AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 2 **13 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A curve has the equation $y = \dfrac{\sin 2x}{2 - \cos 2x}$.

(a) Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x - 2}{(2-\cos 2x)^2}$, where k is a constant. [3]

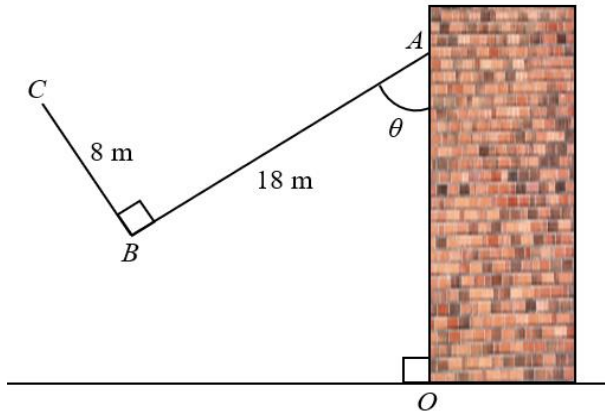
(b) Find the acute angle between the tangent to the curve at $x = \dfrac{\pi}{12}$ and the line $y = 0$. [3]

2 (a) Factorise $x^3 + 27k^3$ as a product of a linear and a quadratic factor. [2]

(b) Hence solve $x^3 + 27 = (x+3)(x+10)$, expressing non-integer roots in surd form. [3]

(c) Find the value of k given that $x^3 + 27k^3$ leaves a remainder of 351 when divided by $x - 2$. [2]

3 The diagram shows a L-shaped rod ABC where AB and BC have length 18 m and 8 m respectively and angle ABC is 90°. The rod is hinged to a wall at A so as to rotate in a vertical plane. The rod AB makes an acute angle θ with the vertical wall surface OA.



(a) Given that G is a point directly below C, show that $OG = p\cos\theta + q\sin\theta$, where p and q are constants to be found. [2]

(b) Express OG in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [3]

(c) Find the length of OG and the corresponding value of θ if G is at maximum displacement from O. [3]

4 A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

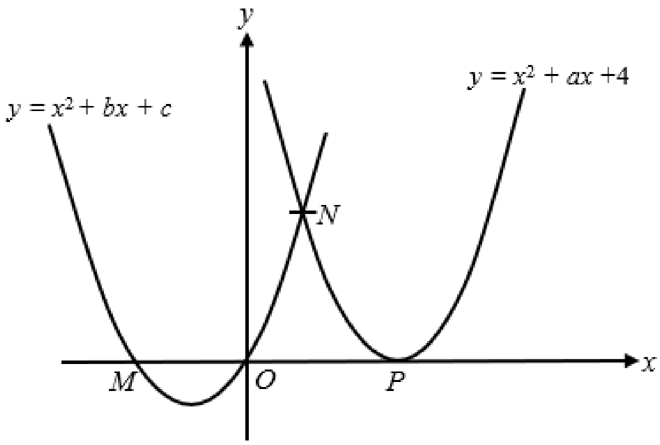
(b) Find the equation of the tangent to the circle at the point $P(7, -5)$. [3]

(c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

5 (a) Prove the identity $\dfrac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]

(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2\cos^3 A = 0$ exactly, for $-\pi \le A \le \pi$ radians. [4]

6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x-axis at P. Points M and O are the x-intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP.



(a) Find the values of a, b and c. [4]

(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]

(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]

7 A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s=\frac{1}{3}t^3-5t^2-3$.

(a) Find the value of t when particle P returns to its initial position. [2]

(b) Find the minimum velocity of particle P. [3]

(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v=24\left(e^{-\frac{t}{6}}-e^{-1}\right)$.

Find the value of t at which the particle Q is instantaneously at rest. [2]

(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]

8 The height, h cm, of a plant is modelled by $h = \dfrac{80}{1+10\mathrm{e}^{-0.4t}}$, where t is the number of months after the first observation.

(a) Show that h is an increasing function. [3]

(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]

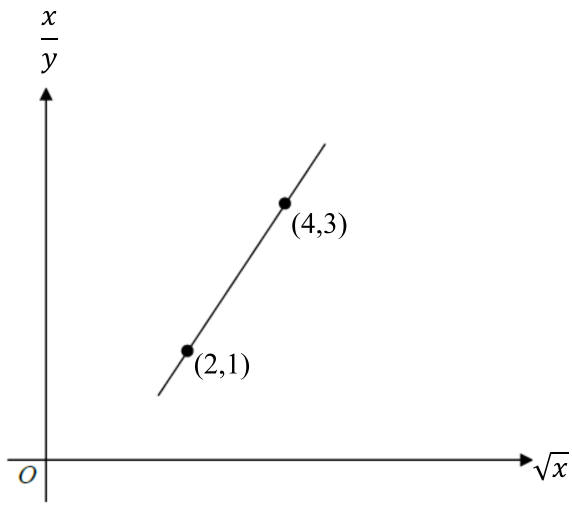
(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \dfrac{1}{a + b\mathrm{e}^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

9 **(a)** Without using a calculator, solve the equation $x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$. Leave your answer in the form $p\sqrt{10}+q\sqrt{3}$, where p and q are fractions. [4]

(b) Without using a calculator, solve the equation $\log_2 x-\log_x 16=0$. [4]

(c) Solve the equation $3^{x+2} - 2\left(3^{-x}\right) = 17$. [4]

10 (a) The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.



Find the equation of the straight line in the form $y=\dfrac{x}{a+b\sqrt{x}}$, where a and b are constants. [3]

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(b) The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

(i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [3]

(iii) Explain how would you use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]

END OF PAPER