1 **Without using a calculator**, express $\frac{4-7\sqrt{3}}{\sqrt{3}+1}$ in the form $\frac{a\sqrt{3}+b}{2}$, where a and b are integers. [3]

2 A programmed robot attempts to throw a basketball into a hoop. The height, h feet, of the basketball from the ground at time t seconds, is given by the equation $h=-2t^2+4t+7$. The basketball hoop is at a height of 10 feet above the ground. Determine, with calculations, if the robot would be able to throw the basketball into the hoop. [3]

3 **(a)** Sketch the graph of $y = 10^x$ and the graph of $y = \lg x$ on the same axes.
Label the graphs and clearly indicate the intercepts. [3]

(b) Solve the equation $e^y\left(2e^y - 5\right) = 3$. [3]

4 Find the coordinates of the points where the line $2x + y = 1$ meets the curve $4x^2 + y^2 = 13$. [4]

5 **(a)** Find the range of values of k for which the equation $4x^2 + 9 = kx$ has no real roots. [3]

(b) Hence, or otherwise, find the smallest prime number k so that the equation $4x^2 + 9 = kx$ has two real and distinct roots. [2]

6 **(a)** A microorganism reproduces where each cell divides into four cells in every hour. If N is the number of cells to begin with and assuming that the cells divide at a constant rate, explain why the number of cells present after h hours is given by $(4)^h N$. [2]

(b) A scientist discovered that the microorganism has a x% increase in cells after 5 hours. Calculate the value of x. [2]

7 Solve, for $0° \le x \le 360°$, the equation $2\cot^2 x = 1 + 5\operatorname{cosec} x$. [4]

8 **(a)** The curve $y = a\cos\frac{x}{2} + c$, where a and c are negative integers, is defined for $0 \leq x \leq 6\pi$. It has an amplitude of 3 and a minimum value of -7.

(i) State the value of a and c. [2]

(ii) Sketch the graph of y. [2]

(b) Given that $\sin 18° = p$, express each of the following in terms of p.

(i) $\sec 162°$ [2]

(ii) $\tan(-198°)$ [2]

9 **(a)** The remainder when $x^3 - 3x^2 - 2x + k$, where k is a constant, is divided by $x + 2$ is twice of the remainder when it is divided by $x - 3$. Find the value of k. [3]

(b) Solve the equation $x^3 + 4x^2 - 8x - 8 = 0$, expressing non-integer roots in surd form. [4]

10 **(a)** Prove that $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(b) Hence find, for $0 < x < 6$, the values of x for which $\dfrac{2}{(\operatorname{cosec}2x - \cot 2x)^2} = 5$. [5]

11 **(a)** Show that $x-1$ is a factor of x^3+3x^2-4. [1]

(b) Express $\dfrac{-2x^2+3x+8}{x^3+3x^2-4}$ as a sum of three partial fractions. [7]

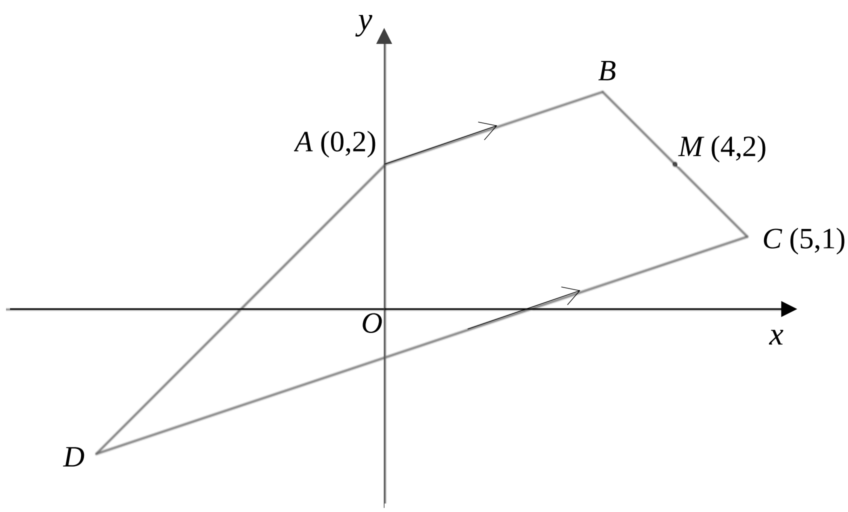
12 **(a)** Solve the equation $\log_3 y + 2\log_{81} y = -3$. [4]

(b) It is given that $\lg p - \lg 2q = \lg(p + 2q)$.

(i) Express p in terms of q. [3]

(ii) State the range of values of p and explain clearly why $0 < q < \frac{1}{2}$. [3]

13



The diagram shows a trapezium *ABCD* in which *AB* is parallel to *DC*. The point *A*(0, 2) lies on the *y*-axis. The point *M*(4, 2) is a mid-point of *BC*. *DC* = 3*AB*.

(a) Find the coordinates of *B*. [2]

(b) Find the coordinates of *D*. [1]

(c) Find the area of *ABCD*. [2]

DA and *BC* extends to meet at a point *P*.

(d) Show that *DP* is perpendicular to *PC*. [2]

(e) Find the coordinates of *P*. [3]

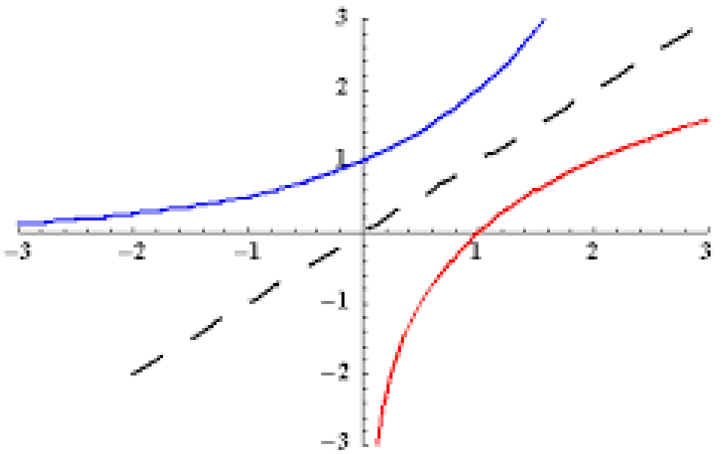
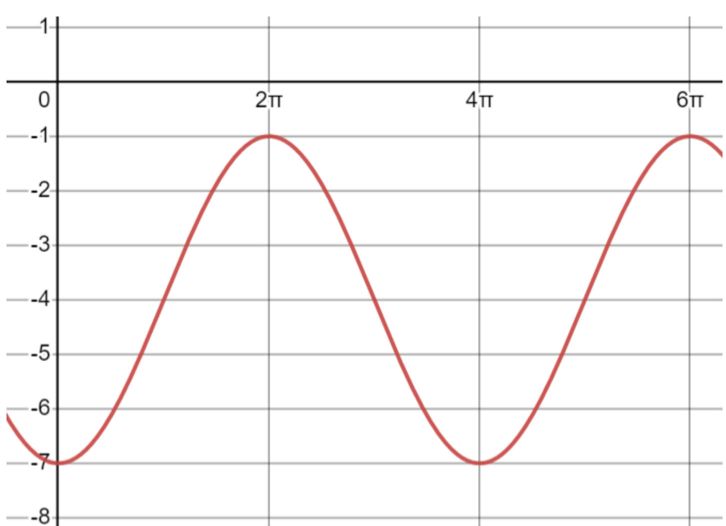
14 **(a)** Write down the first four terms in the expansion of $\left(2-\dfrac{x^2}{3}\right)^{10}$. [4]

(b) **(i)** Juliet claimed that every term is dependent on x in the binomial expansion of $\left(\dfrac{2}{x}+x^2\right)^7$. Do you agree? Explain your answer by considering the general term of the binomial expansion. [3]

(ii) Find the term independent of x in the expansion of $\left(\frac{2}{x}+x^2\right)^7(3-5x)$. [3]

End of paper

Answer Key

1	$\frac{11\sqrt{3}-25}{2}$
2	$h=-2(t-1)^2+9$. Max height the basketball can reach is 9 feet, robot **would not be able** to throw the basketball into the hoop.
3a	 3b) $y=\ln 3=1.10$
4	(1.5, –2) , (–1, 3)
5a	$-12<k<12$ 5b) 13
6a	After 1 hour, number of cells present = $N\times 4=4^1\times N$ 6b) $x=102300$ After 2 hours, number of cells present = $N\times 4\times 4=4^2\times N$ After *h* hours, number of cells present = $(4)^h\ N$
7	x = 19.5° , 160.5°
8ai 8aii	$a=-3$ $c=-4$ $-\frac{1}{\sqrt{1-p^2}}$ 

8bi	$-\dfrac{1}{\sqrt{1-p^2}}$ 8bii) $-\dfrac{p}{\sqrt{1-p^2}}$
9a	$k=-4$ b) $-3\pm\sqrt{5}$
10b	$x=0.564,\ 2.58,\ 3.71,\ 5.72$
11b	$\dfrac{1}{x-1}-\dfrac{3}{x+2}+\dfrac{2}{(x+2)^2}$
12a	$y=\dfrac{1}{9}$ 12bi) $p=\dfrac{4q^2}{1-2q}$
13a	$B(3,3)$ 13b) $D(-4,-2)$ 13c) 16 13e) $P(2,4)$
14a	$1024-\dfrac{5120}{3}x^2+1280x^4-\dfrac{5120}{9}x^6+\ldots$
14bi	Show that when power of x = 0, r = 7/3, which is not a whole number/ integer. Agree, every term is dependent on *x*.
14bii	-3360

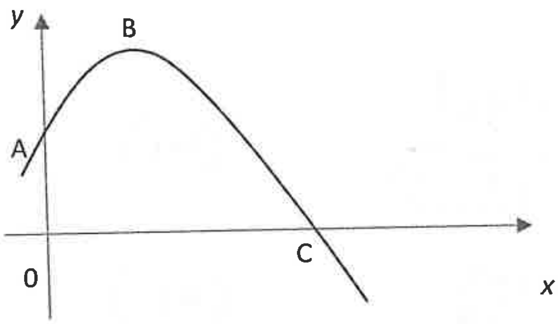
1 Triangle ABC is a right angle triangle with angle $ABC = 90^{o}$. Given $\sin A = 2-\sqrt{3}$ and $BC = 1+2\sqrt{3}$, find the length of the hypotenuse in the simplest exact form. [3]

$$\sin A = \frac{1+2\sqrt{3}}{AC} = 2-\sqrt{3}$$

$$AC = \frac{1+2\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \quad M1 \quad M1$$

$$= 8+5\sqrt{3} \quad A1$$

2



The diagram shows part of the curve $y = a\sin bx + 2$. The curve cuts the coordinate axes at A and C. Point B, whose coordinates are $\left(\frac{\pi}{4}, 8\right)$, lies on the curve such that y is a maximum at B.

(i) Find the coordinates of A. [1]

$(0, 2)$ B1

(ii) State the value of a and of b. [2]

$a = 8-2 = 6$ B1

$$b = \frac{\pi/2}{\pi/4} = 2 \quad B1$$

3 Given that $\sin A = \frac{5}{13}$ and $\cos B = -\frac{4}{5}$, where A and B are in the same quadrant, find the value of each of the following without using calculator.

(i) $\cos A$ [1]

$-12/13$ B1

(ii) $\cos 2B$ [2]

$= 2(-\frac{4}{5})^2 - 1$ M1

$= \frac{7}{25}$ A1

(iii) $\tan 2A$ [2]

$= \frac{2(-5/12)}{1-(-5/12)^2}$ M1

$= -\frac{120}{119}$ A1

(iv) $\cos(A+B)$ [2]

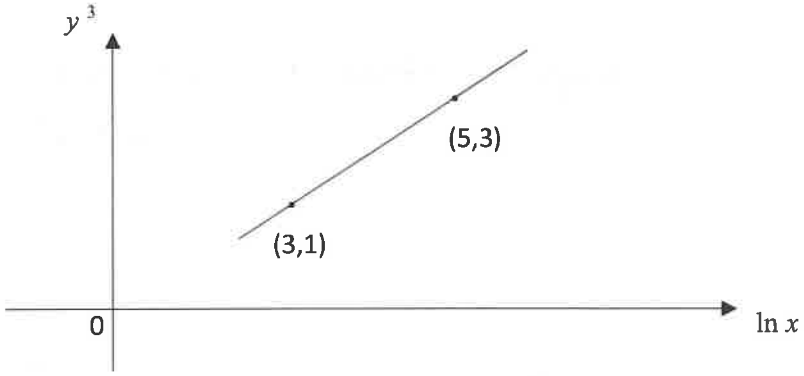
$= \cos A \cos B - \sin A \sin B$

$= (-\frac{12}{13})(-\frac{4}{5}) - (\frac{5}{13})(\frac{3}{5})$ M1

$= \frac{33}{65}$ A1

4 The diagram shows part of the straight line drawn a student by plotting y^3 against $\ln x$.

(i) Find y in terms of x. [3]



grad $= 1$ A1

$Y - 1 = 1(X - 3)$

$Y = X - 2$ M1

$y = \sqrt[3]{\ln x - 2}$ A1

(ii) Find the gradient and the vertical axis-intercept if a student plot $\ln x$ against y^3 instead. [2]

$\ln x = y^3 + 2$

grad $= 1$ B1

and vertical axis-intercept $= 2$ B1

5 **(a)** Given that $6x^3 - 7x^2 - 4x + 2 = (ax - b)(2x^2 + x - 1) + 4x - 3$ for all values of x, find the value of a and of b. [2]

compare coeff of x^3: $2a = 6$
$a = 3$ B1

compare const: $b - 3 = 2$
$b = 5$ B1

(b) State the quotient and the remainder when $6x^3 - 7x^2 - 4x + 2$ is divided by $2x^2 + x - 1$. [2]

Quotient is $3x - 5$ B1

Remainder is $4x - 3$ B1

(c) Express $\dfrac{6x^3 - 7x^2 - 4x + 2}{2x^2 + x - 1}$ in partial fractions. [3]

$= 3x - 5 + \dfrac{4x - 3}{(2x-1)(x+1)}$ M1

$A(x+1) + B(2x-1) = 4x - 3$

let $x = -1$, $-3B = -7$
$B = 7/3$ A1

compare const; $A - B = -3$
$A = -2/3$ A1

will not award if M1 is not achieved

$\therefore 3x - 5 - \dfrac{2}{3(2x-1)} + \dfrac{7}{3(x+1)}$

6 **(a)** Prove that $\dfrac{1-\sin\theta\cos\theta+2\cos^2\theta}{\cos^2\theta}=\tan^2\theta-\tan\theta+3$. [2]

$$LHS = \frac{1}{\cos^2\theta} - \frac{\sin\theta}{\cos\theta} + 2 \quad M1$$

$$= \sec^2\theta - \tan\theta + 2$$

$$= (1+\tan^2\theta) - \tan\theta + 2 \quad M1$$

$$= \tan^2\theta - \tan\theta + 3$$

(b) Solve the equation $1-\sin\theta\cos\theta+2\cos^2\theta=5\cos^2\theta$ for $0^\circ<\theta<360^\circ$. [4]

$$\tan^2\theta - \tan\theta + 3 = 5 \quad M1$$

$$(\tan\theta+1)(\tan\theta-2)=0 \quad M1$$

$\theta = 135^\circ, 315^\circ$ or $\alpha = \tan^{-1}2$

$= 63.43^\circ$

$\therefore \theta = 63.4^\circ, 243.4^\circ$

7 **(a)** Solve the equation $\log_2(x^2-6)+\log_2 4=\log_2 10x$. [3]

$4(x^2-6)=10x$ M1

$2x^2-5x-12=0$ A1

$(2x+3)(x-4)=0$

$x=-3/2$ or 4 A1

NA

(b) Given that $\log_5 x-\log_{25} y=1$, express y in terms of x. [4]

$\log_5 x-\dfrac{\log_5 y}{\log_5 25}=1$ change base M1

$\log_5 x-\dfrac{\log_5 y}{2}=1$

$\log_5 x^2-\log_5 y=2$ Quotient M1 Remove log M1

$\dfrac{x^2}{y}=25$

$y=\dfrac{x^2}{25}$ A1

8 Given that $f(x) = ax^3 - bx^2 - ax + b$.

(i) Show that $(x-1)$ is a factor. [1]

Let $x=1$

$f(1) = a - b - a + b = 0$ (1)

By Factor thm, $x-1$ is a factor

or Remainder $= 0$, $\therefore$ $x-1$ is a factor

When f(x) is divided by x +2, the remainder is -45 and $(x-3)$ is another factor of f(x).

(ii) Find the value of a and of b. [4]

$f(-2) = -8a - 4b + 2a + b = -45$ M1

$2a + b = 15$ — (1)

$f(3) = 0$ $27a - 9b - 3a + b = 0$ M1

$3a = b$ — (2)

subst (2) into (1)

$2a + 3a = 15$

$a = 3$ A1

$b = 9$ A1

(iii) Hence, solve the equation f(x) = 0 [2]

$(x-1)(x-3)(3x+3) = 0$ M1

$x = 1, 3, -1$ A1

9 **(a)** Write down, in the simplest forms, the first 3 terms in the expansion, in ascending powers of x, of $\left(1+\frac{x}{2}\right)^7$. [2]

$= 1 + 7\left(\frac{x}{2}\right) + 21\left(\frac{x}{2}\right)^2 + \ldots$ M1

$= 1 + \frac{7}{2}x + \frac{21}{4}x^2 + \ldots$ A1

(b) Find the coefficients of the term in x^{-6} and x^{-8} in the expansion of $\left(2x+\frac{3}{x}\right)^8$. [3]

General Term $= \binom{8}{r}(2x)^{8-r}\left(\frac{3}{x}\right)^r$

power of $x = 8-2r$ M1

let $8-2r=-6$

$r=7$

coeff of $x^{-6} = 8 \times 2 \times 3^7$

$= 34992$ A1

let $8-2r=-8$

$r=8$

coeff of $x^{-8} = 3^8$

$= 6561.$ A1

(c) Find the coefficient of x^{-6} in the expansion of $\left(1+\frac{x}{2}\right)^7\left(2x+\frac{3}{x}\right)^8$. [3]

$\left(1+\frac{7}{2}x+\frac{21}{4}x^2+\ldots\right)\left(\ldots\frac{34992}{x^6}+\frac{6561}{x^8}\right)$

coeff of $x^{-6} = (1)(34992) + \left(\frac{21}{4}\right)(6561)$

M1 M1

$= 69437\frac{1}{4}$ or $\frac{277749}{4}$

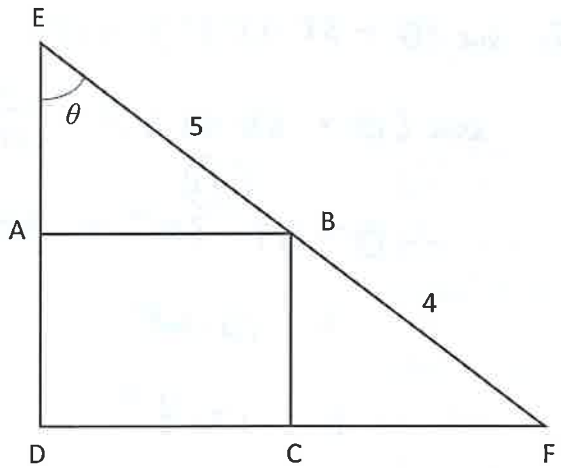
(d) Explain why the term in x^{16} does not exist in the expansion of $\left(1+\frac{x}{2}\right)^7\left(2x+\frac{3}{x}\right)^8$. [1]

Highest power of x in expansion is $\underline{7+8=15}$

1

$\therefore x^{16}$ does not exist

10 In the triangle DEF, angle $EDF = 90^\circ$ and A, B and C are on the sides ED, EF and DF respectively such that $ABCD$ forms a rectangle. Angle $DEF = \theta^\circ$, $EB = 5$ cm and $BF = 4$ cm.



(i) Show that P, the perimeter of the rectangle $ABCD$, is given by

$P = 10\sin\theta + 8\cos\theta$ [2]

$AB = 5\sin\theta$ B1

$BC = 4\cos\theta$ B1

$\therefore P = 2(5\sin\theta + 4\cos\theta)$

$= 10\sin\theta + 8\cos\theta$

(ii) Express P in the form $R\sin(\theta+\alpha)$ where R is a positive constant and α is an acute an acute angle. [2]

$R = \sqrt{10^2+8^2} = 2\sqrt{41}$ B1

$\alpha = \tan^{-1}\frac{4}{5} = 38.659°$ B1

$\therefore P = 2\sqrt{41}\sin(\theta + 38.7°)$

(iii) Solve for the value of θ if $P = 10$ cm. [3]

$2\sqrt{41}\sin(\theta + 38.659°) = 10$

$\sin(\theta + 38.659°) = \frac{10}{2\sqrt{41}}$ M1

A1

$\therefore \theta = 51.34° - 38.659°$

$= 12.68$

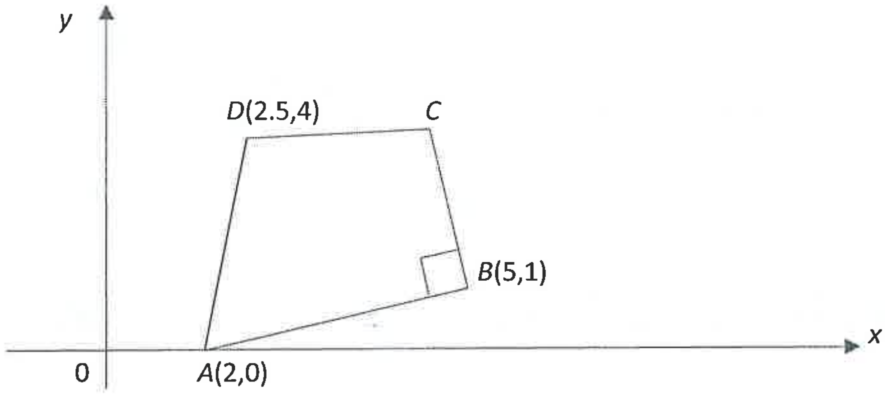
$\approx 12.7°$ A1

(iv) Without further calculations, explain why the perimeter of the rectangle cannot be 13 cm. [1]

Max $P = 2\sqrt{41} < 13$

$\therefore P$ cannot be 13 cm.

11 In the diagram, *ABCD* is a quadrilateral in which *A* is (2,0), *B* is (5,1) and D is (2.5,4). *DC* is parallel to the *x*-axis and *AB* is perpendicular to *BC*.



(i) Find the equation of *BC*. [2]

grad of AB = $\frac{1}{3}$

eq^n of BC: $y-1=-3(x-5)$ M1

$y=-3x+16$ A1

(ii) Find the coordinates of point *C*. [1]

Sub $y=4$

$4=-3x+16$

$\therefore x=4$

C is $(4,4)$ B1

(iii) Calculate the area of the quadrilateral *ABCD*. [2]

Area = $\frac{1}{2}\begin{vmatrix} 2 & 5 & 4 & 2.5 & 2 \\ 0 & 1 & 4 & 4 & 0 \end{vmatrix}$

$=\frac{1}{2}\left[(2+20+16)-(8+10+4)\right]$ M1

$=8$ sq units A1

(iv) Find the equation of the circle passing through the points A, B and C. [3]

mid pt of $AC = \left(\frac{2+4}{2}, \frac{0+4}{2}\right)$

$= (3,2)$ M1

length of $AC = \sqrt{(2-4)^2 + (0-4)^2}$

$= \sqrt{20}$ M1

eqn: $(x-3)^2 + (y-2)^2 = 5$ A1

12 (a) Find the range of values of x for which $x^2 + 1 \leq \frac{13x}{6}$. [2]

$6x^2 - 13x + 6 \leq 0$

$(2x-3)(3x-2) \leq 0$ M1

$\frac{2}{3} \leq x \leq 3/2$ A1

(b) Show that the line $y = (3m-1)x + 2$ will meet the curve $y = x^2 + x + 1$ at two real and distinct points for all real values of m. [4]

$(3m-1)x + 2 = x^2 + x + 1$

$x^2 + (2-3m)x - 1 = 0$ M1

Discriminant $= (2-3m)^2 - 4(1)(-1)$ M1

$= (2-3m)^2 + 4$ A1

min value $= 4$

$\therefore D > 0$

They will meet at 2 real and distinct points

1

(c) The equation of a curve is $y = x^2 - \frac{k}{3} - 1$. The line $y = kx - \frac{k^2}{3}$ is a tangent to the curve at point P. Find the positive value of k. [4]

$x^2 - \frac{k}{3} - 1 = kx - \frac{k^2}{3}$

$x^2 - kx + \frac{k^2}{3} - \frac{k}{3} - 1 = 0$ M1

$(-k)^2 - 4(1)\left(\frac{k^2}{3} - \frac{k}{3} - 1\right) = 0$ M1

$3k^2 - 4k^2 + 4k + 12 = 0$

$-k^2 + 4k + 12 = 0$ M1

$(k+2)(-k+6) = 0$

$k = -2$ or 6 A1

NA.

End of Paper