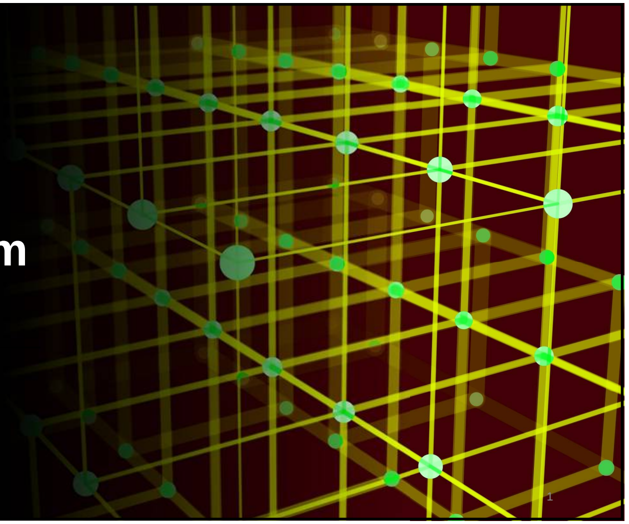
H2 Quantum Physics



Content

- Energy of a photon
- The photoelectric effect
- Wave-particle duality
- Energy levels in atoms
- Line spectra
- X-ray spectra
- The uncertainty principle

Learning Outcomes:

(a) show an appreciation of the particulate nature of electromagnetic radiation

(b) recall and use the equation $E = hf$ for the energy of a photon

(c) show an understanding that the photoelectric effect provides evidence for the particulate nature of electromagnetic radiation while phenomena such as interference and diffraction provide evidence for the wave nature

(d) recall the significance of threshold frequency

(e) recall and use the equation $\frac{1}{2} mv_{max}^2 = eV_s$, where V_s is the stopping potential

(f) explain photoelectric phenomena in terms of photon energy and work function energy

(g) explain why the stopping potential is independent of intensity whereas the photoelectric current is proportional to intensity at constant frequency

(h) recall, use and explain the significance of the equation $hf = \Phi + \frac{1}{2} mv_{max}^2$

(i) describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles

(j) recall and use the relation for the de Broglie wavelength $\lambda = h / p$

(k) show an understanding of the existence of discrete electronic energy levels in isolated atoms (e.g. atomic hydrogen) and deduce how this leads to the observation of spectral lines

(l) distinguish between emission and absorption line spectra

(m) recall and solve problems using the relation $hf = E_2 - E_1$

(n) explain the origins of the features of a typical X-ray spectrum

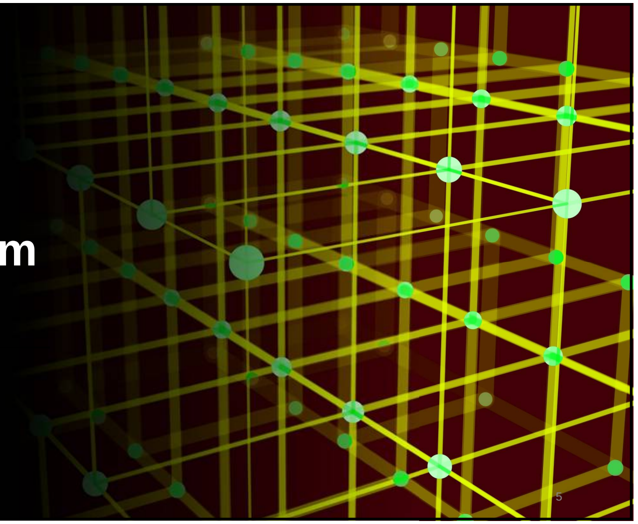
(o) show an understanding of and apply $\Delta p \Delta x \gtrsim h$ as a form of the Heisenberg position-momentum uncertainty principle to new situations or to solve related problems.

Intro: Modern Physics and Quantum Mechanics

- Developed from 1900s to explain atomic-scale phenomena
  - Energy exists in discrete packets (quanta)
  - Wave-particle duality
  - Uncertainty principle
- Changed our understanding of nature:
  - Particles are described by quantum wavefunctions (probabilistic in nature)
  - Light and matter have dual wave-particle nature
- Led to modern technologies:
  - Electronics and computers
  - Medical imaging
  - Lasers

H2 Quantum Physics

- Energy of a photon
- The photoelectric effect



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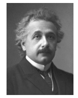
Photons

- Concept introduced by Max Planck and Albert Einstein
- Photon: A discrete packet (quantum) of energy of electromagnetic radiation
- Behaves like a particle
- Energy of a photon: $\boxed{E = hf}$

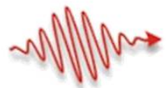
where: E = energy of photon

h = Planck's constant (6.63×10^{-34} J s)

f = frequency of radiation



photon



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Question 1

(have frequencies above 3×10^{19} Hz)

Determine the energy in joule of a high-energy gamma photon of frequency 10^{26} Hz.

Solution:

$$E = hf$$
$$= (6.63 \times 10^{-34}\ \text{J s})(10^{26}\ \text{s}^{-1})$$
$$= 6.63 \times 10^{-8}\ \text{J}$$

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Electron-volts

- Q1 shows that the energy for a high energy photon is far less than 1.0 J
- Joule is not a convenient unit for measuring photon energies
- The electronvolt (eV) is often used instead

1 eV is the energy gained by an electron when it is accelerated through a potential difference of 1 V

- Energy gained = $Q\Delta V$ = (1 e) (1 V) = (1.60×10^{-19} C) (1 V) = 1.60×10^{-19} J
- Therefore,

$$\boxed{1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}}$$

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Question 2

Visible light has wavelength spanning from 400 nm (violet) to 700 nm (red). Find, in eV, the maximum energy of a photon of visible light.

Solution:

$E_{max} = hf_{max} = hc/\lambda_{min}$

$= (6.63 \times 10^{-34})(3.00 \times 10^{8}) / (400 \times 10^{-9})$

$= 4.97 \times 10^{-19}$ J

$= 3.11$ eV

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Question 3

Determine the number of photons emitted per second by a 60 W violet light source. Wavelength of the violet light source is 400 nm.

Solution:

Power of the light source

= Total energy of light source per second

= number of photons per second × energy of a violet photon

$60 = n/t \times 4.97 \times 10^{-19}$ (from Question 2)

$n/t = 1.21 \times 10^{20}$ photons per second

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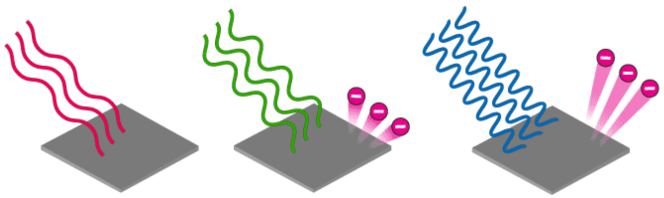
Photoelectric effect

(conduction electrons in a metal which are not attached to a particular atom)

The emission of electrons from a cold metal surface when electromagnetic radiation of a sufficiently high frequency falls on it.



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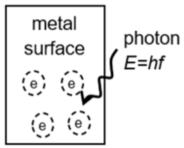
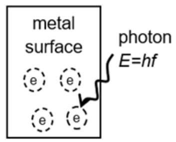
Photoelectric effect (mechanism)

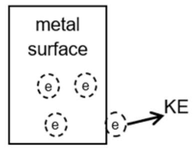


- A photon of energy hf is incident on the metal surface



- All the energy of the photon is being absorbed by the electron

Note: not all photons get to interact with electrons



- Some energy is used to overcome the forces from positive ions holding the electron in the metal, the rest is KE of the emitted electron.

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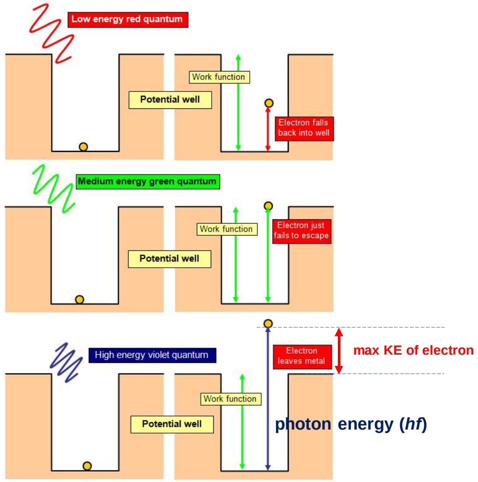
Potential well and Photoelectric Effect



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Einstein's Photoelectric Equation

Using the idea of a photon, by conservation of energy:

$$\text{energy provided by a single photon} = \text{work function energy of a metal} + \text{max KE of emitted electrons}$$

$$\boxed{hf = \Phi + \tfrac{1}{2}mv_{max}^2}$$

The work function energy Φ of a metal is the *minimum* energy of photon to cause emission of electron from surface of a metal.

- It depends on the nature of the material of the metal [e.g., Ca: 2.9 eV, Al: 4.08 eV, Fe: 4.9 eV] and its surface conditions (like contamination, which can reduce Φ).

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Einstein's Photoelectric Equation

$$\boxed{hf = \Phi + \tfrac{1}{2}mv_{max}^2}$$

- The threshold frequency f_o is the lowest frequency of electromagnetic radiation that gives rise to the ejection of electrons from the metal surface.
- The threshold wavelength λ_o is the highest wavelength of electromagnetic radiation that gives rise to the ejection of electrons from the metal surface.
- At this frequency/wavelength, the amount of energy supplied by each photon is just able to overcome the work function energy, hence KE of emitted electrons is 0.

$$\boxed{hf_o = \frac{hc}{\lambda_o} = \Phi}$$

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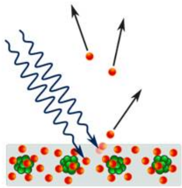
Characteristics of the photoelectric effect

- A single photon can only interact, and hence exchange its energy with a single electron (one-to-one interaction). It is extremely unlikely for any electron to simultaneously receive energy from multiple photons.
- Not all photons (of sufficient energy) get to interact with electrons. Most are reflected from metal surface.
- The photoelectrons are emitted in all random directions with varying speeds.



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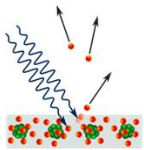
Characteristics of the photoelectric effect

When Incident Radiation Frequency < Threshold Frequency:

- Photon still transfers energy to an electron
- Electron cannot escape the metal surface
- Energy absorbed appears as electron kinetic energy
- Electrons collide with metal ions, losing kinetic energy
- Result: Metal heats up



Example: Metal plate near a table lamp gets hot

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Question 4

The maximum KE of the electrons emitted from a metallic surface is 1.0 eV when the frequency of the incident radiation is 7.5 ×10^{14} Hz. Calculate

(a) the work function energy of the metal in joules,

(b) the threshold wavelength for emission of electrons from the metal.

(a)
$$hf = \Phi + \tfrac{1}{2}mv_{max}^2$$
$$(6.63 \times 10^{-34})(7.5 \times 10^{14}) = \Phi + (1.0)(1.60 \times 10^{-19})$$
$$\Phi = 3.37 \times 10^{-19}\ \text{J}$$

(b)
$$\Phi = \frac{hc}{\lambda_o}$$
$$3.37 \times 10^{-19} = (6.63 \times 10^{-34})(3.0 \times 10^{8} / \lambda_o)$$
$$\lambda_o = 5.90 \times 10^{-7}\ \text{m}$$

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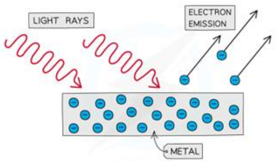
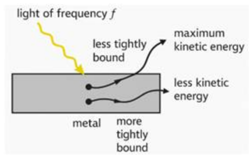
Question 5

Suggest why the emitted electrons are likely to have a range of kinetic energy (instead of just the maximum KE) for any one frequency of the electromagnetic radiation.

- Maximum KE corresponds to the electrons emitted from the metal surface
- Photon may interact with other electrons not at the surface; electrons emitted from deeper in the metal lose energy escaping
- Some electrons may collide with other particles whilst escaping; collisions reduce final kinetic energy





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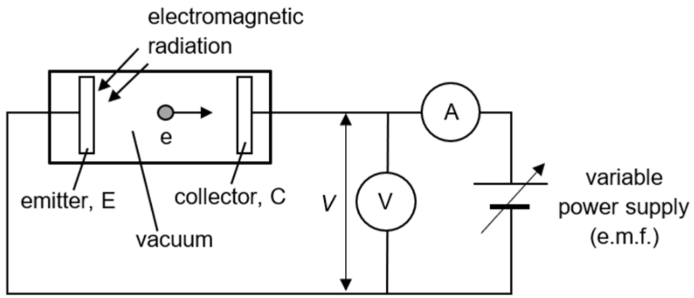
Photoelectric Effect Experimental Setup

Components:

- Evacuated glass chamber
- Emitter (E): Metal electrode (negative terminal)
- Collector (C): Metal electrode (positive terminal)
- Voltage source
- Ammeter
- Monochromatic light source



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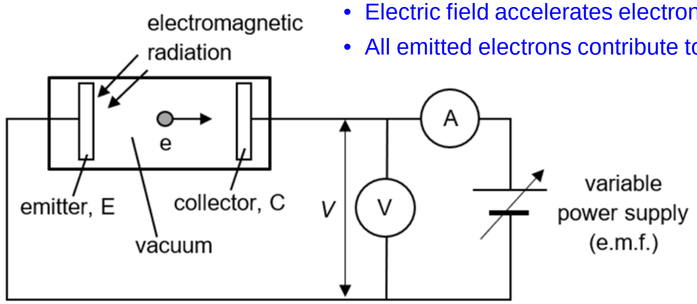
Photoelectric Effect Experimental Setup

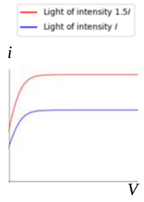
Observations:

1. In darkness: No current
2. Light on (f > threshold):
   - Photoelectrons emitted from E
   - Current detected

Key Points:

- V: Potential of collector relative to emitter
- Photoelectrons emitted with various kinetic energies
- Electric field accelerates electrons towards C
- All emitted electrons contribute to photocurrent i





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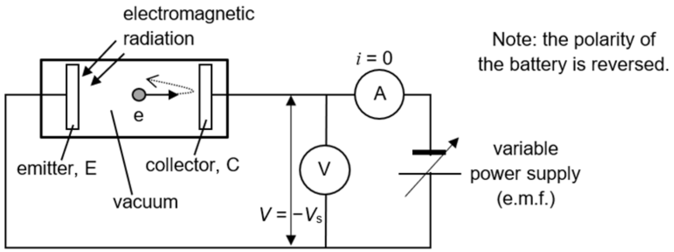
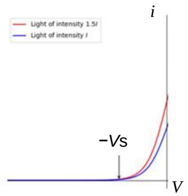
Photoelectric Effect Experimental Setup

- Reversing e.m.f. source polarity creates opposing electric field
- Adjusting field strength prevents less energetic electrons from reaching collector
- Stopping potential (V_s): Minimum reverse voltage to stop current flow
- When $V = -V_s$, current i stops
- V_s determines maximum kinetic energy of emitted electrons





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Using Stopping Potential to Find Maximum Kinetic Energy

Electron movement:

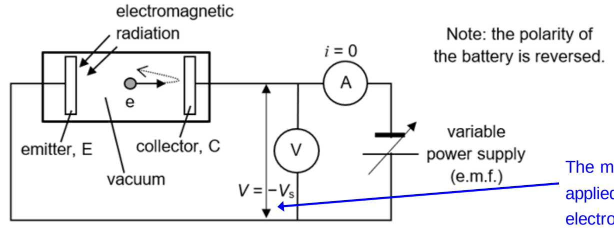
- From emitter to collector
- Potential decreases by V_s
- *EPE* of electron increases, *KE* decreases
- Most energetic electron leaves E with KE_{max} has zero *KE* at C

Energy conservation:

$$\Delta KE + \Delta EPE = 0$$

$$\Rightarrow (0 - KE_{max}) + (-e)(-V_s) = 0$$

$$\Rightarrow KE_{max} = \tfrac{1}{2}mv_{max}^2 = \underbrace{eV_s}_{\text{maximum kinetic energy with which electrons leave the emitter}}$$



The minimum potential, V_s, that is applied to stop the most energetic electrons is the **stopping potential**.

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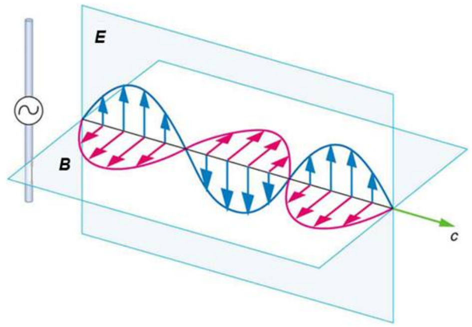
2

Classical wave theory:

Light is a propagating wave of electric (***E***) and magnetic (***B***) fields, with intensity I proportional to the square of its amplitude.

$$\left[I = \frac{1}{2}\varepsilon_0 c E_{max}^2\right]$$



A part of the electromagnetic wave sent out from the antenna at one instant in time.

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Wave Model Predictions vs Experimental Results

Wave Model Prediction	Reason	Experimental Result
1. Photocurrent independent of light frequency. Effect should occur for any frequency.	Wave intensity depends on amplitude, not frequency. $\left[I = \frac{1}{2}\varepsilon_0 c E_{max}^2\right]$	1. Photocurrent depends on light frequency. No current below threshold frequency, regardless of intensity.
2. Time delay expected for faint light before electron emission.	Electrons need time to accumulate enough energy from weak waves.	2. No measurable time delay, even with faint light (above threshold frequency).
3. Stopping potential should • Increase with light intensity • Not depend on light frequency	More intense waves deliver more energy to electrons. Frequency shouldn't affect energy transfer.	3. Stopping potential: • Independent of intensity • Increases with frequency

Classical wave theory failed to explain observed photoelectric behaviour.

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Einstein's Explanation of the Photoelectric Effect

Key Postulates:

1. Light consists of discrete packets of energy called photons or quanta
2. Energy of a single photon: $E = hf$
3. One photon is absorbed by one electron (all-or-nothing process)

Quantitative analysis:

Photoelectric equation: $KE_{max} = \frac{1}{2}mv^2_{max} = hf - \Phi$

Stopping Potential Relationship: $eV_s = hf - \Phi$

- V_s depends only on frequency, not intensity
- Explains linear relationship between V_s and f
- Allows determination of h and Φ

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How Einstein's Theory Explains Observations

1. Threshold frequency
   - electron emission occurs only when energy of each photon $hf > \Phi$
   - explains why photoelectric effect occurs only above f_o
2. Intensity effects
   - Higher intensity = more photons per second
   - Results in more electrons emitted per second (higher current)
   - Doesn't affect individual electron energies
3. No Time Delay
   - Photon absorption is instantaneous
   - Explains immediate electron emission
4. Stopping Potential: $V_s = \frac{h}{e}f - \frac{\Phi}{e}$
   - V_s independent of intensity
   - V_s increases linearly with frequency
5. Maximum KE of Emitted Electrons $KE_{max} = \frac{1}{2}mv^2_{max} = hf - \Phi$
   - Depends on photon energy (hf) minus work function (Φ)
   - Explains why electron energy increases with light frequency

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Example 6

UV light of wavelength 350 nm falls onto Caesium metal of work function 2.1 eV. Find the stopping potential (in eV) required for the ammeter to register zero current.

Solution:

$$hf = \Phi + \tfrac{1}{2}mv^2_{max}$$

$$\tfrac{1}{2}mv^2_{max} = h\frac{c}{\lambda} - \Phi$$

$$= 6.63\times10^{-34}\left(\frac{3.00\times10^{8}}{350\times10^{-9}}\right)\left(\frac{1}{1.60\times10^{-19}}\right) - 2.1$$

$$= 1.45\text{ eV}$$

$$\tfrac{1}{2}mv^2_{max} = eV_s$$

$$V_s = \frac{\frac{1}{2}mv^2_{max}}{e} = \frac{1.45\text{ eV}}{e} = \frac{e(1.45\text{ V})}{e} = 1.45\text{ V}$$

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Example 7

A metal surface is illuminated with a beam of monochromatic electromagnetic radiation. What determines the maximum speed of the photoelectrons that are emitted from the surface.

(A) The intensity of the radiation only

(B) The frequency of the radiation only

(C) The intensity and frequency of the radiation

(D) The frequency of the radiation and the work function energy of the metal

Ans: **D** $\frac{1}{2}mv^2_{max} = hf - \Phi$

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Graphical representations of the Photoelectric effect results

Based on our experimental set up, the variables that can be measured are:

- frequency f of EM radiation
- V, potential of collector relative to emitter
- photocurrent i

Two key graphs to analyse:

- photocurrent i, against V
- stopping potential, V_s, against frequency f graph

A straight line graph!

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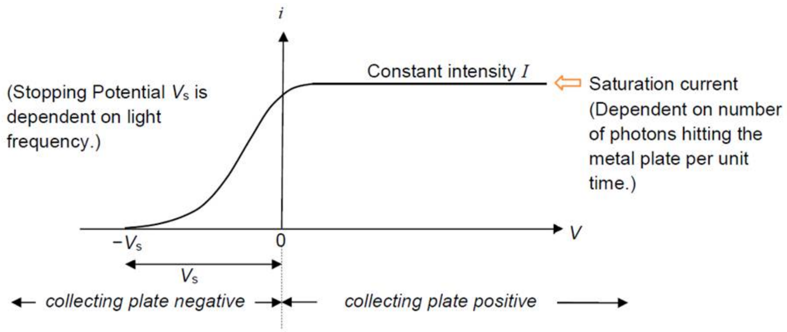
i against V graph for light of a given f



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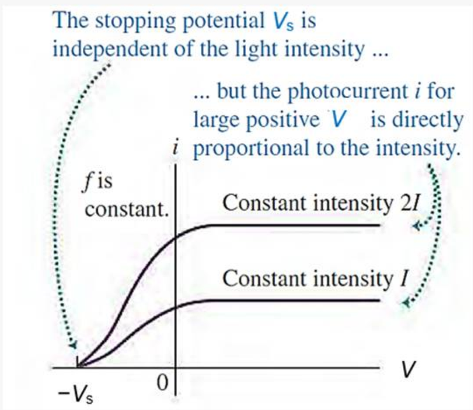

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Photocurrent (i) vs voltage (V) for two different light intensities

- Higher intensity increases photocurrent
- Stopping potential (V_s) remains constant

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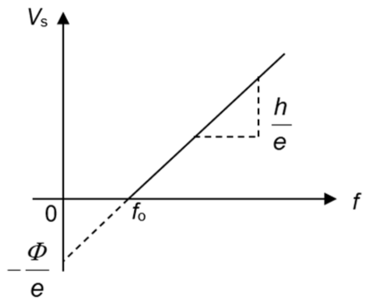
V_s against frequency f graph



From the photoelectric equation:

$$eV_s = hf - \Phi$$

$$\boxed{V_s = \frac{h}{e}f - \frac{\Phi}{e}}$$

Plotting a graph of V_s against frequency f is a linear graph with:

Gradient = $\frac{h}{e}$

Vertical intercept = $-\frac{\Phi}{e}$

Horizontal intercept = f_o

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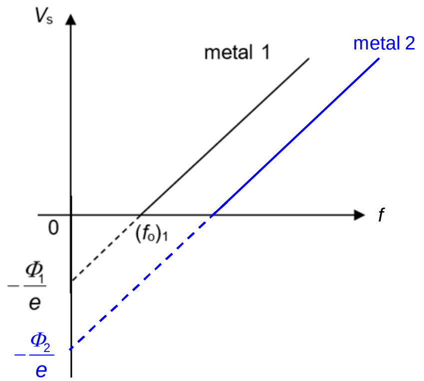
Example 8

In the figure below, draw the graph when the experiment is repeated using a metal of greater work function.



Given $\Phi_2 > \Phi_1$.

Horizontal-intercept is larger because $(f_o)_2 > (f_o)_1$ and $\Phi = hf_o$

Vertical-intercept is lower since $-\frac{\Phi_2}{e} < -\frac{\Phi_1}{e}$

Gradient is the same because $\frac{h}{e}$ is constant

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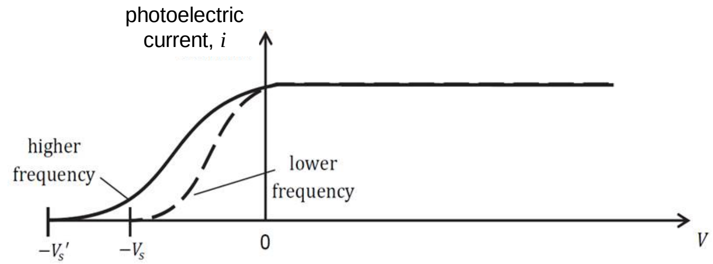
Example 9

Plot a new graph if the frequency of the light is increased but keeping the number of photons reaching the plate per second the same.



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Erratum – Page 12

Hints:

1. Consider the photoelectric equation: $hf = \Phi + eV_s$
2. Effect on stopping potential (V_s): As f increases, V_s will increase.
3. Effect on maximum photoelectric current:
   - Maximum current depends on the number of electrons emitted per second.
   - This is determined by the number of photons hitting the metal surface per second.
   - Since the number of photons per second remains constant, the maximum current will not change.
4. Shape of the new graph:
   - The overall shape will be like the original graph.
   - The saturation current (maximum current) will remain the same.
   - The curve will shift to the left due to the increased stopping potential.
5. Consider the energy of individual photons:
   - Higher frequency means each photon carries more energy ($E = hf$).
   - This results in photoelectrons being emitted with higher maximum kinetic energy.

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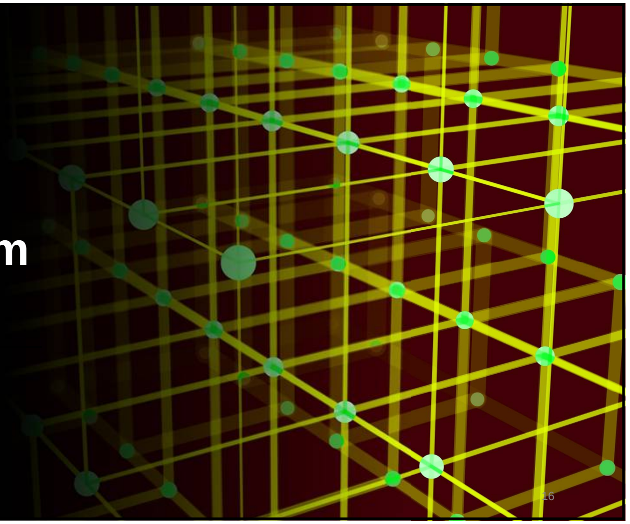
H2 Quantum Physics

- Wave-particle duality



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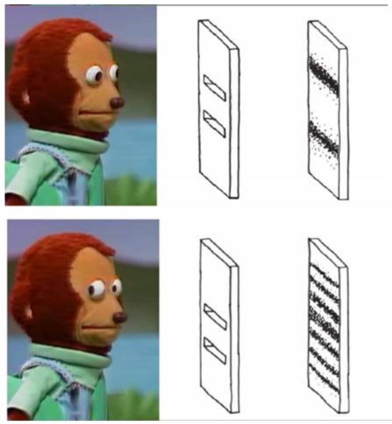
Consider this…

We have learnt earlier that:

Light behaves like a particle (photon) based on evidence from the photoelectric effect.

But, if we were to think of light as particles…

Why do we observe interference patterns when photons pass through a double slit?



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Wave Particle Duality

- Light and electromagnetic radiation exhibit both wave and particle behaviour
- Wave behaviour: Interference and diffraction
- Particle behaviour: scattering and absorption of photons
- Matter (e.g., electrons) can also exhibit wave-like properties

Key Idea: Light waves can behave like particles, and particles of matter can behave like waves.

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de Broglie Wavelength

- If a particle acts like a wave, it should have a wavelength and a frequency
- In 1923, De Broglie postulated that a free particle with rest mass m, moving with nonrelativistic speed v, should have a wavelength λ related to its momentum $p = mv$, as expressed by:

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

*The **de Broglie** wavelength of a particle*

- It relates particle properties (mass, velocity) to wave properties (wavelength)

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Wave Particle Duality

The frequency f, according to de Broglie, is also related to the particle's energy E in exactly the same way as for a photon:

$$\boxed{E = hf}$$

*The **energy** of a particle*

Similarly, for electromagnetic radiation of wavelength, λ, which exhibits particle behaviour, it will have an associated momentum, p, given by

*The **momentum** of a wave* $$p = \frac{h}{\lambda} = \frac{hf}{c} = \frac{E}{c} \qquad \left[\text{from } c = f\lambda \Rightarrow \frac{1}{\lambda} = \frac{f}{c}\right]$$

Caution: **Not all photon equations apply to particles with mass.**

Be careful when applying $E = hf$ to particles with non-zero rest mass, such as electrons. Unlike a photon, they do not travel at speed c, so the equations $E = pc$ and $f = c/\lambda$ do not apply to them!

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Wave-Particle Duality in Summary

- Both matter and light exhibit wave and particle properties
- The nature observed depends on the experimental circumstances
- Applies to photons and atomic/subatomic particles
- De Broglie wavelength connects particle and wave properties
- Energy-frequency relationship applies universally, but with caution for non-zero rest mass particles

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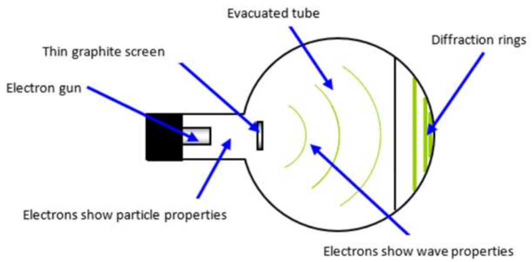
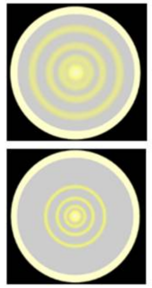
Electron-diffraction experiment





Low accelerating voltage

High accelerating voltage

- Electron gun: produces a beam of electrons with controlled energy
  - Heated cathode emits electrons; positively charged anode accelerates electrons towards it
- Thin film of crystalline graphite: acts as a diffraction medium for the electrons
- Fluorescent screen: shows the diffraction pattern as glowing rings

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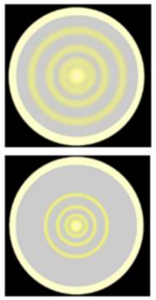
Formation of Circular Rings

- Electrons have a de Broglie wavelength: $\lambda = h/p = h/mv = h/\sqrt{2meV}$ $\left[eV = \frac{p^2}{2m} \Rightarrow p = \sqrt{2meV}\right]$ where V is the accelerating potential in the electron gun
- Graphite acts like a diffraction grating for electrons: spacing between carbon atom layers is analogous to grating spacing (d)
- Diffraction pattern: follows a principle like the grating equation $d\sin\theta = n\lambda$
- Circular rings: formed due to the random orientations of individual microscopic crystals; each ring corresponds to a specific diffraction order (n) of intensity maximum
- Effect of accelerating potential (V):
  - Higher V: Shorter λ, smaller θ, more closely spaced rings
  - Lower V: Longer λ, larger θ, more widely spaced rings



Low accelerating voltage

High accelerating voltage

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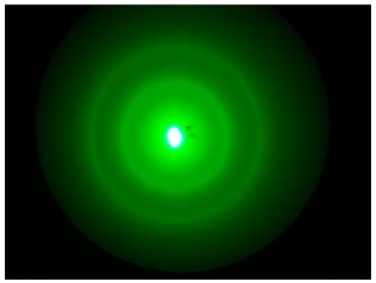
Electron Diffraction: Proof of Wave-Particle Duality

Photograph of the electron diffraction pattern for a particular V:



- Electrons, typically considered particles, exhibit wave-like behaviour
- Confirms de Broglie's hypothesis of matter waves
- Demonstrates wave-particle duality extends beyond light to matter

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Example

Electron Diffraction

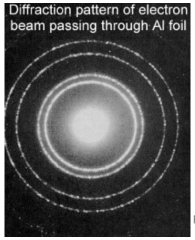
(a) Assuming that the electrons behave as **particles**, predict what would be seen on the screen.

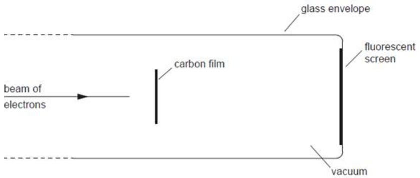
- A bright spot (intensity of spot is 'uniform' distribution)

(b) In this experiment, the electrons do **not** behave as particles.

Describe briefly the pattern that is actually observed on the screen. You may draw a sketch if you wish.

- Concentric rings



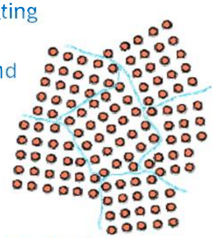


Example

Electron Diffraction

- Why the pattern of electron diffraction is a series of concentric circles?

  - A single crystal acts as a diffraction grating
  - Graphite (carbon) is polycrystalline
    ➔ has many crystals of varying size and are all randomly orientated.

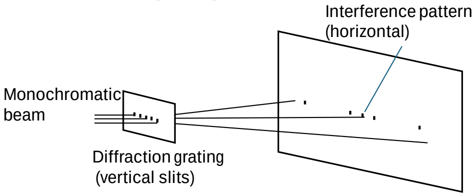


  - Dots from the individual crystal combine to produce concentric circles.

Example

Electron Diffraction

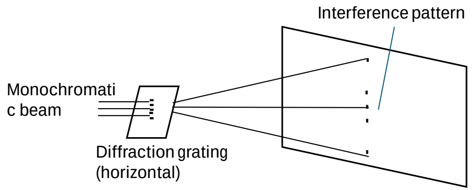
A single crystal acts as a diffraction grating



Example

Electron Diffraction

Another crystal but in vertical orientation

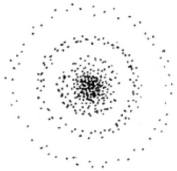


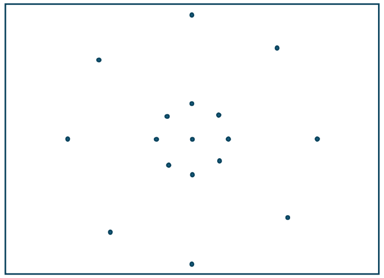
Example

Electron Diffraction

Many crystal randomly orientated
➔ many grating with random orientation

1. Vertical slits grating
2. Horizontal slits grating
3. Grating at 45°





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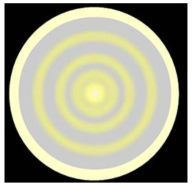
Example

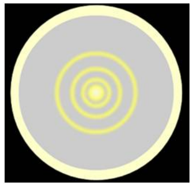
Electron Diffraction

(c) The speed of the electrons is gradually increased. (by increasing the accelerating p.d.)

State and explain what change, if any, is observed in the pattern on the screen.

- Speed increases ➔ momentum increases
- Given de-Broglie wavelength, $\lambda = h/p$
- so λ decreases
- ($d \sin\theta = n\lambda$), θ and hence diameter decreases





Low accelerating voltage (low speed)

High accelerating voltage (high speed)

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Example 10

Estimate the de Broglie wavelength associated with

a) An electron accelerated through a p.d. of 1000 V from rest

b) A teacher (mass 100 kg) running at 10 m s^{-1}

(a) conservation of energy:

$\Delta KE + \Delta EPE = 0$

$\left[\frac{1}{2}(9.11\times10^{-31})v^2 - 0\right] + (-1.60\times10^{-19})(1000-0) = 0$

$v = 1.87\times10^{7}$ m s^{-1}

matter wavelength of electron

$= \frac{h}{mv}$

$= \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(1.87\times10^{7})}$

$= 3.9\times10^{-11}$ m

The wavelength of the electron is in the X-ray region, which is in the same order as that of inter-atomic spacing in a graphite / carbon crystal. Therefore, a noticeable diffraction pattern can be observed by sending the electrons towards a thin film of carbon.

1

1

Example 10

Estimate the de Broglie wavelength associated with

a) An electron accelerated through a p.d. of 1000 V from rest

b) A teacher (mass 100 kg) running at 10 m s^{-1}

(b) matter wavelength of the teacher

$= \frac{h}{mv}$

$= \frac{6.63\times10^{-34}}{(100)(10)}$

$= 6.63\times10^{-37}$ m

Ordinary objects of much bigger masses have extremely small wavelengths. The wave-like behaviour is not observable.

2

2

Example 11

Estimate the minimum change in momentum of a photon of visible light when it is reflected normally off a plane mirror.

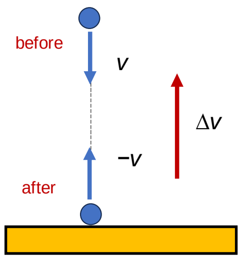
minimum momentum of visible light,

$p_{min} = \frac{h}{\lambda_{max}}$

$= \frac{h}{\lambda_{red}}$

$= \frac{6.63\times10^{-34}}{700\times10^{-9}}$

$= 9.47\times10^{-28}$ kg m s^{-1}



direction of the change in momentum:

perpendicularly away from the plane mirror

minimum change in momentum

$= (\Delta p)_{min} = 2p_{min} = 1.89\times10^{-27}$ kg m s^{-1}

3

3

Example 12

Calculate the number of photons of the light in Example 11 needed per second to accelerate a mass of 1.0 kg by 1.0 m s^{-2}.

force = number of photons per second × change in momentum of 1 photon

$ma = \left(\frac{N}{t}\right)\Delta p$

$(1.0\text{ kg})(1.0\text{ m s}^{-1}) = \left(\frac{N}{t}\right)(1.89\times10^{-27})$

$\left(\frac{N}{t}\right) = 5.29\times10^{26}$ s^{-1}

Futuristic Applications –
Solar sails use the momentum of photons from sunlight for propulsion in space.

Key Advantages: Eliminates the need for rocket propellant; Potentially unlimited range

Challenges: Large sail area needed for significant thrust; Limited effectiveness far from the Sun

The sail is stored inside a spacecraft about the size of microwave oven. Lightweight carbon fibre booms unroll from spacecraft to form rigid tubes that support the ultra thin reflective polymer sail to form a square about the size of half the tennis court. Altitude & positions are changed by angling the sail toward or away from the Sun.

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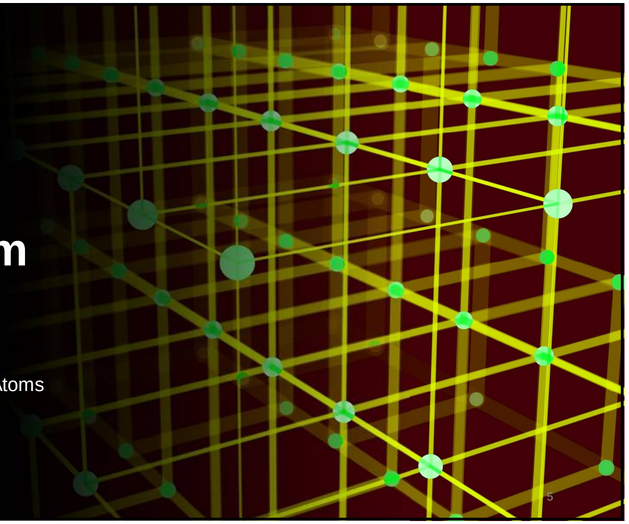
H2 Quantum Physics

- Energy Levels in Isolated Atoms



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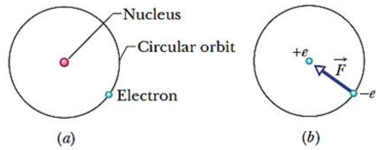
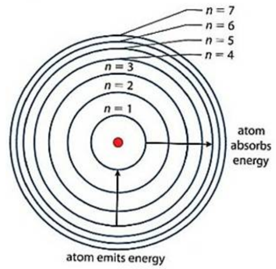
Energy Levels in Isolated Atoms

Bohr Model Features:
- Electrons in specific circular orbits
- Each orbit corresponds to an energy level
- Each level associated with a principal quantum number



Energy Levels in Hydrogen:
- *n* = 1: Ground state (lowest energy, smallest orbit)
- *n* = 2, 3, 4, etc.: Excited states (higher energy, larger orbits)



Limitations:
- Simplified model
- More complex for atoms beyond hydrogen

Key Principle:
- Discrete energy levels apply to all elements
- Characterized by principal quantum numbers

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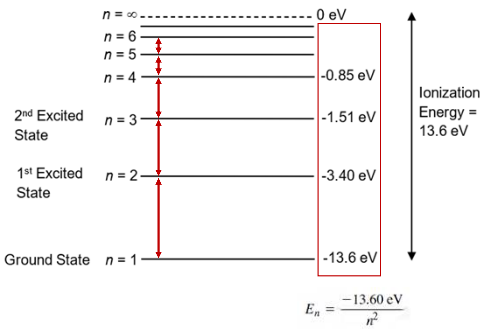
Energy Levels in Hydrogen Atom



***n* : Principal quantum number**
- Ground state is the lowest energy level in which the atom is most stable (at *n* =1)
- Highest energy level (*n* = ∞) corresponds to a state in which the electron is no longer bound to the atom (assigned energy value of 0 eV)

Notice that the energies are negative values.
- Lower *n* has more negative energy values
- Energy difference between adjacent levels get smaller as *n* increases

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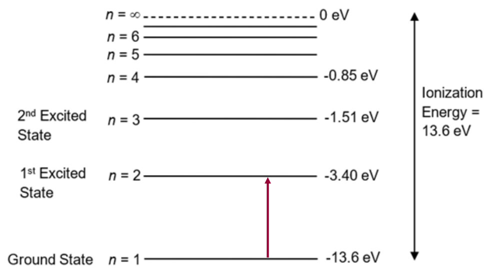
Energy Levels in Hydrogen Atom



Electrons can transit between states by either absorbing or emitting energy exactly equal to the energy difference between states.

E.g., Transition from ground state (*n* = 1) to 1st excited state (*n* = 2)

$$E_{absorbed} = E_2 - E_1$$
$$= (-3.40) - (-13.6)$$
$$= 10.2 \text{ eV}$$

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8

Common Ways to Cause Excitation of Electrons

Photon Absorption
Cool gas is exposed to electromagnetic radiation.
An electron absorbs energy from a single photon.
Photon energy must exactly match the energy difference between levels, governed by the equation: $hf = \Delta E = E_{\text{final}} - E_{\text{initial}}$ This precise requirement means that only photons of particular frequencies can excite electrons in a given atom, leading to the characteristic absorption spectra we observe.

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9

Common Ways to Cause Excitation of Electrons

Photon Absorption	High Speed Collisions
Cool gas is exposed to electromagnetic radiation.	Cool gas is bombarded with fast-moving particles (ions, atoms, or electrons).
An electron absorbs energy from a single photon.	Electron absorbs part or all of the colliding particle's kinetic energy.
Photon energy must exactly match the energy difference between levels, governed by the equation: $hf = \Delta E = E_{\text{final}} - E_{\text{initial}}$ This precise requirement means that only photons of particular frequencies can excite electrons in a given atom, leading to the characteristic absorption spectra we observe.	Kinetic energy of the colliding particle must be at least equal to the energy level difference, described by the inequality: $\frac{1}{2}mv^2_{\text{colliding particle}} \geq \lvert\Delta E\rvert$ This allows for a range of particle energies to cause excitation, as any excess energy can be distributed as kinetic energy among the colliding particles after the interaction.

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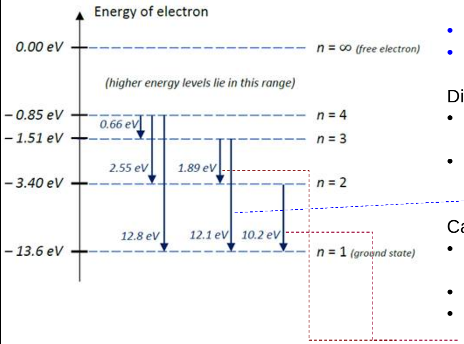
De-excitation of electrons



Two Main Pathways:
1. Direct Transition
2. Cascading Transitions

- De-excitation: Electron moves from higher to lower energy state
- Results in emission of photons
- Typically occurs within nanoseconds

Direct Transition:
- Electron drops directly to ground state in one step (e.g., n = 3 to n = 1)
- Emits single photon with energy equal to full level difference (e.g., $E_3 - E_1$ = 12.1 eV)

Cascading Transitions:
- Electron descends through intermediate levels (e.g., n = 3 to n = 2 & n = 2 to n = 1)
- Emits separate photon for each step
- Each photon's energy = difference between specific levels (e.g., $E_3 - E_2$ = 1.89 eV and $E_2 - E_1$ =10.2 eV)

11

11

Example 13

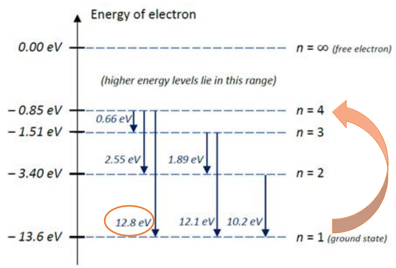
With reference to the diagram earlier, calculate the wavelength of the photon required to bring an electron of a hydrogen atom from the ground state to the n = 4 excited state.

$$E_4 - E_1 = hf = h\frac{c}{\lambda}$$

$$(-0.85)-(-13.6)\ \text{eV} = 6.63\times10^{-34}\left(\frac{3.00\times10^{8}}{\lambda}\right)$$

$$(12.75)(1.60\times10^{-19})\ \text{J} = \frac{1.99\times10^{-25}}{\lambda}$$

$$\lambda = 97.5\ \text{nm}$$



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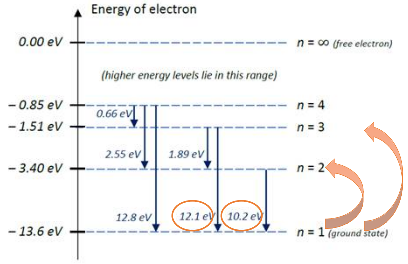
Example 14

An electron with 12.5 eV of kinetic energy is incident on an unexcited electron in a hydrogen atom. Determine the remnant kinetic energy, in joule, of the colliding electron.

Incident electron has enough energy to bring an unexcited electron from ground state to n = 3 state or n = 2 state.

$$KE_{remnant} = KE_{initial} - (E_3 - E_1) = 12.5 - [-1.51 - (-13.6)] = 0.41 \text{ eV} = 6.56 \times 10^{-20} \text{ J}$$

$$KE_{remnant} = KE_{initial} - (E_2 - E_1) = 12.5 - [-3.40 - (-13.6)] = 2.3 \text{ eV} = 3.68 \times 10^{-19} \text{ J}$$



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Example 15

State how many possible transitions are there when a hydrogen atom in its n = 4 state de-excites to the ground state.

The possible transitions are:

$4 \rightarrow 1$ $\quad 3 \rightarrow 2 \quad 2 \rightarrow 1$

$4 \rightarrow 2$ $\quad 3 \rightarrow 1$

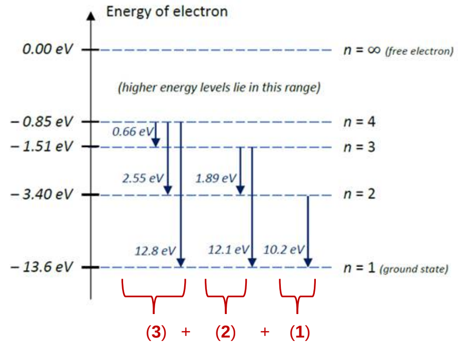
$4 \rightarrow 3$

(3) + **(2)** + **(1)** = **6**

No. of possible transitions = $^4C_2$ = **6**

number of possible transitions from state n

$$= \sum_{i=1}^{(n-1)} i = 1 + 2 + 3 + ... + (n-1) = \frac{(n-1)n}{2}$$



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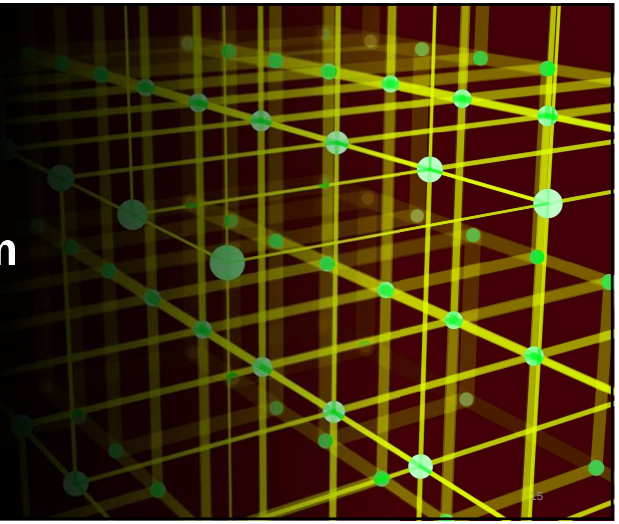
H2 Quantum Physics

- Line Spectra



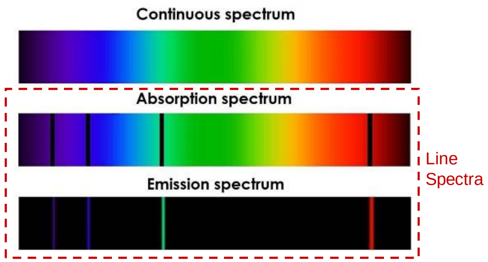
15

Line Spectra

Line spectra are distinctive patterns of light emitted or absorbed by atoms, appearing as discrete lines rather than a continuous spectrum.

These spectra are unique to each element, serving as atomic 'fingerprints'.

They provide evidence for the existence of discrete energy levels within atoms, as the specific wavelengths observed correspond to electrons transitioning between these levels.



The energy of the photon emitted/absorbed is equal to the energy difference between the energy levels:

$$|\Delta E| = hf = hc/\lambda$$

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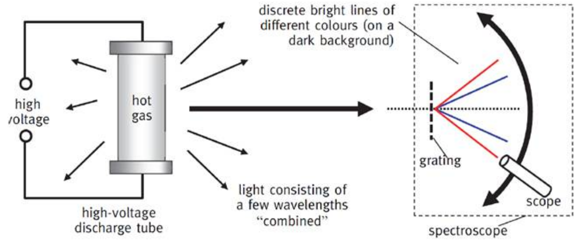
Emission Line Spectra



- Low pressure gas atoms are excited by fast-moving electrons accelerated by high voltage. The excited state is unstable, and the electron will return back into its ground state.
- When electrons transit down from higher energy states to lower energy states, photons with specific frequency/wavelength will be **emitted** such that the energy of the photon is equal to the energy difference between the energy levels.
- When passing through a diffraction grating, the photons get diffracted at specific angles according to their wavelengths (from $d \sin\theta = n\lambda$). This gives rise to the set of characteristic spectral lines.

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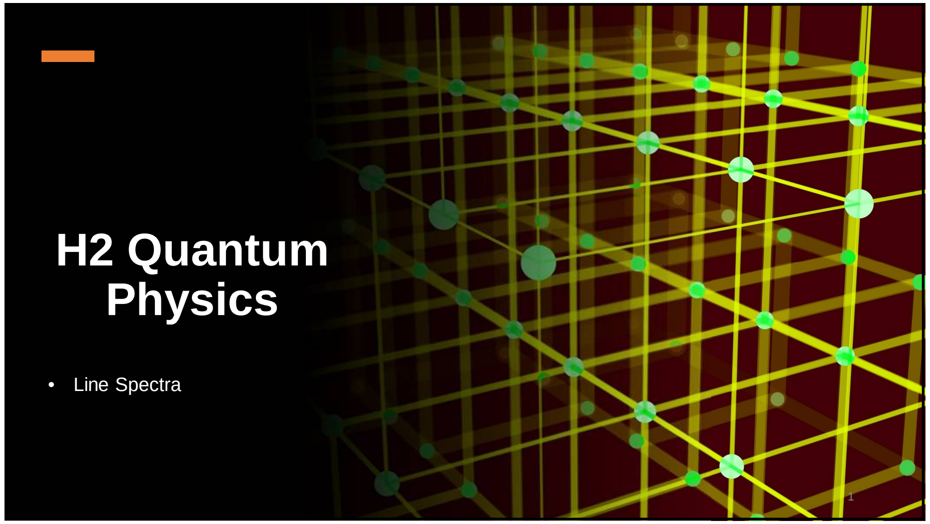
H2 Quantum Physics

- Line Spectra



1

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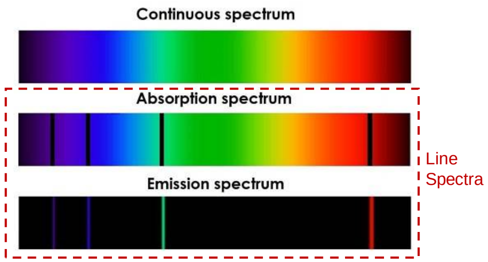

The energy of the photon emitted/absorbed is equal to the energy difference between the energy levels:

$$|\Delta E| = hf = hc/\lambda$$

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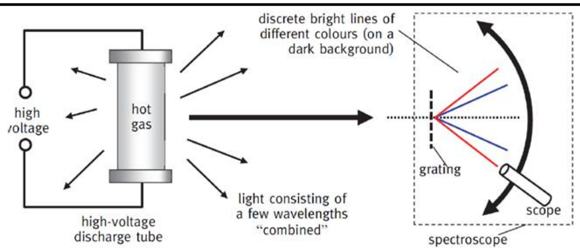
Emission Line Spectra



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- When passing through a diffraction grating, the photons get diffracted at specific angles according to their wavelengths (from $d \sin\theta = n\lambda$). This gives rise to the set of characteristic spectral lines.

3

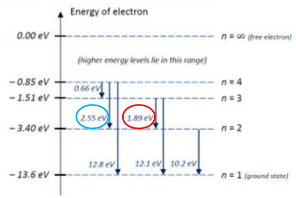
Emission Line Spectra

$$E = hf = hc/\lambda$$

Transition	Energy in eV	Wavelength	Spectral line
$n = 4 \rightarrow n = 3$	0.66 eV = 1.06 × 10^{-19} J	1870 nm	Infrared
$n = 3 \rightarrow n = 2$	1.89 eV = 3.03 × 10^{-19} J	652 nm	Red
$n = 4 \rightarrow n = 2$	2.55 eV = 4.09 × 10^{-19} J	483 nm	Cyan
$n = 2 \rightarrow n = 1$	10.2 eV = 16.3 × 10^{-19} J	121 nm	Ultraviolet
$n = 3 \rightarrow n = 1$	12.1 eV = 19.4 × 10^{-19} J	102 nm	Ultraviolet
$n = 4 \rightarrow n = 1$	12.8 eV = 20.5 × 10^{-19} J	96.3 nm	Ultraviolet

If the light from an excited hydrogen gas is passed through a diffraction grating, we can expect to see the following characteristic spectrum:

Balmer series (within the visible spectrum)



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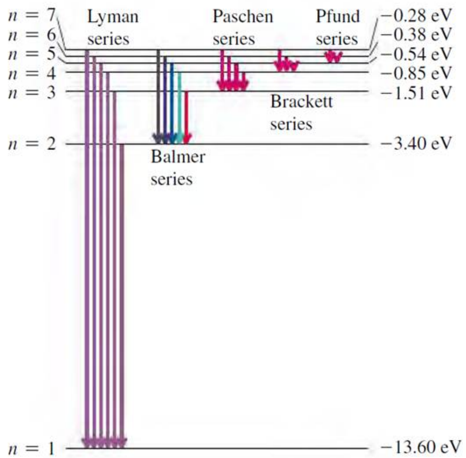
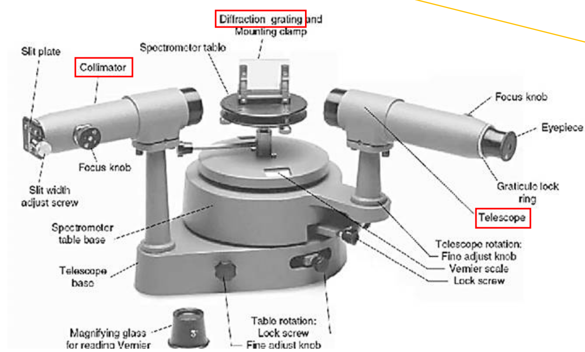
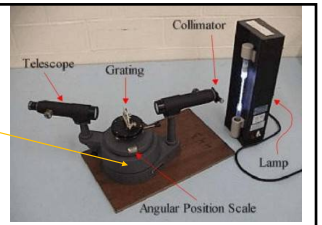
Energy-level diagram for hydrogen, showing some transitions corresponding to the various series:



Page 19 Lecture Notes

Spectrometer

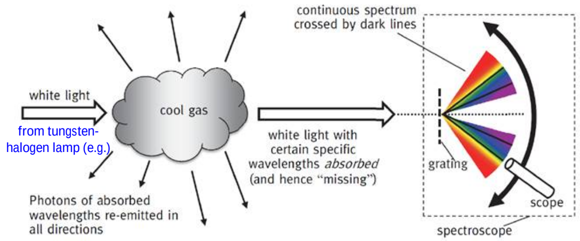




Apparatus

- By measuring the exact angle at which each wavelength is diffracted to, the wavelength and thus energy can be calculated.
- By comparing the wavelengths or energy levels to known records, the substance emitting light can be identified.

Absorption Line Spectra

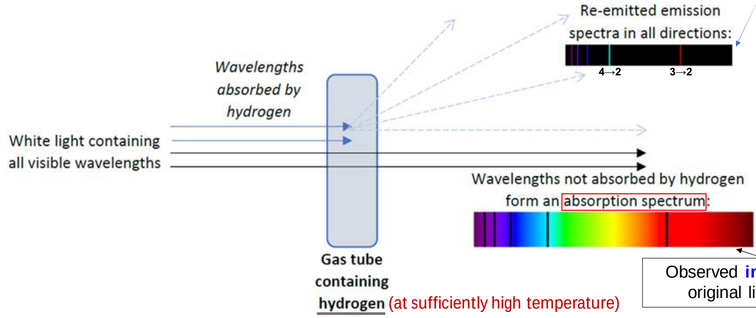


- White light with a continuous range of wavelengths are passed through a cool gas to excite the gas from the ground state.
- For the electrons to be excited to its excited state, photons with specific frequency/wavelength will be **absorbed** such that the energy of the photon is equal to the energy difference between the ground state to its respective excited state.
- When the electrons are de-excited, they get emitted in random directions, causing them to be "missing" from the straight through direction. Therefore, this is observed as black lines on the spectrum.

Absorption Line Spectra

Importantly, this emission of light by the excited gas creates its own emission spectrum, as shown below.

Observed from the **side** of the gas sample: these emission lines correspond to the energy transitions of the electrons as they return to lower energy states.



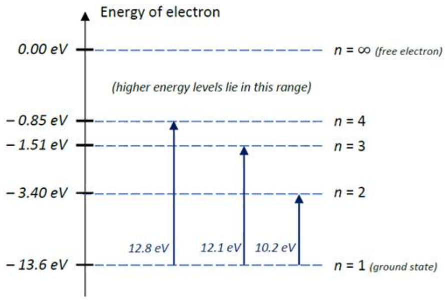
Observed **in line with** the original light source.

Absorption Line Spectra

The (UV) dark lines in the absorption spectra correspond to the following transitions:

Since these transitions always start from the ground state (if the gas were cool), absorption spectra will generally have **fewer** characteristic lines than emission spectra.

Note that if the hydrogen gas were at a sufficiently high temperature, collisions would excite atoms into higher levels. Such excited atoms can absorb photons (e.g., from n = 2 to n = 3), and other absorption lines would appear in the spectrum. Thus, the observed absorption spectrum of a given substance depends on its energy levels and its temperature.



Transitions corresponding to the absorption spectra of **cool** hydrogen.

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Requirements for Line Spectra to be observed

The atoms must be sufficiently isolated so that each atom has discrete energy levels (i.e. atoms are far apart from each other)

- This is best achieved when the emitter/absorber is in the gaseous state so that atoms are far apart from each other (compared to solids and liquids)
- The gas should also be at low pressures as the average spacing between particles of low pressure gases is higher.

$$pV = NkT \Longrightarrow p = \left(\frac{N}{V}\right)kT$$

Hence, lower pressure p reduces the number density $\left(\frac{N}{V}\right)$ of gas atoms, which means more spacing between gaseous atoms.

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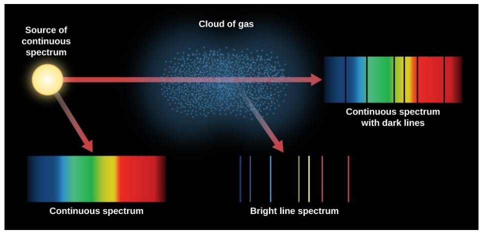
Absorption VS Emission Spectrum



Absorption:

- Black lines on a continuous spectrum (transitions from ground state to an excited state give rise to absorption)

Emission Spectrum:

- Coloured bright lines on a dark background
- Typically has more lines than absorption spectrum (due to possible transitions from one excited state to a less excited state such as n = 3 to n = 2)

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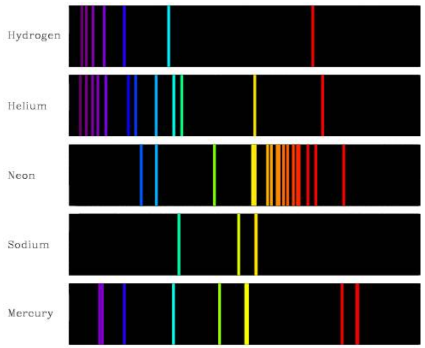
Applications of Line Spectra



- Each element has a unique profile of discrete energy levels.
- Hence, the line spectra can be used to identify elements as the line spectra of each element is unique

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X-rays

X-ray Spectra

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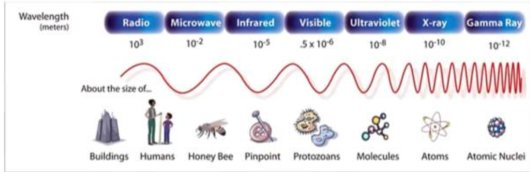
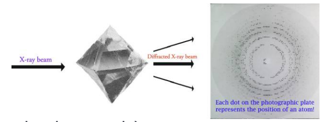
X-rays: High-Energy Electromagnetic Radiation

X-rays are very high-energy electromagnetic radiation with wavelengths ranging from 0.01 nm to 10 nm (10^{-11} m to 10^{-8} m).





- Applications:
  - Study of crystal structures (as λ comparable to interatomic spacing in crystals)
  - Medical Imaging: Diagnostic X-rays, CT scans
- Safety Considerations:
  - A form of ionizing radiation - hazardous to living tissues
  - Exposure should be carefully limited to minimize potential health risks

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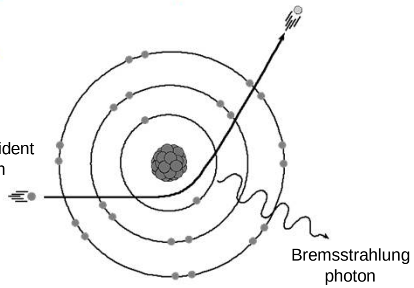
Bremsstrahlung (Braking Radiation)

It is the electromagnetic radiation produced by the deceleration of a charged particle when deflected by another charged particle, typically an electron by an atomic nucleus.



The moving charged particle loses kinetic energy during deflection. By conservation of energy, kinetic energy lost by the moving charged particle is converted into electromagnetic radiation (photons).

Bremsstrahlung produces a continuous spectrum of radiation and is a common mechanism for generating X-rays in X-ray tubes.

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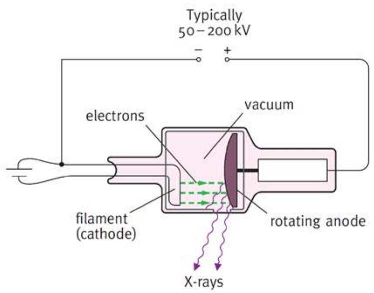
X-ray Production



- The filament is heated up via the use of electric currents and emits electrons. (Thermionic Emission)
- The electrons are accelerated by a high voltage and gain kinetic energy:

$$\boxed{E = e\Delta V}$$

- When the electrons collide with a metal target, the sudden deceleration of electrons due to the collision causes them to be slowed down or stopped by the target
- Electrons lose kinetic energy, which is converted into X-ray photons (Bremsstrahlung)

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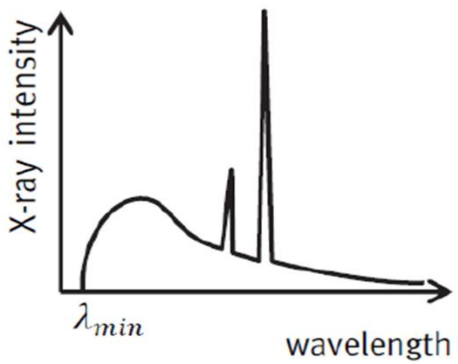
Typical X-ray Spectrum

The X-rays have a range of energies.

When represented on a spectrum, one can see that it is made up of 2 components, the broad continuous spectrum and a few sharp characteristic lines / peaks.

They arise from the different ways in which a single electron loses its energy when it collides with the metal target.



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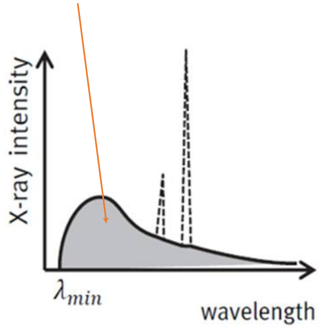
X-ray Spectrum: Broad Continuous Background

- Photons are produced whenever a charged particle (e.g. electron) is accelerated / decelerated (Bremsstrahlung Radiation)
- Energy lost by the electrons = Energy of X-ray Bremsstrahlung photon emitted

$$\boxed{E_{\text{loss of electron}} = hf}$$

- When electrons hit the metal target, they are decelerated by different extents, so a continuous spectrum is produced (as electrons have a distribution of decelerations)
- The maximum energy X-ray photon produced with wavelength λ_{min} is produced when an electron gives up all its energy in one collision



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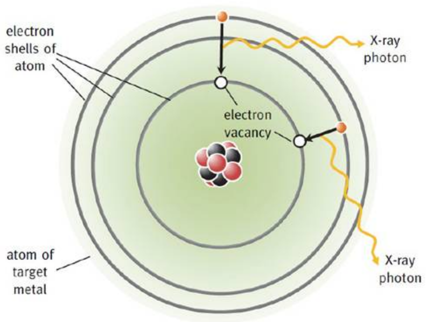
X-ray Spectrum: Characteristic Lines

They are specific peaks in the X-ray spectrum produced by electron transitions between atomic energy levels in the target metal.

Process:

- High-energy electrons knock out inner-shell electrons
- Vacancies created in inner shells
- Electrons from higher energy shells fill these vacancies
- Energy difference emitted as X-ray photons

Similar to visible light emission spectra, but involving higher energy transitions.



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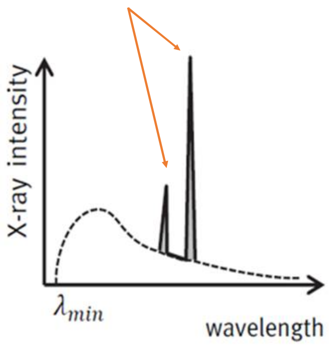
X-ray Spectrum: features of Characteristic X-ray Lines

- Sharp, Specific Wavelengths:
  - Due to quantized energy levels in atoms
  - Each transition produces a specific frequency
- High Intensity Peaks:
  - All atoms in target have same energy levels
  - Many similar transitions occur simultaneously
- Frequency Calculation: $E_2 - E_1 = hf$
  - E_2 = energy of higher electron shell
  - E_1 = energy of lower electron shell
- Varying Peak Intensities:
  - Related to probability of specific transitions



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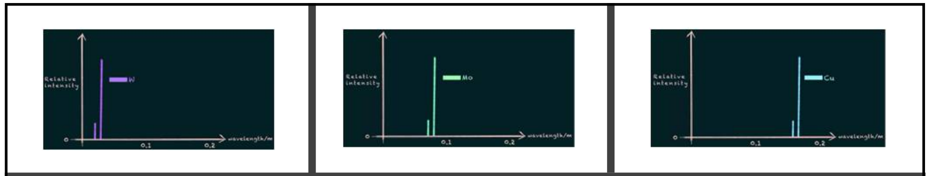
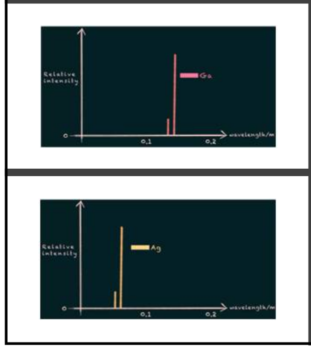
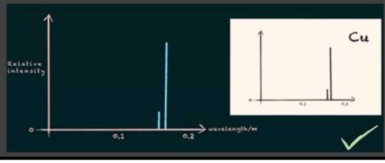



Applications of X-ray Characteristic Lines

X-ray Spectroscopy: Unveiling Material Composition

Each element produces unique X-ray spectral lines, serving as a 'fingerprint' as wavelengths of these spikes depend on the atom's permissible energy levels. Hence, we would be able to recognize a metal through the spikes we see on the graph.



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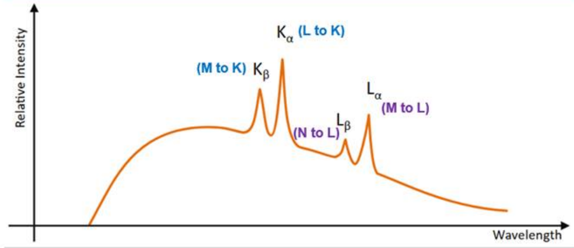
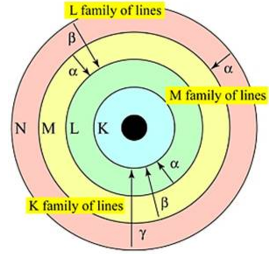
Naming conventions: [Background Knowledge] – Useful for understanding but **not required** for the exam.

The X-ray peaks are named after the orbitals of the vacancies that electrons from the higher energy orbitals can transit to. The subscript Greek alphabets indicate which orbital the electrons from higher orbitals transit from.





K, L, and M refers to the shell from which the electron was knocked out.

α, β, γ refers to the number of shells out that the orbital electron dropped to fill the vacancy.
α = dropped one shell,
β = dropped two shells,
γ = dropped three shells.

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Example 16

Calculate the wavelength of the most energetic X-ray photons that could be produced by a tube operating at 1.0×10^5 V. State any assumptions with justification.

electrical energy of one bombarding electron = energy of the most energetic X-ray photon

$$eV = \frac{hc}{\lambda_{min}}$$

$$1.60 \times 10^{-19} \times 1.0 \times 10^{5} = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{\lambda_{min}}$$

$$\lambda_{min} = 1.2 \times 10^{-11}\ \text{m}$$

- In X-ray production, the initial kinetic energy of the electrons is neglected because these are very small in comparison with the kinetic energy gained by the electrons.
- Relativistic effects are neglected for a first approximation.

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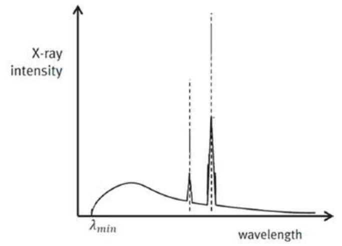
Example 17

An X-ray tube was operated using a high voltage power supply operating at 80 kV. The spectrum of the output X-ray is shown.

1. Determine the wavelength λ_{min}.
2. Sketch the new graph if the power was supplied with greater tube current (measured by number of electrons per second striking the target)

1. electrical energy of one bombarding electron = energy of the most energetic X-ray photon



$$eV = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{eV}$$

$$= \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{(1.60 \times 10^{-19})(80 \times 10^{3})}$$

$$= 1.55 \times 10^{-11} \text{ m}$$

2

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Example 17

An X-ray tube was operated using a high voltage power supply operating at 80 kV. The spectrum of the output X-ray is shown.

1. Determine the wavelength λ_{min}.
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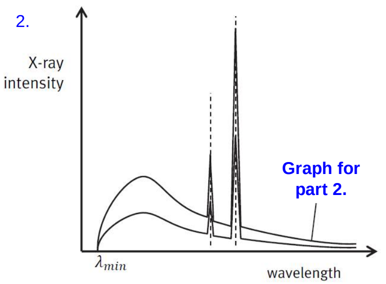
2.



Notes:

- Same target metal means that the characteristic lines remain the same wavelengths. The height of these peaks would increase proportionally to the increase in current.
- Power supply operates at the same potential difference of 80 kV means that λ_{min} remains the same.
- Greater tube current increases the number of electrons hitting the metal target per unit time. More X-ray photons are produced (higher intensity) for the same range of wavelengths. The shape of the continuous spectrum would remain the same.

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Example 18

In Uranium, the energy of an incoming photon required to remove an electron from its inner most shell (n = 1) completely from the atom is 115.59 keV. If the X-ray wavelength produced by the electron transition from E_2 to E_1 is 0.126 (= 0.126×10^{-10} m), determine the wavelength of the incoming photon needed to remove an electron from the E_2 completely.

Energy of X-ray photon emitted

= hc / λ

= $[(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})] / (0.126 \times 10^{-10})$

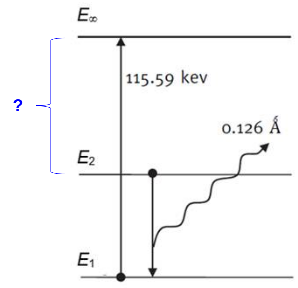
= 1.58×10^{-14} J = 98.66 keV

Energy of the required photon

= $E_\infty - E_2$

= 115.59 – 98.66

= 16.93 keV



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Energy of the required photon

$= E_\infty - E_2$

= 115.59 – 98.66

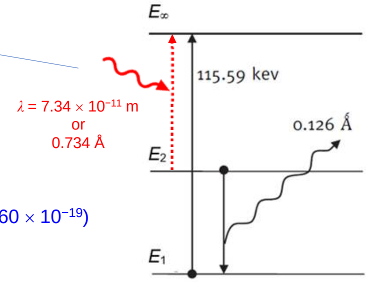
= 16.93 keV

Wavelength of the photon

$\lambda = hc$ / (Energy of required photon)

$= [(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})] / (16.93 \times 10^{3} \times 1.60 \times 10^{-19})$

$= 7.34 \times 10^{-11}$ m



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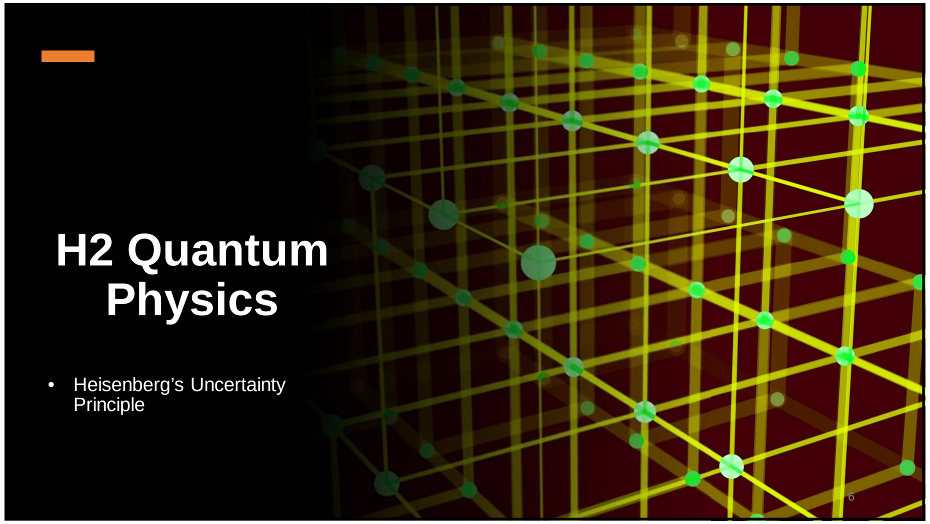

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The Heisenberg position-momentum uncertainty principle

Quantum mechanics shows that we can't know everything about a particle with perfect accuracy. In 1927, Werner Heisenberg introduced the uncertainty principle, which says that there are limits to how exactly we can measure things like a particle's position and speed at the same time.

The Heisenberg position-momentum uncertainty principle states that for a particle, the product of the uncertainties in position (Δx) and momentum (Δp) in the same direction cannot be smaller than Planck constant h:

$$\boxed{\Delta x \Delta p \geq h}$$

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The Heisenberg position-momentum uncertainty principle

Suppose we design an experiment to determine simultaneously the position and momentum of a particle. This principle says that it is impossible to simultaneously determine both the exact position and exact momentum of a particle. The more precisely one quantity is measured, the less precisely the other can be known.

Heisenberg emphasized that these uncertainties are a fundamental aspect of quantum mechanics, arising from the wave-like nature of matter at the quantum scale, rather than from any limitations/imperfections in the measuring instruments.

In practice, the quantum uncertainties for large objects are so small that Newtonian mechanics can ignore them.

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Example 19

The speed of an electron is measured to be 5.00×10^3 m s^{-1} to an accuracy of 0.0020%. Find the uncertainty in determining the position of this electron.

The momentum of the electron is

$$p = mv = \left(9.11\times10^{-31}\text{kg}\right)\left(5.00\times10^{3}\text{m s}^{-1}\right) = 4.555\times10^{-27}\text{ kg m s}^{-1}$$

The uncertainty in p is 0.0020% of this value, hence

$$\Delta p = 0.000020\times\left(4.555\times10^{-27}\text{ kg m s}^{-1}\right) = 9.11\times10^{-32}\text{ kg m s}^{-1}$$

The minimum uncertainty in position Δx can be calculated via the uncertainty principle,

$$\Delta x \geq \frac{h}{\Delta p} = \frac{6.63\times10^{-34}\text{J s}}{\left(9.11\times10^{-32}\text{ kg m s}^{-1}\right)} = 7.3\times10^{-3}\text{m} \quad \text{(2 s.f.)}$$

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Example 20

A bullet (m = 50 g) and an electron (m_e = 9.11 $\times 10^{-31}$ kg) are both moving at 300 m s^{-1}. The uncertainty in the speed measurement is 0.010%. If position and speed are measured at the same time, with what accuracy can we measure the position of each object?

For the electron:

$\Delta p = m\Delta v = 9.11\times10^{-31}\times0.00010\times300 = 2.733\times10^{-32}$ kg m s^{-1}

$\Delta x \geq h / (\Delta p) = 6.63\times10^{-34} / (2.733\times10^{-32}) = 2.4\times10^{-2}$ m [A macroscopic and measurable distance.]

For the bullet:

$\Delta p = m\Delta v = 50\times10^{-3}\times0.00010\times300 = 1.50\times10^{-3}$ kg m s^{-1}

$\Delta x \geq h / (\Delta p) = 6.63\times10^{-34} / (1.50\times10^{-3}) = 4.4\times10^{-31}$ m

{ ~ 10^{21} times smaller than that of the atom! }
[Negligibly small compared to classical measurement uncertainties.]

It illustrates why quantum mechanical principles are crucial for understanding and working with microscopic particles, while classical mechanics suffices for most macroscopic objects in everyday experience.

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Example 21

According to the Bohr model of the hydrogen atom, the electron in the ground state moves in a circular orbit of radius 0.529×10^{-10} m, and the speed of the electron in this state is 2.2×10^6 m s^{-1}. Assuming that the maximum uncertainty in its radial momentum is its own momentum, estimate the minimum uncertainty in the radial position of the electron. Comment on your results.

$$\Delta p \leq mv = \left(9.11\times10^{-31}\text{ kg}\right)\left(2.2\times10^{6}\text{ m s}^{-1}\right) = 2.0042\times10^{-24}\text{ kg m s}^{-1}$$

Using the uncertainty principle, the minimum uncertainty in the radial position of the electron is

$$\Delta r = \frac{h}{\Delta p} = \frac{6.63\times10^{-34}\text{ J s}}{2.0042\times10^{-24}\text{ kg m s}^{-1}} = 3.3\times10^{-10}\text{ m}$$

Compare this uncertainty with the radius of the orbit: $\dfrac{\Delta r}{r} = \dfrac{3.3\times10^{-10}\text{ m}}{0.529\times10^{-10}\text{ m}} = 6.24$

The minimum uncertainty in the radial position is **624**% of the orbital radius. This is a very significant uncertainty.

This result highlights a fundamental limitation of the Bohr model. The Bohr model assumes a well-defined circular orbit with a precise radius, which is inconsistent with the uncertainty principle.

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According to the Bohr model of the hydrogen atom, the electron in the ground state moves in a circular orbit of radius 0.529×10^{-10} m, and the speed of the electron in this state is 2.2×10^6 m s^{-1}. Assuming that the maximum uncertainty in its radial momentum is its own momentum, estimate the minimum uncertainty in the radial position of the electron. Comment on your results.

Interpretation of Atomic Structure: The large uncertainty suggests that we cannot think of electrons in atoms as classical particles with well-defined positions and momenta simultaneously.

Measurement Implications: This result implies that any attempt to measure the electron's position in the hydrogen atom would significantly disturb its momentum, and vice versa, consistent with the uncertainty principle.

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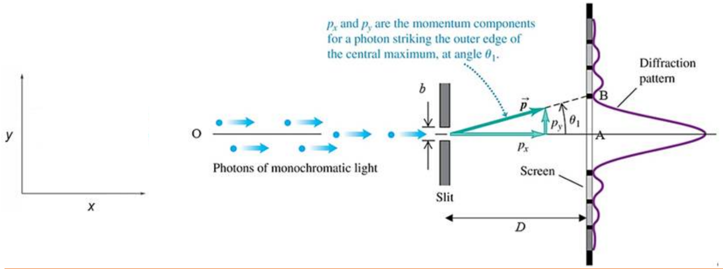
Use of single slit diffraction of photons to further appreciate HUP



The uncertainty principle relates to what we are able to predict about the properties of a photon passing through the slit.

We know that it must pass through the slit, but we do not know where. There is therefore an uncertainty in the particle position and the size of this uncertainty is Δy, where Δy is the width b of the slit.

If the particle strikes the screen in the centre, its motion between the slit and the screen must have been along the horizontal line OA. However, if it arrives at a point away from the centre of the pattern – say the point B – its velocity must have been at an angle to the horizontal.

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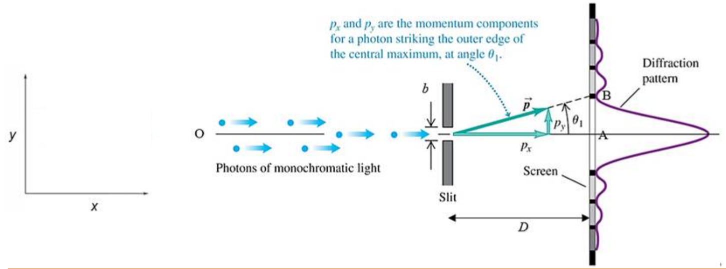
Use of single slit diffraction of photons to further appreciate HUP



In this case, the particle's velocity, and hence its momentum, must have had a component in the y direction.

Since we know that the majority of photons will hit the screen between the central maximum and the first minimum, the width of this diffraction pattern is a measure of the uncertainty in our prediction of this component of momentum, i.e. the largest momentum any of the photons can have in the y-direction is p_y, with an associated angle θ_1, where it will strike the screen at B.

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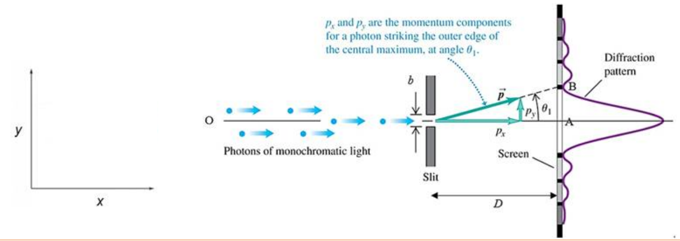
Use of single slit diffraction of photons to further appreciate HUP



- The angular position of the first minimum is given by

$$b\sin\theta_1 = \lambda \Rightarrow \theta_1 \approx \frac{\lambda}{b} \quad \text{when } \theta_1 \text{ is small}$$

- The uncertainties in the position and momentum of an individual photon on the y-axis:
  - Uncertainty in y for the diffracted photons: $\Delta y \approx b$
  - Uncertainty in p_y for the diffracted photons: $\Delta p_y \approx p_y\,(\text{at } \theta_1) = p_x \tan\theta_1 \approx p_x\theta_1 \approx p_x\frac{\lambda}{b} = \frac{h}{b}$

$$\Rightarrow \boxed{\Delta p_y \Delta y \approx h}$$

- Decreasing the slit width Δy, increases $\Delta p_y \Longrightarrow$ more spreading of waves

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Quantum Physics

The End

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