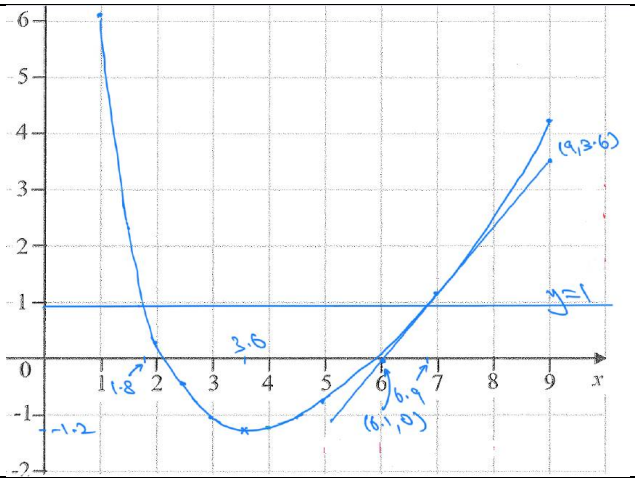
2023 4N(A) Mathematics Prelim Exam Paper 2 Mark Scheme

S/n	Solution	Mark Allocation
1a	$540 = 2^2 \times 3^3 \times 5$	B1
1b	Greatest Integer $= 2^2 \times 3^2$ $= 36$	B1 (B0 if left in indices form)
2	12 mths ------ 3.2% 6 mths ------ $\frac{3.2}{12} \times 6$ $= 1.6\%$ Thus r = 1.6%, and n = 3x2 = 6. $A = P\left(1 + \frac{r}{100}\right)^n$ $10000 + I = 10000\left(1 + \frac{1.6}{100}\right)^6$ $10000 + I = 10999.229$ $I = \$999.229$ $I = \$999\ (3sf)$	B1 (either r or n correct) M1 (correct formula for compound interest) A1
3a	$\angle STR = \angle PQR$ $= 48°$ $\angle RST = 180° - 60° - 48°$ $= 72°$	B1
3b	$\frac{PQ}{ST} = \frac{QR}{TR} = \frac{PR}{SR}$ $\frac{7.3}{ST} = \frac{8}{10}$ $\frac{ST}{7.3} = \frac{10}{8}$ $ST = \frac{10}{8} \times 7.3$ $= 9.125\text{ cm}$ Accept alternative working $\frac{ST}{\sin 60°} = \frac{10}{\sin 72°}$ $ST = \frac{10 \sin 60°}{\sin 72°}$ $ST = 9.1059$ $ST = 9.11\text{ cm (3sf)}$	M1 (correct values in ratio) A1 (exact) (A0 if in 3sf)
4	Volume of 4 balls $= 4 \times \frac{4}{3} \times \pi r^3$ $= \frac{16}{3}\pi r^3$ Volume of cylinder $= \pi r^2 h$ $= \pi r^2 (8r)$ $= 8\pi r^3$	M1 (either formula of sphere or cylinder correct with values substituted)

	Percentage of empty space $= \dfrac{8\pi r^3 - \dfrac{16}{3}\pi r^3}{8\pi r^3} \times 100\%$ $= \dfrac{\dfrac{8}{3}\pi r^3}{8\pi r^3} \times 100\%$ $= 33\dfrac{1}{3}\%$ Thus is it 1/3 of the volume of the tube.	M1 (percentage formed) A1 (A0 if no conclusion of 1/3)
5a	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(2)(-7)}}{2(2)}$ $x = \dfrac{4 \pm \sqrt{72}}{4}$ $x = 3.121 \text{ or } x = -1.1213$ $x = 3.12 \text{ or } x = -1.12 \ (3sf)$	M1 (correct substitution in formula) A1, A1 (minus 1m if one or both answers are not in 3sf)
5b	From eqn 2: $x = 2y - 3$ ---2a Sub eqn 2a into eqn 1: $4(2y-3) + y = 6$ $8y - 12 + y = 6$ $9y = 18$ $y = 2$ Sub $y = 2$ into eqn 2a $x = 1$ Thus $x = 1$, $y = 2$.	M1 (substitution into one eqn) M1 (manipulation to solve for one variable) A1 (A0 if 1 value is incorrect)
6a	Angle in a semi-circle property, since CE is the diameter of the circle.	B1
6b	$\angle ABC = 180° - 68°$ $= 112°$	B1 (cao)
6c	$\angle CAD = 180° - 90° - 68°$ $= 22°$ Since ΔOAB is isosceles, $\angle OBA = \angle BAO = 30° + 22°$ $= 52°$	M1 (finding $\angle CAD$ or other suitable angle leading to $\angle OBA$) A1 (cao)
7a	In ΔABD, $\sin \angle ADB = \dfrac{8}{12}$ $\angle ADB = \sin^{-1}\left(\dfrac{8}{12}\right)$ $\angle ADB = 0.72972$ = 0.730 radians (3sf)	M1 (forming sine ratio) A1

7b	In ΔABC, $\tan 1 = \frac{8}{BC}$ $BC = \frac{8}{\tan 1}$ $BC = 5.13674$ $BC = 5.14\ cm\ (3sf)$	M1 (forming sine ratio) A1
7c	In ΔABD, $12^2 = 8^2 + BD^2$ $BD^2 = 12^2 - 8^2$ $BD = \pm\sqrt{80}$ $BD = 8.94427$ (reject negative value) Area $\Delta ABC = \frac{1}{2}(AD)8$ $= \frac{1}{2}\left(\sqrt{80} - 5.13674\right)8$ $= 15.2301$ $= 15.2\ cm^2\ (3sf)$	 B1 (BD value correct) M1 (correct substitution of values, no penalty if AD does not have extra accuracy) A1 (cao)
8a	$(3a-b)^2 + b(2a-b)$ $= 9a^2 - 6ab + b^2 + 2ab - b^2$ $= 9a^2 - 4ab$ *or* $a(9a-4b)$	M1, M1 (correct expansion for each term) A1
8bi	$12 - 3x^4$ $= 3\left[4 - x^2\right]$ $= 3\left[(2)^2 - \left(x^2\right)^2\right]$ $= 3\left(2 - x^2\right)\left(2 + x^2\right)$	M1 (factorise common term) A1
8bii	$4y - 10 + 6yz - 15z$ $= 2(2y-5) + 3z(2y-5)$ $= (2y-5)(2+3z)$	M1 (factorise common term for each term) A1

9a	$y = 1.2$	B1 (B0 if not left in 1dp)
9b		B1 (at least 6 plots correct) B1 (smooth curve) B1 (shape – composite graph)
9c	Minimum Point = $(3.5, -1.2)$	B1 (accept $3.5 \le x \le 4$, $-1.4 \le y \le -1.1$, must be correct to 0.1 decimal only)
9d	Gradient $= \dfrac{3.6-0}{9-6.1}$ $= 1.24\ (3sf)$	M1 (draw tangent at $x = 1.5$) A1 (accept $1.1 \le m \le 1.5$)
9e	$\dfrac{x^2}{9} + \dfrac{12}{x} = 7$ $\dfrac{x^2}{9} + \dfrac{12}{x} - 6 = 7 - 6$ $y = 1$ From the graph, $x = 1.8$ or $x = 6.9$	M1 (obtain $y = 1$ graphically or algebraically) A1 (any 1 answer correct) (accept $1.6 \le x \le 1.9$ and $6.7 \le x \le 6.9$, must be correct to 0.1 decimal only)
10a	Choosing age less than 30yrs old, walking speed $= \dfrac{4.82 \times 1000}{1 \times 60 \times 60}$ $= 1.3388$ $= 1.34\ m/s\ (3sf)$	B1 (choose 4.82 km/h) B1 (must be in m/s)
10b	By measuring the arc length *AB*, from 4T1 to 5N1, I estimated it to be about 16.5 cm. 1cm ----- 20m 16.5cm ----- 16.5×20 $= 330\ m$	M1 (accept $14 \le arc\ AB \le 18$) M1 (proportion is calculated correctly)
10c	Time to walk from classroom to field $= \dfrac{400}{1.3388}$ $= 298.775$ $= 4\,\text{min}\ 59\,\text{sec}$	M1 (distance / 10a speed)

<table>
<tr><td></td><td>Time to form up BT70 logo
$= \frac{330}{1.3388}$
$= 246.489$
$= 4\,\text{min}\ 6\,\text{sec}$

Assume time to walk back from field to classroom is the same as time to walk from classroom to field = 4min 59sec

Total time taken after flag-raising to end of activity
= 2(4min 59sec) + 4min 6sec
= 14min 4sec

Time permitted for photo-taking factoring time wastage
= 55min – 14min 4sec
= 40min 56sec

It is reasonable to assume that photo taking factoring time wastage will not exceeed 40min 56sec. Thus, we conclude that the students should be able to return by 8.30am.</td><td>M1 (10b distance / 10a speed)

M1 (reasonable attempt to calculate total time taken)

A1 (reasonable explanation and conclusion)</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>11a i</td><td><table><tr><th></th><th>Mathematics test</th></tr><tr><td>Lowest mark</td><td>40</td></tr><tr><td>Highest mark</td><td>89</td></tr><tr><td>Range</td><td>49</td></tr><tr><td>Lower Quartile</td><td>58</td></tr><tr><td>Upper Quartile</td><td>81</td></tr><tr><td>Interquartile Range</td><td>23</td></tr><tr><td>Median mark</td><td>71</td></tr></table></td><td>B1 (range cao)
B1 (UQ – LQ)√
B1 (Median cao)</td></tr>
<tr><td>11a ii</td><td>No, I do not agree, because the spread of the lowest quartile (bottom 25%) is much wider than the spread of highest quartile (top 25%).</td><td>B1 (any reasonable explanation)</td></tr>
<tr><td>11a iii</td><td>Students score higher marks in the Mathematics test than English test, because the $\text{Median}_{\text{math}} > \text{Median}_{\text{english}}$.
or
The spread of marks is wider for the Mathematics test than English test, because $\text{IQR}_{\text{math}} > \text{IQR}_{\text{english}}$ and $\text{Range}_{\text{math}} > \text{Range}_{\text{english}}$</td><td>B1 (any 1 comparison) √

(no need to state Range)</td></tr>
<tr><td>11b i</td><td>Probability of both blue balls
$= \frac{4}{7} \times \frac{3}{6}$
$= \frac{2}{7}$ or 0.286 (3sf)</td><td>B1</td></tr>
<tr><td>11b ii</td><td>Probability of both balls of different colours
= 1 – Probability of both blue balls – Probability of both red balls
$= 1 - \frac{2}{7} - \frac{1}{7}$</td><td>M1 (or equivalent)

A1</td></tr>
</table>

	$= \frac{4}{7}$ or 0.571 (3sf)	
12a	$\angle BAC = 144° - 68°$ $= 76°$ Using cosine rule, $BC = \pm\sqrt{370^2 + 510^2 - 2(370)(510)\cos 76°}$ $BC = 552.900$ $BC = 553m\ (3sf)$	B1 M1 (cosine rule applied correctly) A1
12b	Using sine rule, $\frac{\sin \angle ABC}{510} = \frac{\sin 76°}{552.900}$ $\sin \angle ABC = \frac{510 \sin 76°}{552.900}$ $\angle ABC = \sin^{-1}\left(\frac{510 \sin 76°}{552.900}\right)$ $\angle ABC = 63.509°$ Thus, Bearing $= 360° - 63.509° - 112°$ = 184.5° (1d.p)	M1 (sine rule applied correctly) A1 B1√
12c	In ΔACD, $\sin 76° = \frac{CD}{510}$ $CD = 510 \sin 76°$ $CD = 494.850$ $CD = 495m\ (3sf)$ Thus the shortest distance is 495m	M1 A1 (need to state shortest distance)