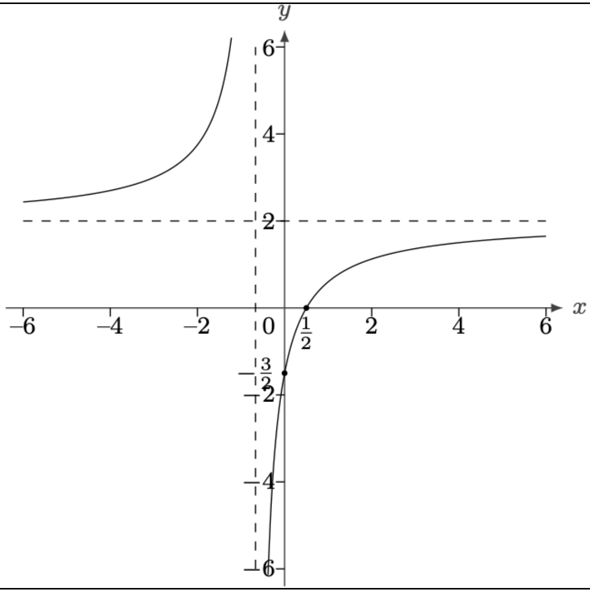
2022 Prelim MAA HL Paper 1 Mark Scheme
Section A

No	Solutions	Mark Scheme
1	$y=\int(\sin 2x)\mathrm{d}x$ $=\left[-\frac{\cos 2x}{2}\right]+c$ When $x=0, y=\frac{1}{2}$, $\frac{1}{2}=-\frac{1}{2}+c$ $c=1$ $y=-\frac{\cos 2x}{2}+1$, $y=-\cos^2 x+\frac{3}{2}$ $y=\sin^2 x+\frac{1}{2}$	
2(a)	Let the probability of getting an odd number and even number be *x* and *y* respectively. $3x+3y=1$ $y=2x$ $9x=1$ $x=\frac{1}{9}$ Probability of getting a 1 is $\frac{1}{9}$.	
2(b)	Required probability $=\left(\frac{1}{9}\right)^2\times\frac{2}{9}\times 3$ $=\frac{2}{243}$	
3(a)	$f(x)=\frac{ax-3}{3x+2}$ $x=-\frac{2}{3}$	
3(b)	$\frac{a}{3}=2$ $a=6$	

3(c)		
4(a)	$f(x)=2x-\frac{4}{x}$ $f'(x)=2+\frac{4}{x^2}$ $f'(2)=2+1=3$	
4(b)	$h(x)=g\left(f(x)\right)$ $h(2)=g\left(f(2)\right)$ $h(2)=g\left(2\right)$ $h(2)=1$	
4(c)	$h(x)=g\left(f(x)\right)$ $h'(x)=g'\left(f(x)\right)\times f'(x)$ $h'(2)=g'\left(f(2)\right)\times f'(2)$ $h'(2)=g'\left(2\right)\times f'(2)$ $h'(2)=4\times 3$ $h'(2)=12$ Equation of tangent: $y-1=12(x-2)$ or $y=12x-23$	

5	Method 1 Sub in $x=-1$ $-p-13+8p-15=0$ $7p=28$ $p=4$ $ab=-15$ $pab=-15\times 4$ $abp=-60$ Method 2 $(x+1)(px^2+apx+bpx+ab)$ $px^3+apx^2+bpx^2+abx+px^2+apx+bpx+ab$ $x^3\to p$ $x^2\to ap+bp+p=-13$ $x\to ab+ap+bp=-8p$ $0x\to ab=-15$ $a=3,b=-5,p=4$ $abp=3\times -5\times 4=-60$	
6	$\lvert v\rvert=a^2+b^2+25$ $27=a^2+b^2+25$ $2=a^2+b^2$ $u.v=1\times a+b\times -7+5\times -2$ $0=a-7b-10$ Solve for a and b $a=7b+10$ $a^2+b^2=2$ $b=-\frac{7}{5}$ $a=\frac{1}{5}$	\

7.	$x^2+\frac{4}{m}x+4=0$ $\alpha\beta=4$ $\alpha+\beta=-\frac{4}{m}$ $2k^2=4$ $3k=-\frac{4}{m}$ $k=\pm\sqrt{2}$ $m=\pm\frac{2\sqrt{2}}{3}$ Finding Roots $k=\frac{-2\pm2\sqrt{1-m^2}}{m}$ leads to no value for m	
8.a)	$\frac{e^x-1-x}{x(e^x-1)}$	
8.b)	Using l'hopital Sub in 0 and show indeterminate $\lim_{x\to0}\frac{e^x-1-x}{x(e^x-1)}=\frac{0}{0}$ Differentiate numerator and denominator. Product rule Show indeterminate 2nd time $\lim_{x\to0}\frac{e^x-1}{xe^x+e^x-1}=\frac{0}{0}$ Differentiate Simplify and eliminate e^x $\lim_{x\to0}\frac{e^x}{xe^x+e^x+e^x}=\frac{1}{x+2}=\frac{1}{2}$	

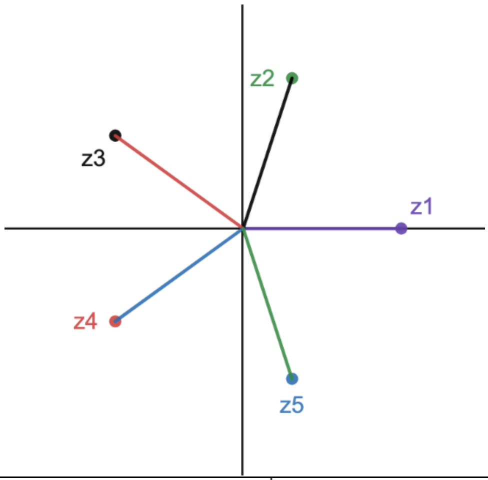
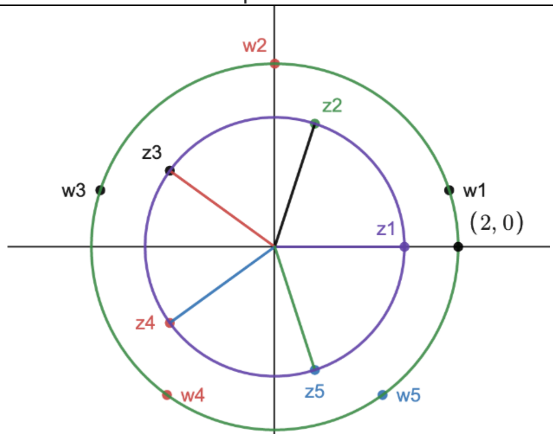
9.	$\frac{dy}{dx}+5\frac{y}{x}=\frac{\ln x}{x^2}$ Finding integrating factor $e^{\int\frac{5}{x}}=e^{5\ln x}=x^5$ $yx^5=\int x^3\ln x dx$ Integrating the RHS using parts $\int x^3\ln x dx=\frac{x^4\ln x}{4}-\int\frac{x^3}{4}dx$ $=\frac{x^4\ln x}{4}-\frac{x^4}{16}(+c)$ $yx^5=\frac{x^4\ln x}{4}-\frac{x^4}{16}(+c)$ $y=\frac{\ln x}{4x}-\frac{1}{16x}+\frac{C}{x^5}$	

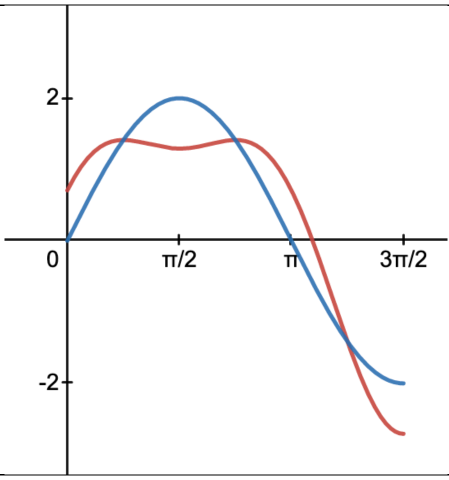
Section B

No	Solutions	Mark Scheme
10(a) (i)	$f(x)=\ln(ax)$ **Method 1:** $f^{-1}(1)=\frac{\mathrm{e}}{2}$ $f\left(\frac{\mathrm{e}}{2}\right)=1$ $\ln\left(a\times\frac{\mathrm{e}}{2}\right)=1$ $a=2$ **Method 2:** $f^{-1}(1)=\frac{\mathrm{e}}{2}$ $\frac{\mathrm{e}^1}{a}=\frac{\mathrm{e}}{2}$ $a=2$	
10(a) (ii)	$f(x)=\ln(2x)$ $y=\ln(2x)$ $x=\frac{1}{2}\mathrm{e}^y$ $f^{-1}(x)=\frac{1}{2}\mathrm{e}^x$	
10(b) (i)	$\ln(54x),\ \ln(px),\ \ln(qx),\ \ln(2x)$ $3d=\ln(2x)-\ln(54x)$ $3d=\ln\left(\frac{2x}{54x}\right)$ $3d=\ln\frac{1}{27}$ $3d=-3\ln 3$ $d=-\ln 3$ $d=\ln\left(\frac{1}{3}\right)$	
10(b) (ii)	$\ln(px)-\ln(54x)=\ln\frac{1}{3}$ $\ln\left(\frac{px}{54x}\right)=\ln\frac{1}{3}$ $\frac{54}{p}=3$ $p=18$	

	$\ln(qx)-\ln(54x)=2\ln\frac{1}{3}$ $\ln\left(\frac{qx}{54x}\right)=\ln\frac{1}{9}$ $q=54\times\frac{1}{9}$ $q=6$	
10(b) (iii)	Method 1: $S_5=\frac{5}{2}\left[2\ln(54x)+4\ln\frac{1}{3}\right]$ $S_5=5\left[\ln(54x)+2\ln\frac{1}{3}\right]$ $S_5=5\ln\left(\frac{54x}{3^2}\right)$ $S_5=5\ln(6x)$ Method 2: $S_5=\frac{5}{2}\left[\ln(54x)+(\ln 2x+\ln\frac{1}{3})\right]$ $S_5=\frac{5}{2}\left[\ln\frac{54x\times 2x}{3}\right]$ $S_5=\frac{5}{2}\left[\ln(36x^2)\right]$ $S_5=\frac{5}{2}\left[2\ln(6x)\right]$ $S_5=5\ln(6x)$	
10(b) (iv)	$S_\infty=\frac{5}{1-\frac{1}{3}}$ $3S_5=\frac{5}{1-\frac{1}{3}}$ $3\times5\ln(6x)=\frac{5\times3}{2}$ $\ln(6x)=\frac{1}{2}$ $6x=e^{\frac{1}{2}}$ $x=\frac{e^{\frac{1}{2}}}{6}$	

No	Solutions	Mark Scheme
11		
(a)(i)	$\frac{1}{z}=\cos\theta-i\sin\theta$ $z+\frac{1}{z}=\cos\theta+i\sin\theta+\cos\theta-i\sin\theta$ $=2\cos\theta$	
(ii)	Method 1 $\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$ $\left(z+\frac{1}{z}\right)^5=z^5+5z^3+10z+\frac{10}{z}+\frac{5}{z^3}+\frac{1}{z^5}$ $\left(z+\frac{1}{z}\right)^5=\cos 5\theta+i\sin 5\theta+5(\cos 3\theta+i\sin 3\theta)+10(\cos\theta+i\sin\theta)$ $+10(\cos\theta-i\sin\theta)+5(\cos 3\theta-i\sin 3\theta)+(\cos 5\theta-i\sin 5\theta)$ $32\cos^5\theta=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ $\cos^5\theta=\frac{1}{16}(\cos 5\theta+5\cos 3\theta+10\cos\theta)$ Method 2 $\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$ $\left(z+\frac{1}{z}\right)^5=z^5+5z^3+10z+\frac{10}{z}+\frac{5}{z^3}+\frac{1}{z^5}$ $\left(z+\frac{1}{z}\right)^5=\left(z^5+\frac{1}{z^5}\right)+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)$ $z^n+\frac{1}{z^n}=2\cos n\theta$ $32\cos^5\theta=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ $\cos^5\theta=\frac{1}{16}(\cos 5\theta+5\cos 3\theta+10\cos\theta)$	
(iii)	$\cos\alpha=\frac{2\sqrt{2}}{3}$ $\cos^5\alpha=\left(\frac{2\sqrt{2}}{3}\right)^5$ $=\frac{32\times 4\sqrt{2}}{3^5}$ $\frac{16\sqrt{2}}{27}(\cos 5\theta+5\cos 3\theta+10\cos\theta)=\frac{16\sqrt{2}}{27}\times 16\times\frac{2^5\times 4\sqrt{2}}{3^5}$ $=\frac{2^4}{3^3}\times 2^4\times\frac{2^5\times 2^2\times 2}{3^5}$ $=\frac{2^{16}}{3^8}=\left(\frac{4}{3}\right)^8=\left(\frac{65536}{6561}\right)$	

(b)(i)	$z_1 = \sqrt{2}\left(\cos(0) + i\sin(0)\right)$ $z_2 = \sqrt{2}\left(\cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)\right)$ $z_3 = \sqrt{2}\left(\cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)\right)$ $z_4 = \sqrt{2}\left(\cos\left(\frac{6\pi}{5}\right) + i\sin\left(\frac{6\pi}{5}\right)\right) = \sqrt{2}\left(\cos\left(-\frac{4\pi}{5}\right) + i\sin\left(-\frac{4\pi}{5}\right)\right)$ $z_5 = \sqrt{2}\left(\cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)\right) = \sqrt{2}\left(\cos\left(-\frac{2\pi}{5}\right) + i\sin\left(-\frac{2\pi}{5}\right)\right)$ 	
(ii)		
(iii)	$v^5 = w^5\left(\sqrt{2}\right)^5\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$ Or $= 32i$	

12		
(a) (i)	$f(x)=2\sin x+\frac{1}{\sqrt{2}}\cos 2x$ $f'(x)=2\cos x+\frac{1}{\sqrt{2}}(-2\sin 2x)$ $f'(x)=2\cos x-\sqrt{2}\sin 2x$	
(ii)	$2\cos x-\sqrt{2}\sin 2x=0$ $2\cos x-2\sqrt{2}\sin x\cos x=0$ $2\cos x\left(1-\sqrt{2}\sin x\right)=0$ $\cos x=0$ $x=-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}$ $\sin x=\frac{1}{\sqrt{2}}$ $x=\frac{\pi}{4},\frac{3\pi}{4}$	
(iii)	$-2-\frac{\sqrt{2}}{2}\le f(x)\le\sqrt{2}$	
(b)	$2\sin x+\frac{1}{\sqrt{2}}\cos 2x=2\sin x$ $\cos 2x=0$ $2x=\frac{\pi}{2},\frac{3\pi}{2}$ $x=\frac{\pi}{4},\frac{3\pi}{4}$	
c (i)		

(ii)	$[g(x)]^2-[f(x)]^2=[2\sin x]^2-\left[2\sin x+\frac{1}{\sqrt{2}}\cos 2x\right]^2$ $=4\sin^2 x-\left(4\sin^2 x+\frac{4}{\sqrt{2}}\sin x\cos 2x+\frac{1}{2}\cos^2 2x\right)$ $=-2\sqrt{2}\cos 2x\sin x-\frac{1}{2}\cos^2 2x$	
(iii)	$a=\frac{\pi}{4}$ $b=\frac{3\pi}{4}$ $\pi\int_a^b y^2dx$ $y^2=-2\sqrt{2}\cos 2x\sin x-\frac{1}{2}\cos^2 2x$ $\cos^2 2x=\frac{1}{2}\cos 4x+\frac{1}{2}$ $\frac{1}{2}\cos^2 2x=\frac{1}{4}\cos 4x+\frac{1}{4}$ $2\cos 2x\sin x=\sin(2x+x)-\sin(2x-x)$ $2\cos 2x\sin x=\sin(3x)-\sin(x)$ $-2\sqrt{2}\cos 2x\sin x=-\sqrt{2}(\sin(3x)-\sin(x))$ $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(-\sqrt{2}(\sin(3x)-\sin(x))-\frac{1}{4}\cos 4x-\frac{1}{4}\right)dx$ $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(-\sqrt{2}\sin 3x+\sqrt{2}\sin x-\frac{1}{4}\cos 4x-\frac{1}{4}\right)dx$ $p=-\sqrt{2}$ $q=\sqrt{2}$ $n=-\frac{1}{4}$ $m=-\frac{1}{4}$	

(iv)	$\pi\left[\sqrt{2}\dfrac{\cos 3x}{3}-\sqrt{2}\cos x-\dfrac{\sin 4x}{16}-\dfrac{x}{4}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$ $\pi\left[\left[\sqrt{2}\dfrac{\cos\left(\frac{9\pi}{4}\right)}{3}-\sqrt{2}\cos\left(\dfrac{3\pi}{4}\right)-\dfrac{\sin\left(\frac{12\pi}{4}\right)}{16}-\dfrac{\left(\frac{3\pi}{4}\right)}{4}\right]-\left[\sqrt{2}\dfrac{\cos\left(\frac{3\pi}{4}\right)}{3}-\sqrt{2}\cos\left(\dfrac{3\pi}{4}\right)-\dfrac{\sin\left(\frac{4\pi}{4}\right)}{16}-\dfrac{\left(\frac{\pi}{4}\right)}{4}\right]\right]$ $\pi\left[\left[\sqrt{2}\dfrac{\cos\left(\frac{9\pi}{4}\right)}{3}-\sqrt{2}\cos\left(\dfrac{3\pi}{4}\right)-\dfrac{\sin(3\pi)}{16}-\dfrac{3\pi}{16}\right]-\left[\sqrt{2}\dfrac{\cos\left(\frac{3\pi}{4}\right)}{3}-\sqrt{2}\cos\left(\dfrac{3\pi}{4}\right)-\dfrac{\sin\left(\frac{4\pi}{4}\right)}{16}-\dfrac{\pi}{16}\right]\right]$ $\pi\left[\left[\dfrac{1}{3}+1-\dfrac{3\pi}{16}\right]-\left[-\dfrac{1}{3}-1-\dfrac{\pi}{16}\right]\right]$ $\pi\left[\left[\dfrac{8}{3}-\dfrac{\pi}{8}\right]\right]$ $V=\pi\left(\dfrac{8}{3}-\dfrac{\pi}{8}\right)$	