around the nucleus, each corresponding to a particular energy level and principal quantum number. For instance, the ground state ( $n = 1$ ) represents the lowest energy orbit, while  $n = 2, 3, 4$ , and so on represent progressively higher energy levels and larger orbits. While more complex atoms have more intricate arrangements involving additional quantum numbers, the principle of discrete energy levels characterized by principal quantum numbers holds true for all elements.

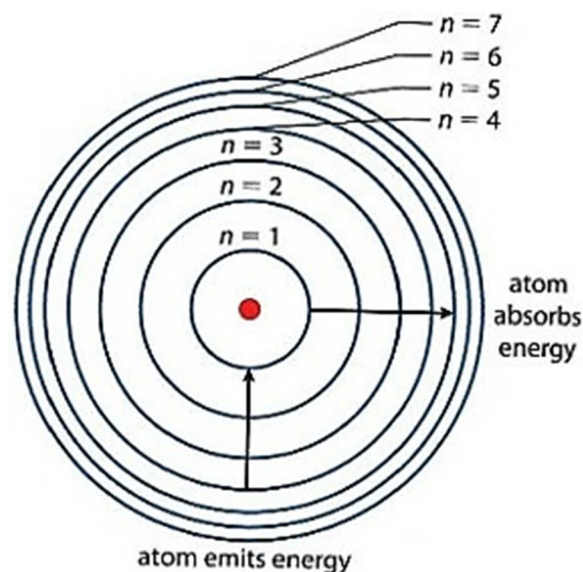


(a) Circular orbit of an electron in the Bohr model of the hydrogen atom.

(b) The Coulomb force on the electron is directed radially inward toward the nucleus.

### Electron Transitions and Spectral Lines

Electrons can move between these energy levels by absorbing or emitting energy. When an electron moves to a higher energy level (increasing  $n$ ), it absorbs energy in a process called **excitation**. For example, an electron might jump from  $n = 1$  to  $n = 2$  or even to  $n = 3$  or higher. However, electrons in excited states are inherently unstable and remain in these higher energy levels for only a very short time, typically on the order of nanoseconds. Almost immediately, the electron falls to a lower energy level (decreasing  $n$ ), releasing energy in a process known as **de-excitation**. The energy changes in these transitions occur in the form of photons, particles of light. The energy of each photon is equal to the energy difference between the levels involved in the transition, following the equation  $E = hf$ , where  $h$  is Planck constant and  $f$  is the

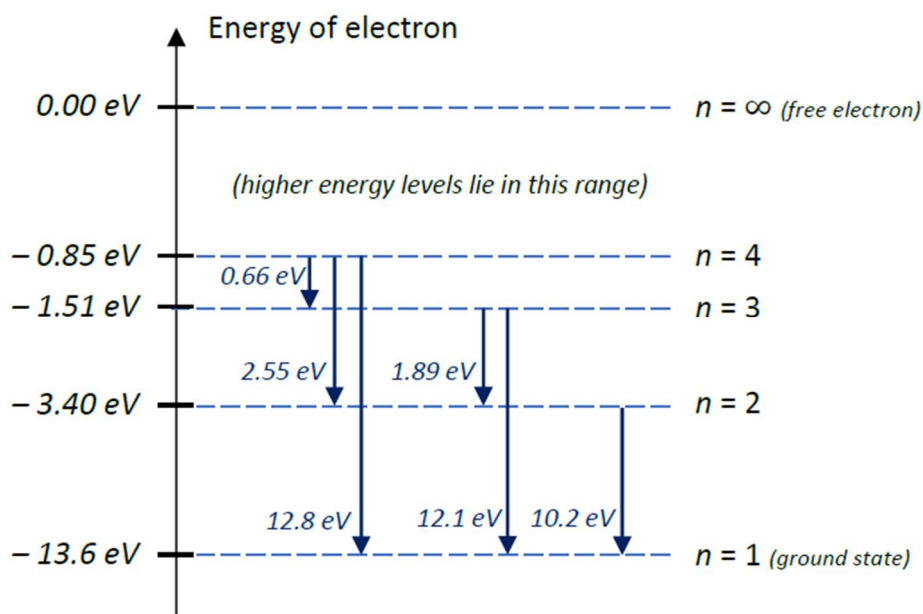


frequency of the light. The larger the difference in  $n$  between the initial and final states, the greater the energy of the emitted photon.

These electron **transitions** give rise to **spectral lines**. Each possible transition between energy levels produces a photon of specific energy, corresponding to light of a particular frequency or wavelength. The collection of these specific wavelengths forms the atom's spectrum. When excited atoms emit photons, we observe an **emission spectrum**, which appears as bright lines on a dark background. Conversely, when atoms absorb specific wavelengths from white light, we see an **absorption spectrum**, appearing as dark lines in a continuous spectrum.

### Uniqueness and Applications of Atomic Spectra

Importantly, each element has a unique set of energy levels, resulting in a distinct spectral pattern. For example, the following diagram shows the possible paths “back to ground state” for a hydrogen atom that has already been excited to the  $n = 4$  state:

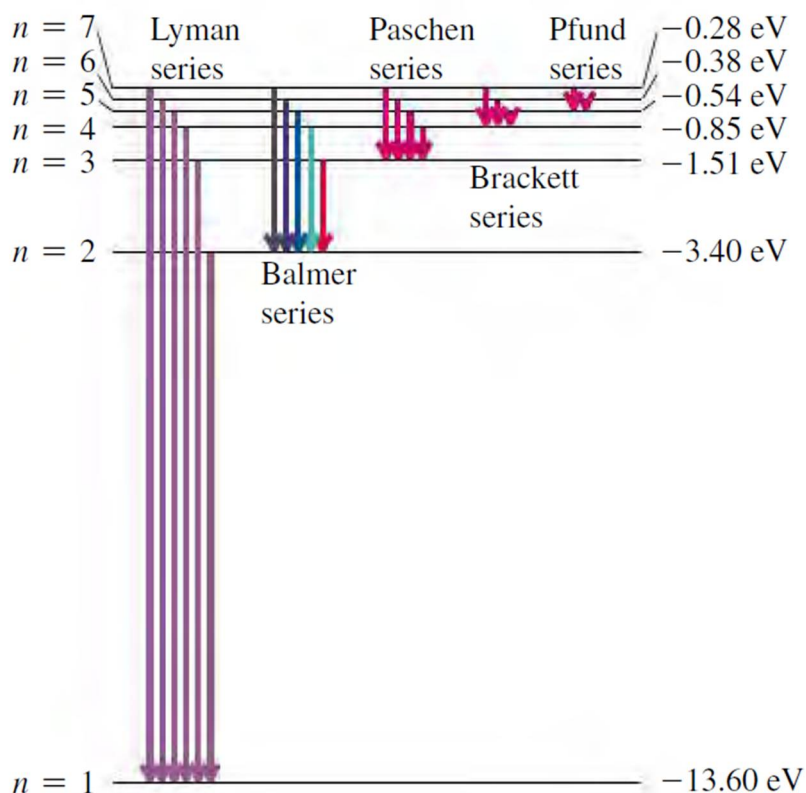


Energy levels of an electron ‘orbiting’ the hydrogen atom showing transitions from  $n = 4$  to the ground state ( $n = 1$ ).

This uniqueness allows for the identification of elements based on their spectra, a principle with wide-ranging applications. In astronomy, spectroscopy is used to identify elements present in distant stars and galaxies. In chemistry and physics, spectral analysis is a powerful tool for material identification and characterization.

Understanding the link between discrete atomic energy levels and spectral lines is crucial in quantum physics. It not only explains the fundamental behavior of atoms but also provides a powerful method for studying the composition of matter across the universe.

Energy-level diagram for hydrogen, showing some transitions corresponding to the various series:



#### Key Features of the Atomic Energy-Level Diagram

1. The principal quantum number,  $n$ , can only take positive integer values (1, 2, 3, ...). It is the primary determinant of an electron's energy in an atom.
2. The ground state ( $n = 1$ ) represents the lowest energy level, where the atom is most stable. Under normal conditions, an electron occupies this state.
3. As  $n$  approaches infinity, it represents an energy state where the electron is no longer bound to the atom (ionization). By convention, this state is assigned an energy of 0 eV.
4. Energy levels become less negative (more positive) as  $n$  increases. Lower, more negative energy states correspond to greater stability.
5. The energy difference between adjacent levels decreases as  $n$  increases, causing higher energy levels to converge towards a continuum.
6. Ionization energy is the minimum energy required to remove an electron from the ground state to  $n = \infty$ . For hydrogen, this is 13.6 eV.
7. Electrons can transition between energy levels by absorbing or emitting energy, not only as photons but also through other processes like collisions.
8. An atom is in an excited state when its electron occupies any energy level above the ground state ( $n > 1$ ).
9. For hydrogen, the energy of each level is given by  $E_n = -13.6 \text{ eV}/n^2$ , directly relating energy to the principal quantum number.
10. Spectral series result from transitions between specific energy levels. For hydrogen, the Lyman series involves transitions to  $n = 1$ , Balmer to  $n = 2$ , and Paschen to  $n = 3$ .
11. While this diagram accurately represents hydrogen, more complex atoms have additional factors influencing their energy level structures.

**19.4.1 Electron Excitation in Atoms: Two Main Mechanisms**

| Photon Absorption                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | High Speed Collisions                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>Cool gas is exposed to electromagnetic radiation.</p> <p>An electron absorbs energy from a single photon.</p> <p>Photon energy must exactly match the energy difference between levels, governed by the equation:</p> $hf = \Delta E = E_{\text{final}} - E_{\text{initial}}$ <p>where <math>h</math> is Planck constant, <math>f</math> is frequency, and <math>E</math> is energy.</p> <p>This precise requirement means that only photons of particular frequencies can excite electrons in a given atom, leading to the characteristic absorption spectra we observe.</p> | <p>Cool gas is bombarded with fast-moving particles (ions, atoms, or electrons).</p> <p>Electron absorbs part or all of the colliding particle's kinetic energy.</p> <p>Kinetic energy of the colliding particle must be at least equal to the energy level difference, described by the inequality:</p> $\frac{1}{2}mv_{\text{colliding particle}}^2 \geq  \Delta E $ <p>where <math>m</math> is mass and <math>v</math> is velocity of the colliding particle.</p> <p>This allows for a range of particle energies to cause excitation, as any excess energy can be distributed as kinetic energy among the colliding particles after the interaction.</p> |

**19.4.2 De-excitation of electron**

Refer to the figure on page 19-18 for hydrogen atom.

This can occur in two ways:

1. Direct transition: The electron drops directly from its excited state to the ground state in a single step, emitting a photon with energy equal to the full difference between these levels.
2. Cascading transitions: The electron may descend through a series of intermediate energy levels before reaching the ground state. In this case, it emits a separate photon for each transition, with each photon's energy corresponding to the energy difference between the levels involved in that particular step.

The specific transition pathway depends on the atom's structure and the selection rules governed by quantum mechanics. The collection of all possible transitions gives rise to the characteristic emission spectrum of the element. The time scale for these de-excitation processes is typically very short, often on the order of nanoseconds, unless the electron becomes 'trapped' in a metastable\* state.

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\* A metastable state is an excited energy level in an atom where an electron can remain for a significantly longer time than in other excited states, typically ranging from microseconds to seconds, or even longer in some cases.

Q13: Calculate the wavelength of the photon required to bring an electron of a hydrogen atom from the ground state to the  $n = 4$  excited state.

!

$$\therefore \lambda = 97.5 \text{ nm}$$

Q14: An electron with 12.5 eV of kinetic energy is incident on an unexcited electron in a hydrogen atom. Determine the remnant kinetic energy, in joule, of the colliding electron.

Incident electron has enough energy to bring an unexcited electron from ground to  $n = 3$ .

Q15: State how many possible transitions are there when a hydrogen atom in its  $n = 4$  state de-excites to the ground state.

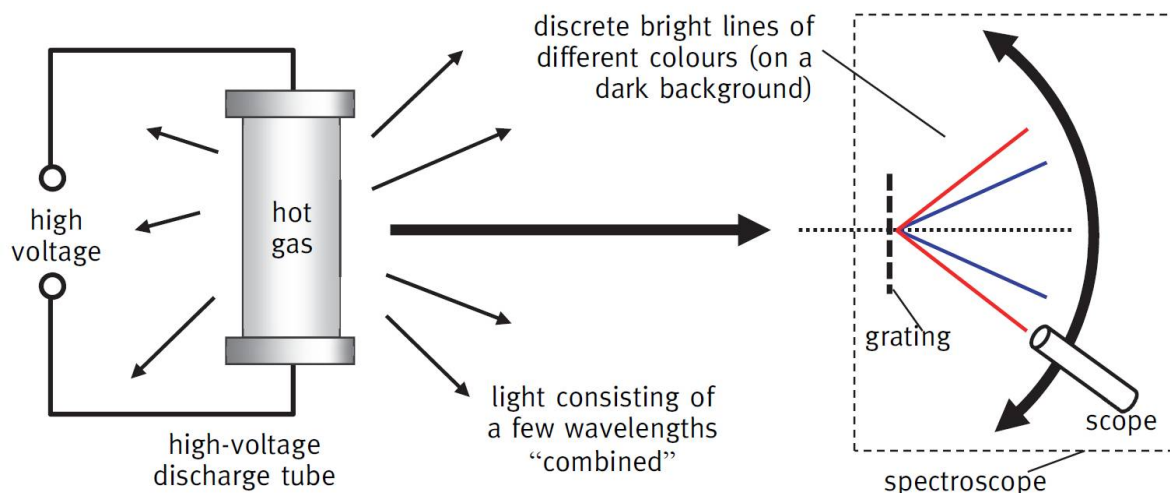
## 19.5 Line Spectra

Another 'mysterious' experimental observation that classical physics struggled to explain during the early 1900s was the phenomenon of line spectra obtained from gaseous elements. When light emitted by excited gases was passed through a diffraction grating, scientists observed distinct, bright lines of specific colors against a dark background, rather than the continuous spectrum predicted by classical electromagnetic theory. Each element produced its own unique pattern of lines, like a spectral fingerprint.

### 19.5.1 Emission Line Spectra

*Emission line spectra consist of discrete bright lines of different colours on a dark background.*

The typical setup used to observe emission line spectra is shown below.



High potential difference (voltage) is applied to the low pressure gas to bring the gas atoms to excited state. An electron in the excited state is unstable. After a short and random interval (about  $10^{-8}$  s), the electron will return to the lowest level.

When the electron transits down from higher excited state to lower states, photons of energies corresponding to the difference between the energy levels will be emitted. Due to the existence of discrete energy levels in isolated atoms, these emitted photons have specific energies which correspond to specific frequencies/wavelengths. These photons are directed at the diffraction grating and get diffracted at specific angles according to their wavelengths. This gives rise to the set of characteristic spectral lines when viewed through a spectroscope. The set of spectral lines emitted by an element is unique to that element.

The following table shows the corresponding wavelengths (using  $E = hf = hc/\lambda$ ) of these transitions:

| Transition                | Energy in eV                                       | Wavelength | Spectral line |
|---------------------------|----------------------------------------------------|------------|---------------|
| $n = 4 \rightarrow n = 3$ | $0.66 \text{ eV} = 1.06 \times 10^{-19} \text{ J}$ | 1870 nm    | Infrared      |
| $n = 3 \rightarrow n = 2$ | $1.89 \text{ eV} = 3.03 \times 10^{-19} \text{ J}$ | 652 nm     | Red           |
| $n = 4 \rightarrow n = 2$ | $2.55 \text{ eV} = 4.09 \times 10^{-19} \text{ J}$ | 483 nm     | Cyan          |
| $n = 2 \rightarrow n = 1$ | $10.2 \text{ eV} = 16.3 \times 10^{-19} \text{ J}$ | 121 nm     | Ultraviolet   |
| $n = 3 \rightarrow n = 1$ | $12.1 \text{ eV} = 19.4 \times 10^{-19} \text{ J}$ | 102 nm     | Ultraviolet   |
| $n = 4 \rightarrow n = 1$ | $12.8 \text{ eV} = 20.5 \times 10^{-19} \text{ J}$ | 96.3 nm    | Ultraviolet   |

If the light from an excited hydrogen gas is passed through a diffraction grating, we can expect to see the following characteristic spectrum:



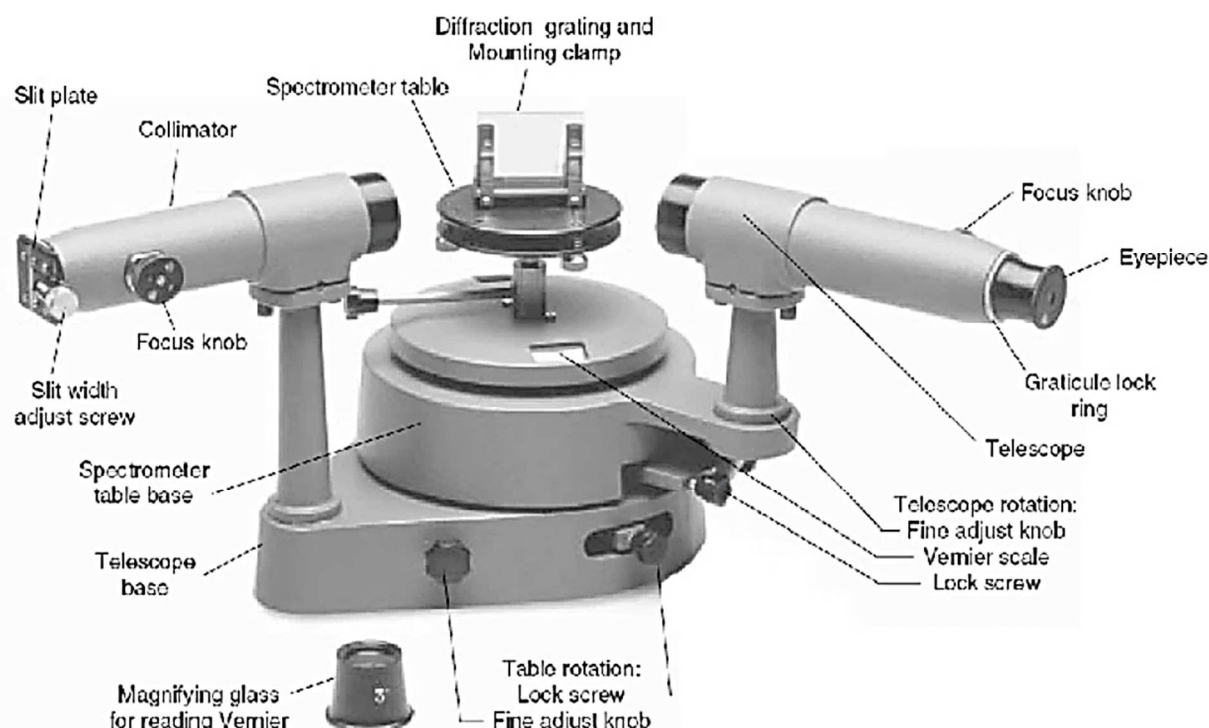
Sample hydrogen spectrum, with energy level transitions labelled. These visible wavelengths are part of the Balmer series, named after the researcher who identified this spectrum in 1885.

There are also other energy level transitions, but these are not within the visible spectrum.

These four lines will be present whenever a substance containing hydrogen is excited to at least the  $n = 6$  level. This characteristic spectrum has been used to identify whether distant stars contain hydrogen. For example, the Orion nebula predominantly exhibits the red wavelength corresponding to the  $n = 3 \rightarrow n = 2$  transition.

### Spectrometer

To view the spectrum clearly, the light has to be split, such as by using a diffraction grating. This principle is used in a device known as a spectrometer. By measuring the exact angle at which each wavelength is diffracted to, the wavelength and thus energy can be calculated. By comparing the wavelengths or energy levels to known records, the substance emitting light can be identified. An example of a spectrometer is shown below.



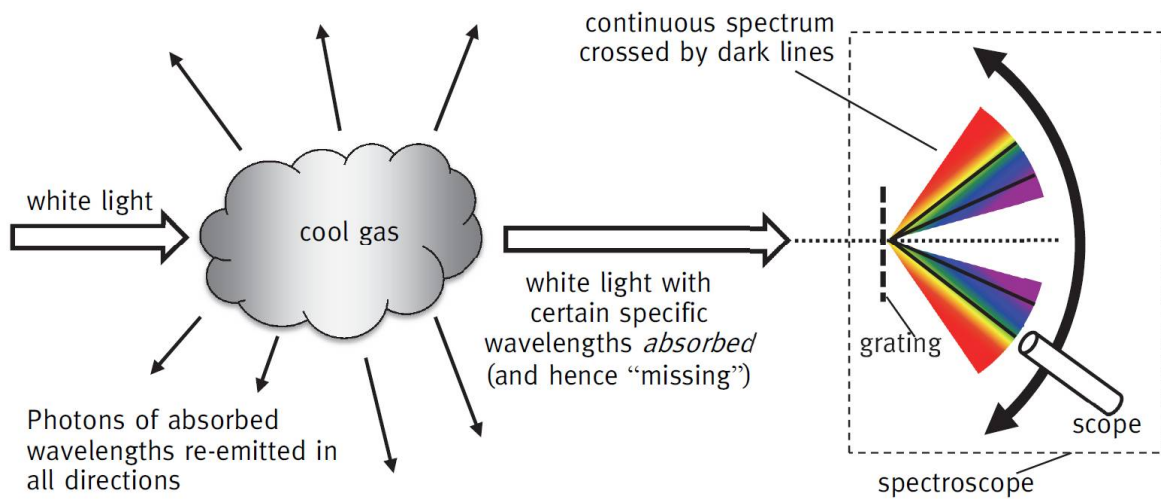
### 19.5.2 Absorption Line Spectra

*Absorption line spectra are continuous spectrum crossed by dark lines.*

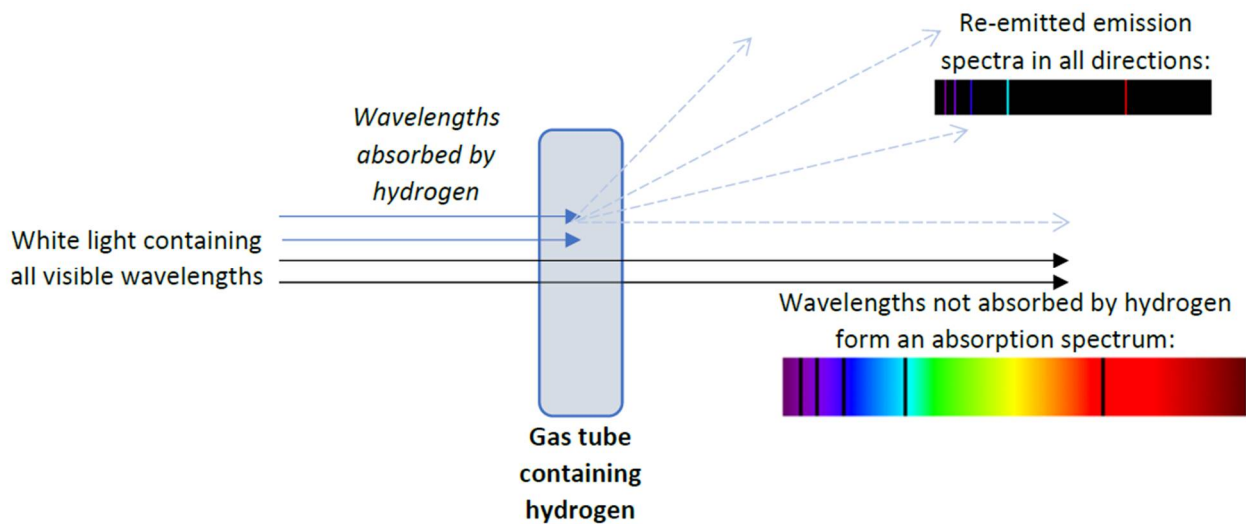
The absorption line spectrum is produced by passing a continuous spectrum of white light through a cool vapour or gas. As white light passes through the gas, the electrons of the vapour (gas atoms) absorb photons of certain frequencies and get excited from ground state to a higher energy level(s). The photons that are absorbed must have energy exactly equal to the energy difference between the two energy levels of the gas atoms. Due to the existence of discrete energy levels in isolated atoms, the absorbed photons have specific energies, which correspond to specific frequencies and specific wavelengths.

When the electrons become de-excited, the photons are re-emitted in random directions, causing them to be missing from the "straight through" direction. Thus dark lines corresponding to the absorption spectrum are observed when viewed through a detector placed on the other side of the gas. The dark lines in the absorption spectrum always correspond to the specific wavelengths

absorbed by the atoms. The set of absorption spectral lines absorbed by an element is unique to that element, serving as a spectral fingerprint that allows scientists to identify the composition of distant stars and interstellar gases.

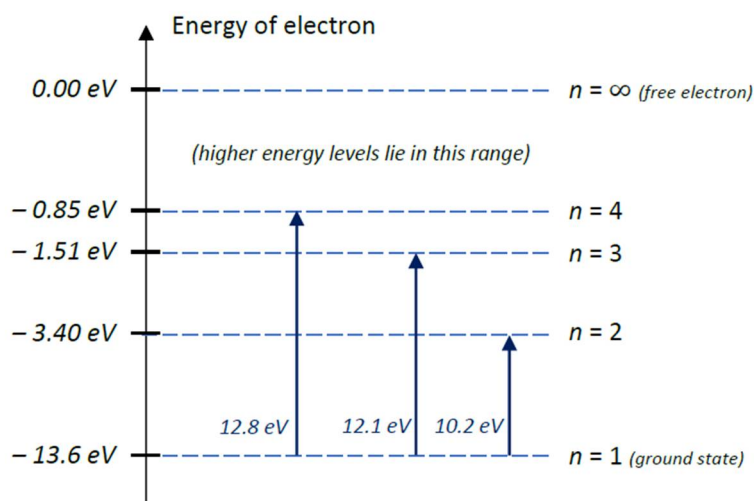


Importantly, this emission of light by the excited gas creates its own emission spectrum, as shown below. If observed from the side of the gas sample, rather than in line with the original light source, one would see bright spectral lines characteristic of the gas. These emission lines correspond to the energy transitions of the electrons as they return to lower energy states.



Paths taken by different frequencies of light after a continuous spectrum is passed through a gas tube, where some light is absorbed and re-emitted in all directions.

The dark lines in the absorption spectra correspond to the following transitions:



Since these transitions always start from the ground state, absorption spectra will generally have fewer characteristic lines than emission spectra.

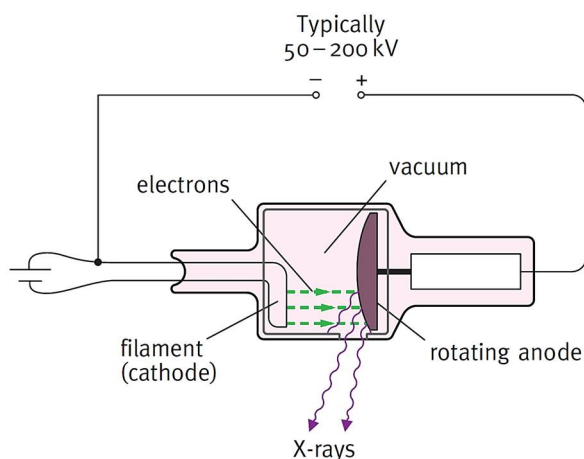
## 19.6 X-ray Spectra

X-rays are very high-energy electromagnetic radiation with wavelengths ranging from  $10^{-11}$  m to  $10^{-8}$  m. The wavelength is in the same order of interatomic spacing in crystals, making it suitable to study crystal structures.

Since X-rays are ionizing radiation, they can be very dangerous and exposure should be limited.

### 19.6.1 Production of X-rays

X-rays tubes are devices which operate on high voltage in order to generate X-ray photons. A simplified diagram of an X-ray tube is shown below.



The filament is heated up by passing electric current through it, and emits electrons. The emitted electron gains kinetic energy given by

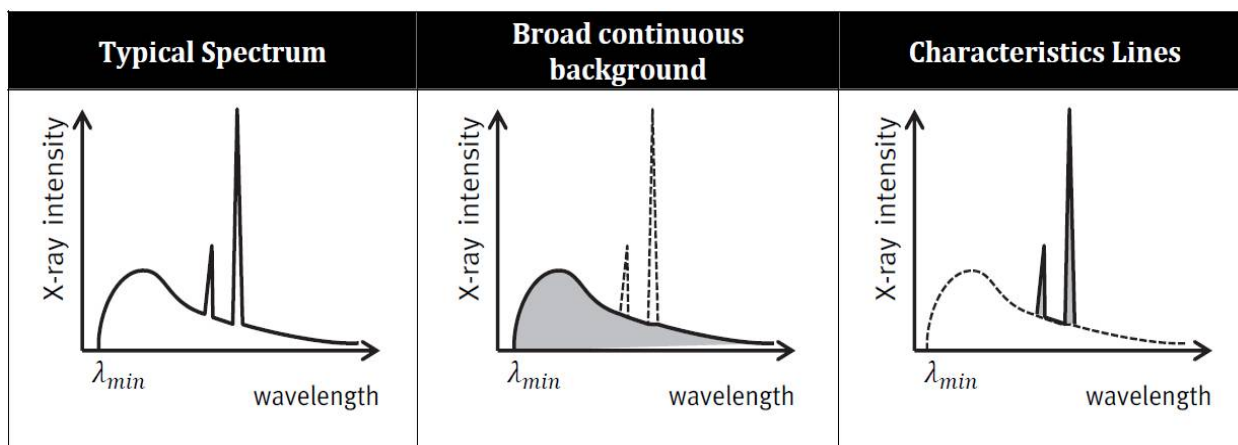
$$E = e\Delta V$$

where  $\Delta V$  is the potential difference in which the electron is accelerated across.

When the electrons that are accelerated by the high voltage collide with a metal target, the sudden deceleration of electrons due to collision causes them to be slowed down or stopped by the target. This process results in the emission of the X-ray photons.

### 19.6.2 X-ray Spectra

The X-rays have a range of energies. When represented on a spectrum, one can see that it is made up of 2 components, the broad continuous spectrum and a few sharp lines/ peaks. They arise from the different ways in which a single electron loses its energy when it collides with the metal target.

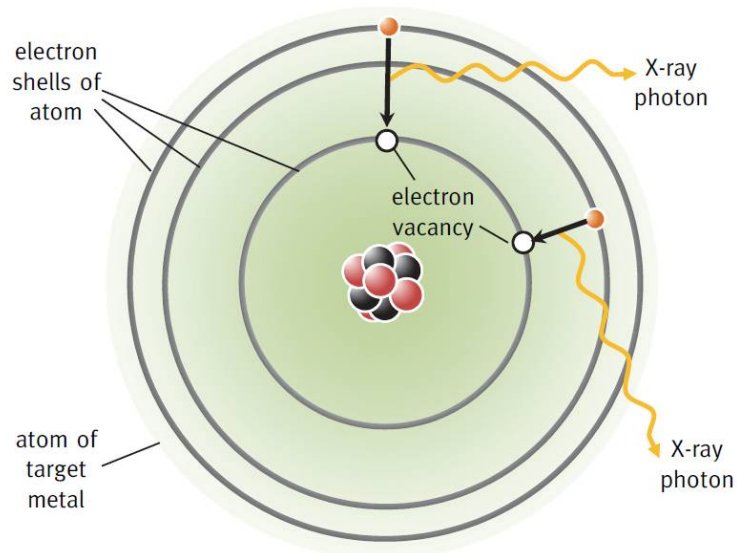


- $\lambda_{\min}$  – It is the X-ray photon with the most photon energy. It is when all the energy of one electron is given up in one collision and is converted into a single X-ray photon.
- Continuous background spectrum – Electromagnetic radiation (photons) are produced whenever a charged particle (electron) is decelerated/accelerated. High-energy electrons fired at the metal target **decelerate suddenly** upon interactions with lattice, losing energy and emitting X-ray photons. The photon frequency can be calculated via  $E_{\text{loss of electron}} = hf$ . The electrons hitting the metal target have a distribution of decelerations so a continuous spectrum is produced. This is also known as **Bremsstrahlung** radiation, which stands for 'braking radiation' in German.
- Characteristic lines – The characteristic peaks in the X-ray spectrum are associated with X-ray photons produced by inner-shell electron transitions from higher-energy electron shells to lower-energy electron shells within the atom of the target metal. This part of the process is similar to the (visible light) emission spectra of excited gases.

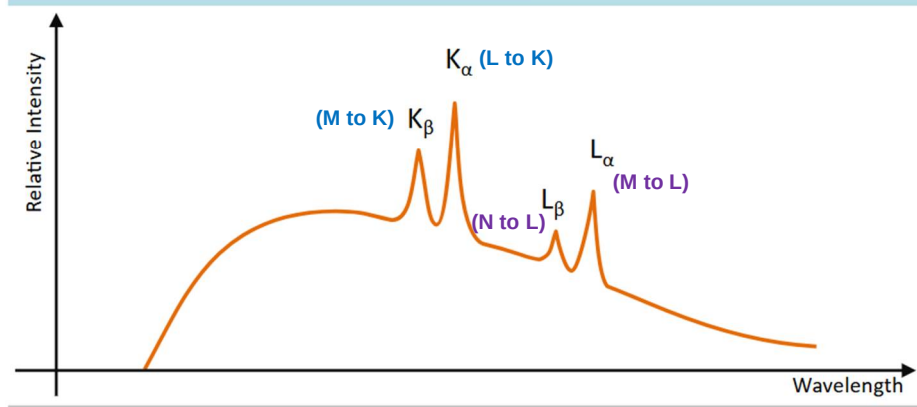
High-energy electrons fired at the metal target knock out bound inner-shell electrons, which leave electronic vacancies that allow electrons in higher-energy electron shells to transit into. The electron loses energy as a result of the electron transition, producing photons of a specific frequency corresponding to the difference in electron energy levels.

These X-ray spectral peaks have sharp/specific (characteristic) wavelengths due to the fixed-energy (quantised) transitions between electron shells (whose energy levels are discrete). The photon frequency can be calculated via  $E_2 - E_1 = hf$  where  $E_1$  is the energy of the lower-energy electron shell and  $E_2$  is the energy of the higher-energy electron shell. The peak intensities are also high because all the atoms of the metal target have the same set of inner-shell electron energy levels.

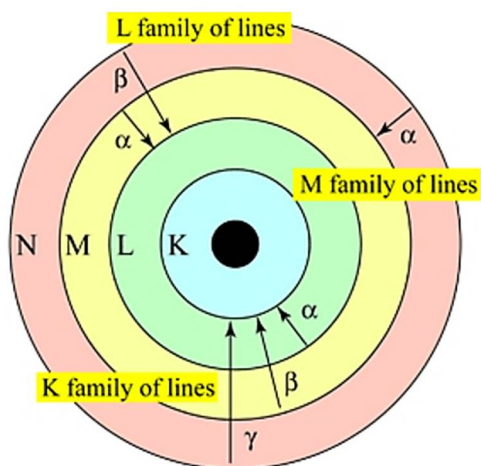
Note that the characteristic X-ray peaks produced have slightly different intensities, and this is related to the relative frequency of the associated electronic transition.



The X-ray peaks are named after the orbitals of the vacancies that electrons from the higher energy orbitals can transit to. The subscript Greek alphabets indicate which orbital the electrons from higher orbitals transit from.



Naming conventions: [Background Knowledge] – Useful for understanding but **not** required for the exam.



K, L, and M refers to the shell from which the electron was knocked out.

$\alpha$ ,  $\beta$ ,  $\gamma$  refers to the number of shells out that the orbital electron dropped to fill the vacancy.  
 $\alpha$  = dropped one shell,  
 $\beta$  = dropped two shells,  
 $\gamma$  = dropped three shells.

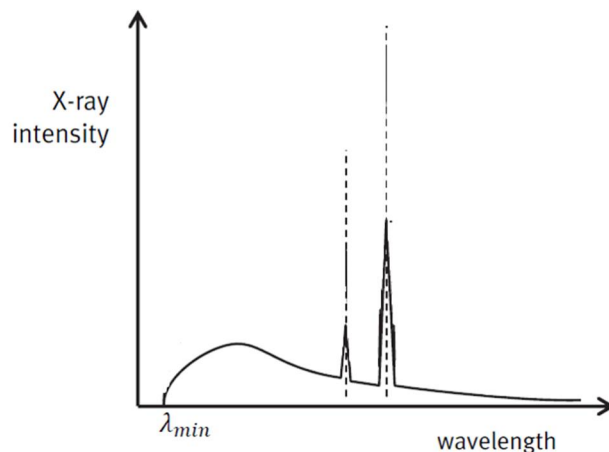
Q16: Calculate the wavelength of the most energetic X-ray photons that could be produced by a tube operating at  $1.0 \times 10^5$  V. State any assumptions with justification.

electrical energy of one bombarding electron = energy of the most energetic X-ray photon

In X-ray production, the work function energy of the target metal and the initial kinetic energy of the electrons are neglected because these are very small in comparison with the kinetic energy gained by the electrons.

Q17: An X-ray tube was operated using a high voltage power supply operating at 80 kV. The spectrum of the output X-ray is shown.

1. Determine the wavelength  $\lambda_{\min}$ .
2. Sketch the new graph if the power was supplied with greater tube current (measured by number of electrons per second striking the target)



1. electrical energy of one bombarding electron = energy of the most energetic X-ray photon

2. There will be more electrons per second with the same maximum energy hitting the metal target. The shape of the continuous spectrum would remain the same. The intensity of the continuous spectrum at all wavelengths would increase proportionally to the increase in current.  
 $\lambda_{\min}$  remains the same because it depends on the maximum energy of the electrons, which is determined by the p.d., not the current.  
 Characteristics lines remain at the same wavelengths since the target metal is the same.  
 The intensity (height) of these peaks would increase proportionally to the increase in current.

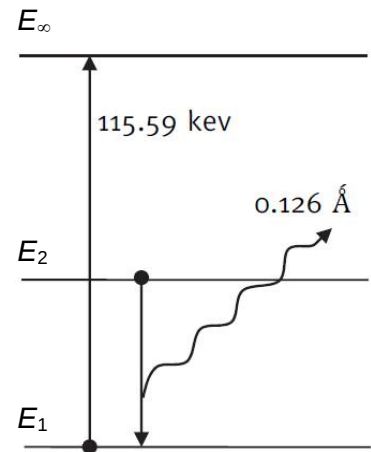
Q18: In Uranium, the energy of an incoming photon required to remove an electron from its inner most shell ( $n = 1$ ) completely from the atom is 115.59 keV. If the X-ray wavelength produced by the electron transition from  $E_2$  to  $E_1$  is  $0.126 \text{ \AA}$  ( $=0.126 \times 10^{-10} \text{ m}$ ), determine the wavelength of the incoming photon needed to remove an electron from the  $E_2$  completely.

Energy of X-ray photon emitted

$$\begin{aligned} &= hc/\lambda \\ &= (6.63 \times 10^{-34}) \times (3.00 \times 10^8) / (0.126 \times 10^{-10}) \\ &= 1.58 \times 10^{-14} \text{ J} = 98.66 \text{ keV} \end{aligned}$$

Energy of the required photon

Wavelength of the photon



## 19.7 The Heisenberg position-momentum uncertainty principle

The Heisenberg position-momentum uncertainty principle states that for a particle, the product of the uncertainties in position ( $\Delta x$ ) and momentum ( $\Delta p$ ) in the same direction cannot be <sup>†</sup>smaller than Planck constant  $h$ :

$$\Delta x \Delta p \geq h$$

This principle implies that it is impossible to simultaneously determine both the exact position and exact momentum of a particle. The more precisely one quantity is measured, the less precisely the other can be known. Heisenberg emphasized that these uncertainties are a fundamental aspect of quantum mechanics, arising from the wave-like nature of matter at the quantum scale, rather than from any limitations in measuring instruments.

<sup>†</sup> More precisely,  $\Delta x \Delta p \geq \frac{h}{4\pi}$ . [Not in Exam Syllabus]

Q19: The speed of an electron is measured to be  $5.00 \times 10^3 \text{ m s}^{-1}$  to an accuracy of 0.0020%. Find the uncertainty in determining the position of this electron.

The momentum of the electron is

$$p = mv = (9.11 \times 10^{-31} \text{ kg})(5.00 \times 10^3 \text{ m s}^{-1}) = 4.555 \times 10^{-27} \text{ kg m s}^{-1}$$

The uncertainty in  $p$  is 0.0020% of this value, hence

$$\Delta p =$$

The minimum uncertainty in position  $\Delta x$  can be calculated via the uncertainty principle,

$$\Delta x \geq$$

Q20: A bullet ( $m = 50 \text{ g}$ ) and an electron ( $m_e = 9.11 \times 10^{-31} \text{ kg}$ ) are both moving at  $300 \text{ m s}^{-1}$ . The uncertainty in the speed measurement is 0.010%. If position and speed are measured at the same time, with what accuracy can we measure the position of each object?

For the electron:

$$\Delta p =$$

$$\Delta x \geq$$

[A macroscopic and measurable distance.]

For the bullet:

$$\Delta p =$$

$$\Delta x \geq$$

{  $\sim 10^{21}$  times smaller than that of the atom! }

[Negligibly small compared to classical measurement uncertainties.]

This example clearly illustrates why quantum mechanical principles are crucial for understanding and working with microscopic particles, while classical mechanics suffices for most macroscopic objects in everyday experience.

Q21: According to the Bohr model of the hydrogen atom, the electron in the ground state moves in a circular orbit of radius  $0.529 \times 10^{-10} \text{ m}$ , and the speed of the electron in this state is  $2.2 \times 10^6 \text{ m s}^{-1}$ . Assuming that the maximum uncertainty in its radial momentum is its own momentum, estimate the minimum uncertainty in the radial position of the electron. Comment on your results.

$$\Delta p \leq mv = (9.11 \times 10^{-31} \text{ kg})(2.2 \times 10^6 \text{ m s}^{-1}) = 2.0042 \times 10^{-24} \text{ kg m s}^{-1}$$

Using the uncertainty principle, the minimum uncertainty in the radial position of the electron

$\Delta r$  is

$\Delta r =$

Compare this uncertainty with the radius of the orbit:  $\frac{\Delta r}{r} =$

The minimum uncertainty in the radial position is \_\_\_\_\_% of the orbital radius. This is a very significant uncertainty.

This result highlights a fundamental limitation of the Bohr model. The Bohr model assumes a well-defined circular orbit with a precise radius, which is inconsistent with the uncertainty principle.

Interpretation of Atomic Structure: The large uncertainty suggests that we cannot think of electrons in atoms as classical particles with well-defined positions and momenta simultaneously.

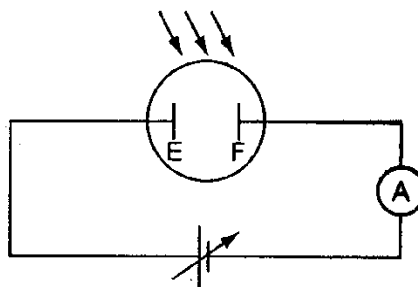
Measurement Implications: This result implies that any attempt to measure the electron's position in the hydrogen atom would significantly disturb its momentum, and vice versa, consistent with the uncertainty principle.

Topic 19 **Quantum Physics** Tutorial Questions**Photoelectric Effect**

- 1 Estimate the rate of emission of photons emitted by a 4.00 mW red laser (632 nm).  
[1.27 × 10<sup>16</sup> s<sup>-1</sup>]
- 2 Light of wavelength  $3.82 \times 10^{-7}$  m is incident on a substance and electrons are emitted with a maximum speed of  $6.87 \times 10^5$  m s<sup>-1</sup>. Calculate the work function energy of the substance.  
[N07 H2 P3 Q7c] [3.06 × 10<sup>-19</sup> J]
- 3 When light of wavelength 350 nm falls on a potassium surface, electrons are emitted that have a maximum kinetic energy of 1.31 eV. Determine
  - (a) the work function energy of potassium,
  - (b) the cut-off wavelength,
  - (c) the frequency corresponding to the cut-off wavelength.
 [3.59 × 10<sup>-19</sup> J, 554 nm, 5.42 × 10<sup>14</sup> Hz]
- 4 In an experiment on photoelectric effect, a beam of light is used and the voltage across the electrodes required to stop the photocurrent is 1.5 V.
  - (a) Determine the maximum speed of the photoelectrons.
  - (b) Sketch a graph to illustrate how the photocurrent varies with the voltage across the electrodes and label it (b).
  - (c) With reference to photoelectrons, explain the significance of the sloping section of your graph for negative values of potential difference.
  - (d) The metal is replaced by another with a greater work function energy. Add a new line to the graph above to indicate how the photocurrent now varies with the voltage and label it (d).
  - (e) With this new metal, the intensity of the light is now doubled. Add another line to the graph to indicate how the photocurrent now varies with the voltage and label it (e).
 [7.26 × 10<sup>5</sup> m s<sup>-1</sup>]
- 5 In a photoelectric emission experiment, ultraviolet radiation of wavelength 254 nm and intensity 210 W m<sup>-2</sup>, was incident on a silver surface in an evacuated tube, so that an area of 12 mm<sup>2</sup> was illuminated. A photocurrent of  $4.8 \times 10^{-10}$  A was collected at an adjacent electrode.
  - (a) What was the rate of incidence of photons on the silver surface?
  - (b) What was the rate of emission of electrons?
  - (c) The photoelectric quantum yield is defined as the ratio of  

$$\frac{\text{number of photoelectrons emitted per second}}{\text{number of photons incident per second}}$$
    - (i) Find the quantum yield of this silver surface at wavelength of 254 nm.
    - (ii) Give two reasons why this value might be expected to be much less than one.
  - (d) When the experiment was repeated with the radiation of wavelength 313 nm, no photoelectron was emitted. Explain this observation.
 [J83 P1 Q16 part] [3.22 × 10<sup>15</sup>, 3 × 10<sup>9</sup>, 9.3 × 10<sup>-7</sup>]

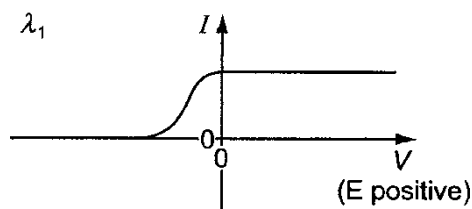
- 6 The diagram shows a circuit used for photoelectric emission experiments.



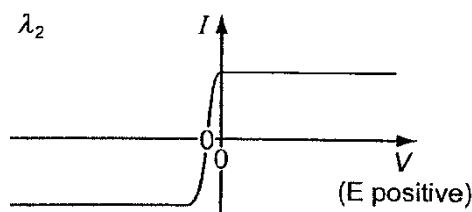
The two electrodes E and F are made of different metals. The work function energy of electrode E is  $\phi_E$  and the work function energy of electrode F is  $\phi_F$ .

Current-voltage ( $I$ - $V$ ) characteristics are obtained when both electrodes are illuminated with monochromatic light.

When the wavelength of the light is  $\lambda_1$ , the  $I$ - $V$  characteristic is as shown.



When the wavelength of the light is  $\lambda_2$  the  $I$ - $V$  characteristic is as shown.



Explain whether

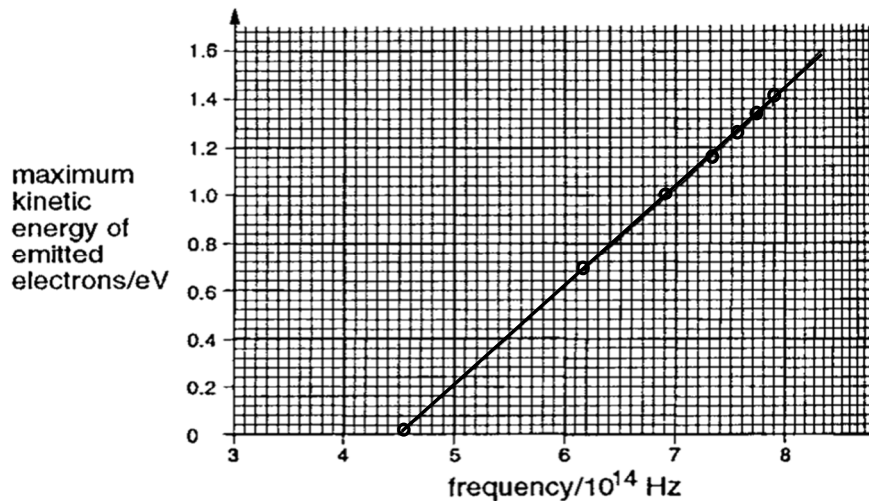
- $\lambda_2$  is greater or less than  $\lambda_1$ .
- $\phi_E$  is greater or less than  $\phi_F$ .

[N07 H2 P1 Q36 mod]

- 7 (a) In a photoelectric effect demonstration, photoelectrons are being produced at a rate of  $2.7 \times 10^{13}$  per second. Each electron has energy of  $1.2 \times 10^{-19}$  J.
- Calculate the current giving this rate of production of photoelectrons.
  - Determine the e.m.f. generated.
- (b) State, with a reason, what modifications to the apparatus of the demonstration would be required, if separately, to increase
- the energy of the photoelectron,
  - the rate of production of photoelectrons.

[N92/III/6 mod]

- 8 (a) With reference to the photoelectric effect experiment, sketch a graph to show the variation of the stopping potential with the frequency of radiation.
- (b) Explain the change in the graph if
- the metal surface is replaced by one with a greater work function energy,
  - the intensity of light increases.
- 9 (a) The graph below shows the maximum kinetic energies, in electron volts, of electrons emitted from a sodium surface by light of different frequencies from a hydrogen light source.



In calculations on electron emission, an equation which is often used is

**photon energy = work function energy + maximum kinetic energy of the emitted electrons**

With the help of the graph, determine the numerical values for these three terms in joules when the incident radiation from the hydrogen lamp has a frequency of  $6.2 \times 10^{14}$  Hz.

[ $4.11 \times 10^{-19}$ ,  $2.96 \times 10^{-19}$  J,  $1.15 \times 10^{-19}$  J]

- (b) The frequencies of light from the lamp, shown by the small circles on the graph are the frequencies obtained in this range. Explain how this shows the existence of discrete energy levels in hydrogen atoms. Without giving numerical values, sketch the pattern of these levels.

[N94/III/6 mod]

### Wave-particle duality • Energy levels in atoms • Line spectra

- 10 (a) Express the de Broglie wavelength of an electron in terms of its kinetic energy,  $E$ , the electron mass  $m$ , and the Planck constant  $h$ .
- (b) Estimate the wavelength of a beam of electrons which was accelerated through a p.d of 3600 V.

[ $2.05 \times 10^{-11}$  m]

- 11** The measured wavelength,  $\lambda_m$ , of selected lines in the hydrogen spectrum are given by empirically by

$$\frac{1}{\lambda_m} = R \left( \frac{1}{4} - \frac{1}{m^2} \right)$$

where  $R$  is a constant and has the value  $1.097 \times 10^7 \text{ m}^{-1}$  and  $m$  is an integer taking the values of 3, 4, 5, ... , etc.

- Calculate the value of the wavelength when  $m = 4$ .
- Calculate the minimum wavelength given by this equation.
- Draw a diagram showing the approximate position of the line on a horizontal axis of wavelength. Mark the value calculated in (a) and the red and violet ends of the spectrum.
- Explain why it is that although there is an infinite number of lines in this spectrum, the spectrum is nevertheless seen as a line spectrum.

[J88/II/12 (c) part] [ $4.86 \times 10^{-7} \text{ m}$ ,  $3.65 \times 10^{-7} \text{ m}$ ]

- 12** Neutrons may be used to study the atomic structure of matter. Diffraction effects are noticeable when the de Broglie wavelength of the neutrons is comparable to the spacing between the atoms. This spacing is typically  $2.6 \times 10^{-10} \text{ m}$ .

- Suggest why using neutrons may be preferable to using electrons when investigating matter.
- Calculate the speed of a neutron having a de Broglie wavelength of  $2.6 \times 10^{-10} \text{ m}$ . The mass of a neutron is  $1.7 \times 10^{-27} \text{ kg}$ .

[ $1.5 \times 10^3 \text{ m s}^{-1}$ ]

- 13** An atom is assumed to have zero energy in the ground state; and its energy in the first, second and third excited states are:  $1.635 \times 10^{-18} \text{ J}$ ,  $1.936 \times 10^{-18} \text{ J}$  and  $2.019 \times 10^{-18} \text{ J}$  respectively.

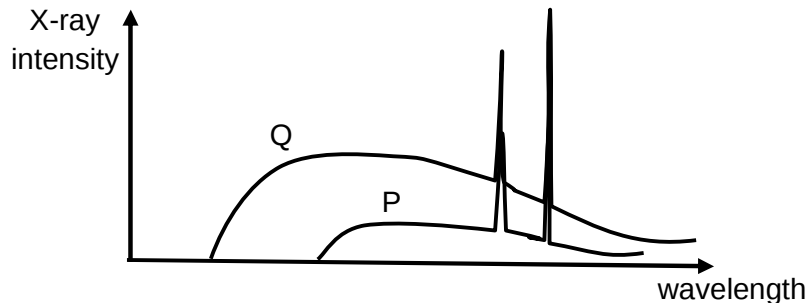
- Determine the wavelength of the photon which would excite the atom from the first excited state to the second excited state.
- A blue line of wavelength  $5.18 \times 10^{-7} \text{ m}$  is observed in the spectrum of this atom. Determine the transition between energy levels that will give rise to this spectral line.
- Suggest whether light incident on a metal surface exerts a pressure on the surface.
- The atom in the ground state is bombarded with
  - electrons of energy  $1.800 \times 10^{-18} \text{ J}$
  - photons of energy  $800 \times 10^{-18} \text{ J}$
 State whether any transition takes place in each case.

[661 nm,  $3.84 \times 10^{-19} \text{ J}$ ]

- 14**
- Describe how line spectra can be explained using the idea of discrete electron energy levels in isolated atoms.
  - State the conditions under which atoms are sufficiently isolated so that line spectra can be observed.
  - Explain the difference between how the emission line spectrum and the absorption line spectrum are produced.
  - Explain what can be deduced from observing the line spectrum.
  - Suggest a practical application of observing the line spectrum.

## X-ray Spectrum

- 15** X-ray spectra are taken from two X-ray tubes P and Q. The intensity of the X-rays is plotted against the wavelength in both cases and is shown below.



- (a) Explain whether or not the target material in both tubes is the same.  
 (b) Explain whether X-ray tube Q has the higher or lower voltage applied to it compared with X-ray tube P.
- 16** The potential difference between the target and cathode of an X-ray tube is 50 kV and the current in the tube is 20 mA. Only 1% of the total energy supplied is emitted as X-ray radiation.
- (a) Calculate the maximum frequency of emitted radiation. [ $1.2 \times 10^{19}$  Hz]  
 (b) Determine the rate at which heat must be removed from the target in order to keep its temperature steady. [990 W]

## The uncertainty principle

- 17** Suppose that the x-component of the velocity of a  $2 \times 10^{-4}$  kg mass is measured to an accuracy of  $\pm 10^{-6} \text{ m s}^{-1}$ . Determine the limit of the accuracy with which we can locate the particle along the x-axis. [ $3.3 \times 10^{-24}$  m]
- 18** A measurement establishes the position of a proton with an accuracy of  $\pm 1.50 \times 10^{-11} \text{ m}$ . Find the minimum uncertainty in the proton's position 1.00 s later. [26000 m]
- 19** What is the uncertainty in the location of a photon of wavelength 300 nm if this wavelength is known to an accuracy of one part in a million? [0.30 m]
- 20** A pulse of radio wave lasts for  $1.0 \times 10^{-5} \text{ s}$ . A photon of the radio wave may be considered to be at a point anywhere within this pulse, although the location of the point is not known. Calculate
- (a) the length of the pulse,  
 (b) the uncertainty in the position of the photon,  
 (c) the uncertainty in the momentum of the photon.

[N07 H2 P3 Q7 (d) & (e) mod]  
 [3000 m, 3000 m,  $2.2 \times 10^{-37} \text{ kg m s}^{-1}$ ]