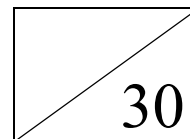


Name : _____ ()

Class: Sec 4 _____



**SINGAPORE CHINESE GIRLS' SCHOOL
SECONDARY 4
O-LEVEL PROGRAMME**



.....
Parent's / Guardian's Signature

ADDITIONAL MATHEMATICS

Common Test 1

Wednesday

15 February 2023

45 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class at the top of this page.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions on the question paper.

If working is needed for any question it must be shown with the answer.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 30.

This question paper consists of **9** printed pages and **1** blank page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. Given that $\tan \theta = p$, where θ is acute, express each of the following in terms of p .

(a) $\sin\left(\frac{\pi}{2} - \theta\right)$ [1]

(b) $\sec 2\theta$ [2]

2. **Without using a calculator,** find $\sin 15^\circ$ in the form $\frac{\sqrt{a} - \sqrt{b}}{4}$, where a and b are integers. [3]

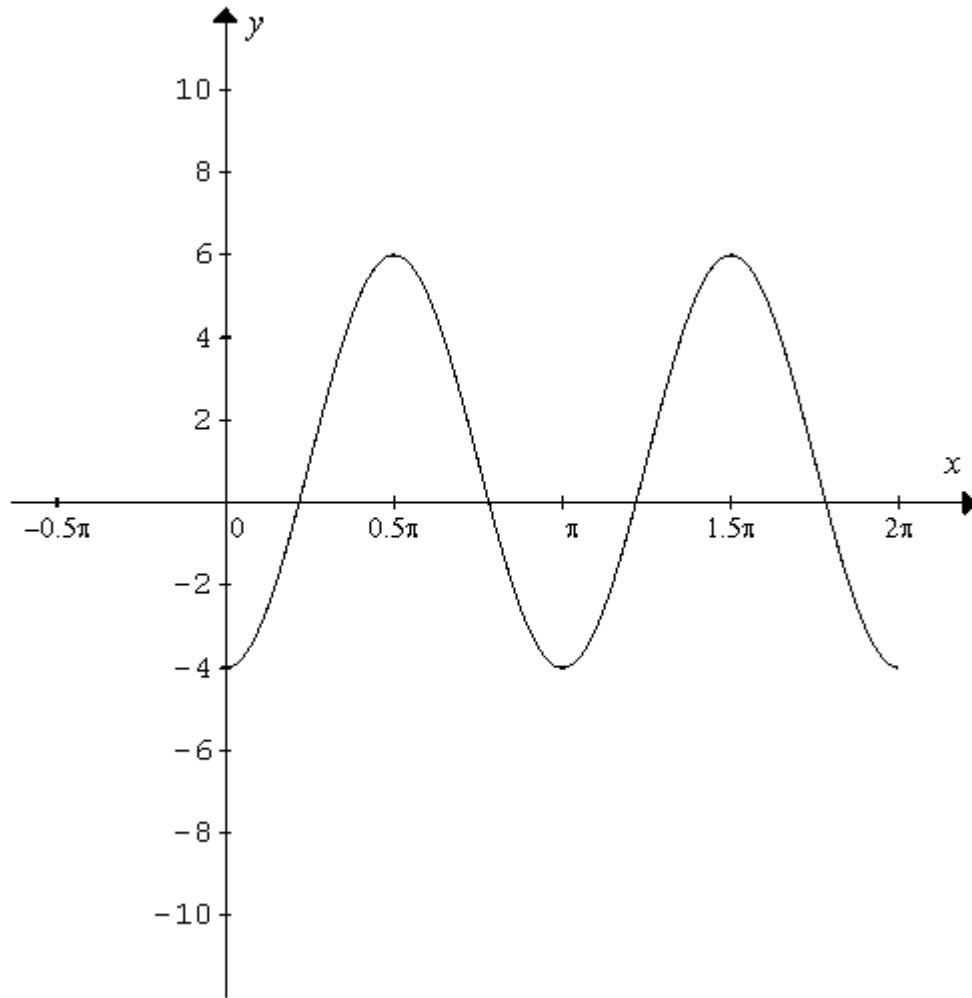
3. (a) Solve the equation $2 \tan A = 5 \sin A$ for $-90^\circ \leq A \leq 270^\circ$. [4]

- (b) Solve the equation $6 \cos^2 B + \sin B = 5$ for $0 \leq B \leq 2\pi$. [4]

4. Prove that $\cot \theta - \tan \theta = 2 \cot 2\theta$.

[3]

5. The diagram shows part of the graph of $y = a \cos bx + c$, where a , b and c are constants.



- (a) Write down the value of a , of b and of c .

[3]

$$a =$$

$$b =$$

$$c =$$

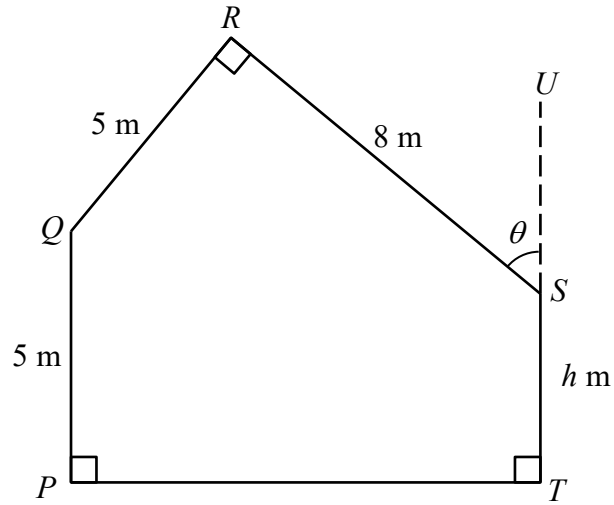
- (b) On the given diagram, sketch the graph of $y = 2 \tan\left(\frac{x}{2}\right)$ for $0 \leq x \leq 2\pi$.

Hence, state the number of solutions, for $0 \leq x \leq 2\pi$, of the equation

$$a \cos bx + c = 2 \tan\left(\frac{x}{2}\right).$$

[3]

6. The diagram shows a pentagon $PQRST$ where $PQ = QR = 5$ m, $RS = 8$ m and $ST = h$ m. U lies on TS produced such that angle $RSU = \theta$. Angles QRS , QPT and PTS are right angles.



- (a) Show that $h = 5 + 5 \sin \theta - 8 \cos \theta$. [2]

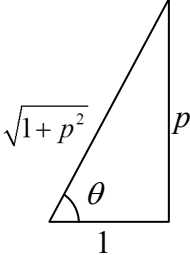
- (b) Express h in the form $5 + R \sin(\theta - \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

(c) Hence, find the value of θ when $h = 7$.

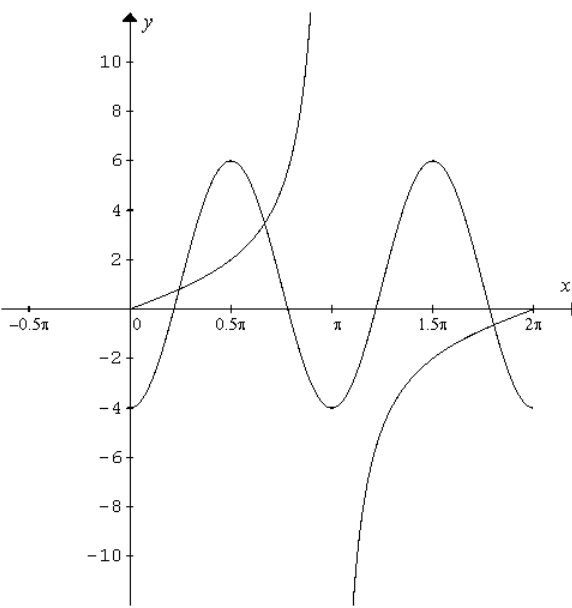
[2]

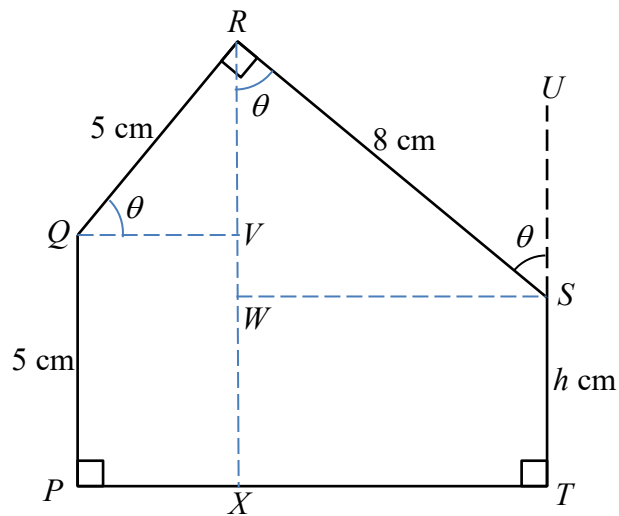
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Answers

1. (a) (b)	$\sin\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sqrt{1+p^2}}$ $\sec 2\theta = \frac{1}{\cos 2\theta}$ $= \frac{1}{2\cos^2 \theta - 1}$ $= \frac{1}{2\left(\frac{1}{1+p^2}\right) - 1}$ $= \frac{1}{\frac{2 - (1+p^2)}{1+p^2}}$ $= \frac{1+p^2}{1-p^2}$ 	B1 M1 (correct double angle with substitution) A1 (3 marks)
2.	$\sin 15^\circ$ $= \sin(45^\circ - 30^\circ) \text{ or } \sin(60^\circ - 45^\circ)$ $= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$ Or $\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$ $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{\sqrt{6} - \sqrt{2}}{4}$	M1 B1 (any one correct ratio) A1 (3 marks)
3. (a)	$2 \tan A = 5 \sin A$ $\frac{2 \sin A}{\cos A} = 5 \sin A$ $2 \sin A - 5 \sin A \cos A = 0$ $\sin A(2 - 5 \cos A) = 0$ $\sin A = 0 \quad \text{or} \quad \cos A = \frac{2}{5}$ Basic angle = 66.421° $A = 0^\circ, 180^\circ, 66.4^\circ, -66.4^\circ$	M1 (attempt to solve using relevant manipulation) M1 (Factorisation) A1, A1 (per set)

(b)	$6 \cos^2 B + \sin B = 5$ $6(1 - \sin^2 B) + \sin B = 5$ $6 \sin^2 B - \sin B - 1 = 0$ $(3 \sin B + 1)(2 \sin B - 1) = 0$ $\sin B = -\frac{1}{3} \text{ or } \sin B = \frac{1}{2}$ $\text{Basic angle} = 0.33983, 0.52359 \left(\text{or } \frac{\pi}{6} \right)$ $B = 3.48, 5.94, 0.524, 2.62$ $\text{or } B = 3.48, 5.94, \frac{\pi}{6}, \frac{5\pi}{6}$	B1 (replace $\cos^2 A$) M1 (Factorisation) A1, A1 (for any correct pair) (8 marks)
4.	<u>Method 1</u> $\begin{aligned} \text{LHS} &= \cot \theta - \tan \theta \\ &= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} \\ &= \frac{\cos^2 \theta - \sin^2 \theta}{\frac{1}{2}(2 \sin \theta \cos \theta)} \\ &= \frac{2 \cos 2\theta}{\sin 2\theta} \\ &= 2 \cot 2\theta \\ &= \text{RHS} \end{aligned}$ <u>Method 2</u> $\begin{aligned} \text{RHS} &= 2 \cot 2\theta \\ &= \frac{2}{\tan 2\theta} \\ &= 2 \left(\frac{1 - \tan^2 \theta}{2 \tan \theta} \right) \\ &= \frac{1}{\tan \theta} - \tan \theta \\ &= \cot \theta - \tan \theta \\ &= \text{LHS} \end{aligned}$	B1 (convert $\tan \theta$ or $\cot \theta$) M1 (express as a single fraction) A1 (Convert to double angle) B1 (convert to $\tan 2\theta$) B1 (apply double angle formula) B1 (convert to partial fraction and conclusion) (3 marks)

5. (a) (b)	$a = -5$ $b = 2$ $c = 1$  3 solutions	B1 B1 B1 B1 (shape and 1 cycle) B1 (asymptote and x-intercepts) $\left(\frac{\pi}{2}, 2\right), \left(\frac{3\pi}{2}, -2\right)$ Must be indicated on the sketch (no penalty for this test) B1 (6 marks)
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6. (a)	$RW = 8 \cos \theta \text{ m}$ $RV = 5 \sin \theta \text{ m}$ $h = RX - RW$ $= 5 + 5 \sin \theta - 8 \cos \theta$	B1 (correct RW or RV) B1
(b)	Let $5 \sin \theta - 8 \cos \theta = R \sin(\theta - \alpha)$ $R = \sqrt{5^2 + 8^2}$ $= \sqrt{89}$ $\tan \alpha = \frac{8}{5}$ $\alpha = 1.0121$ $h = 5 + \sqrt{89} \sin(\theta - 1.01)$ Or $h = 5 + 9.43 \sin(\theta - 1.01)$	M1 (Finding R) M1 (finding α) A1
(c)	$h = 5 + \sqrt{89} \sin(\theta - 1.0121) = 7$ $\sin(\theta - 1.0121) = \frac{2}{\sqrt{89}}$ Basic angle = 0.21362 $\theta - 1.0121 = 0.21362$ $\theta = 1.23$	M1 A1 (7 marks)