Year 6 Mathematics: analysis and approaches - MARKSCHEME
Higher Level
Paper 3
Preliminary Examinations

Friday 28 August 2023

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions on the answer sheets provided. Write your name and class on each answer sheet, and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is [**55 marks**].

Question	Marks
1	
2	

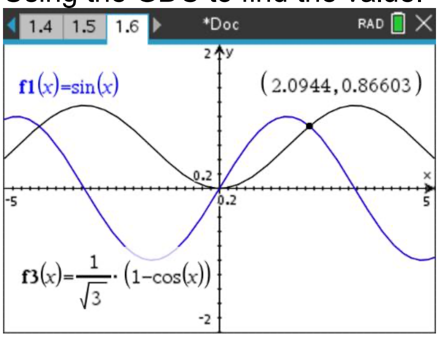
Name: ____________________

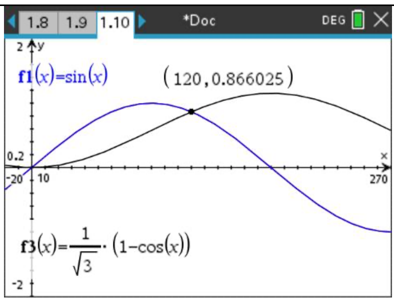
Class: ____________________

Index: ____________________

/ 55

1.

(a)(i)	At E $y = 0 = a(1-\cos\theta)$ $\cos\theta = 1$ $\theta = 2\pi$ So $OE = x = a(2\pi - \sin 2\pi)$ $OE = 2a\pi$
(a)(ii)	Max height at $\theta = \pi$ $y = a(1-\cos\pi) = 2a$
(b)(i)	$\frac{dy}{d\theta} = a\sin\theta$ $\frac{dx}{d\theta} = a - a\cos\theta$
(b)(ii)	$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$ $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{a\sin\theta}{a - a\cos\theta} = \frac{\sin\theta}{1-\cos\theta}$
(c) (i)	$\tan 30^\circ = \frac{1}{\sqrt{3}}$ Or gradient $= \frac{1}{\sqrt{3}}$. $\frac{\sin\theta}{1-\cos\theta} = \frac{1}{\sqrt{3}}$ $\sin\theta = \frac{1}{\sqrt{3}}(1-\cos\theta)$
(c) (ii)	Using the GDC to find the value.  $\theta = 2.0944$ $\theta = 2.09 \times \frac{180}{\pi}$ $\theta = 120$

	 $\theta = 120$
(c)(iii)	$BF = y = a(1-\cos\theta)$ $BF = a(1-\cos(2.09))$ $BF = 1.5a = \frac{3}{2}a$
(c)(iv)	$OF = x = a(2.0944 - \sin(2.0944))$ $OF = 1.23a$
(d)	BC= their OE – 2 their OF $BC = 2a\pi - 2(1.23a) = 3.8232a$ $AF = \frac{BF}{\tan 30^\circ} = 2.5981a$
(e)	$AD = BC + 2AF$ $AD = 3.8232a + 2(2.5981a)$ $20 = 9.0164a$ $a = 2.22$ Can have FT.
(f)	$\frac{\mathrm{d}y}{\mathrm{d}\theta} = 8\sin 2\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 4 - 4\cos\theta$ $L_{PQ} = \int_a^b \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta$ $L_{PQ} = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sqrt{(4-4\cos\theta)^2 + (8\sin 2\theta)^2}\,\mathrm{d}\theta$ $L_{PQ} = 26.8163$ $L_{PQ} = 26.8\text{m}(3\text{sf})$
(g)	Part (f) + MP +QN $MP = 4\left(\frac{\pi}{2} - \sin\frac{\pi}{2}\right) = 2.2832$

	$QN = 25 - 4\left(\frac{3\pi}{2} - \sin\frac{3\pi}{2}\right) = 2.15044$ $MPTQN = 26.8163 + 2.2832 + 2.15044$ $MPTQN = 31.24994$ $MPTQN = 31.2(3\text{sf})$

2.

(a)(i)	$u=(1-x)^n \Rightarrow \frac{du}{dx}=n(1-x)^{n-1}(-1)$ $\frac{dv}{dx}=x^{n-1} \Rightarrow v=\frac{x^n}{n}$ $\left[(1-x)^n\frac{x^n}{n}\right]_0^1-\int_0^1\frac{x^n}{n}n(1-x)^{n-1}(-1)\,dx$ $\int_0^1 x^{n-1}(1-x)^n\,\mathrm{d}x=\int_0^1 x^n(1-x)^{n-1}\,\mathrm{d}x$
(a)(ii)	Replace one of the terms $\int_0^1 x^n(1-x)^{n-1}\,\mathrm{d}x+\int_0^1 x^{n-1}(1-x)^n\,\mathrm{d}x$ $\int_0^1 x^{n-1}(1-x)^{n-1}(x+1-x)\,\mathrm{d}x$ $\int_0^1 x^{n-1}(1-x)^{n-1}\,\mathrm{d}x=I_{n-1}$
(b)(i)	$u=x^n \Rightarrow \frac{du}{dx}=nx^{n-1}$ $\frac{dv}{dx}=(1-x)^n \Rightarrow v=\frac{(1-x)^{n+1}}{(-1)(n+1)}$ $\left[x^n\frac{(1-x)^{n+1}}{(-1)(n+1)}\right]_0^1-\int_0^1\frac{(1-x)^{n+1}}{(-1)(n+1)}nx^{n-1}\,dx$ $\frac{n}{n+1}\int_0^1 x^{n-1}(1-x)^{n+1}\,dx$
(b)(ii)	$\frac{n}{n+1}\int_0^1 x^{n-1}(1-x)^n(1-x)\,dx$ $\frac{n}{n+1}(\int_0^1 x^{n-1}(1-x)^n\,dx-\int_0^1 x^n(1-x)^n\,dx)$ $I_n=\frac{n}{n+1}(\frac{1}{2}I_{n-1}-I_n)$ $\frac{2n+1}{n+1}I_n=\frac{n}{2(n+1)}I_{n-1}$ $I_n=\frac{n}{2(2n+1)}I_{n-1}$

(c)	n=1 $LHS = I_1 = \int_0^1 x(1-x)\,\mathrm{d}x$ $= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$ $RHS = \frac{(1!)^2}{3!} = \frac{1}{6}$ Assume n=k is true n=k+1 $I_{k+1} = \frac{k+1}{2[2(k+1)+1]} I_k$ $= \frac{k+1}{2[2k+3]} \frac{(k!)^2}{(2k+1)!}$ $= \frac{(k+1)^2 (k!)^2}{(2k+2)(2k+3)(2k+1)!}$ $= \frac{((k+1)!)^2}{(2k+3)!} = RHS$ Since n=1 is true and n=k+1 is true when n=k is true, hence by induction the statement is true for positive integers. (Note: must get at least 5 marks to get the last A1)
(d)	$\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sin\theta\cos\theta$ $= \int_0^{\pi/2} 2\sin^2\theta\cos^2\theta\,\mathrm{d}\theta$ $= \frac{1}{4}\int_0^{\pi/2} (1-\cos 4\theta)\,\mathrm{d}\theta$ $= \frac{1}{4}\left[\theta - \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}}$ $= \frac{1}{4}\left(\frac{\pi}{2} - \frac{1}{4}\sin 2\pi\right)$ $= \frac{\pi}{8}$