SCHOOL OF THE ARTS SINGAPORE

Year 6 Mathematics: analysis and approaches
Higher level
Paper 1
Preliminary Examinations

Friday 25 August 2023

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your name and class on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Section A		Section B	
Question	**Marks**	**Question**	**Marks**
1		10	
2		11	
3		12	
4			
5			
6			
7			
8			
9			

Name:

Class:

Index:

110

13pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

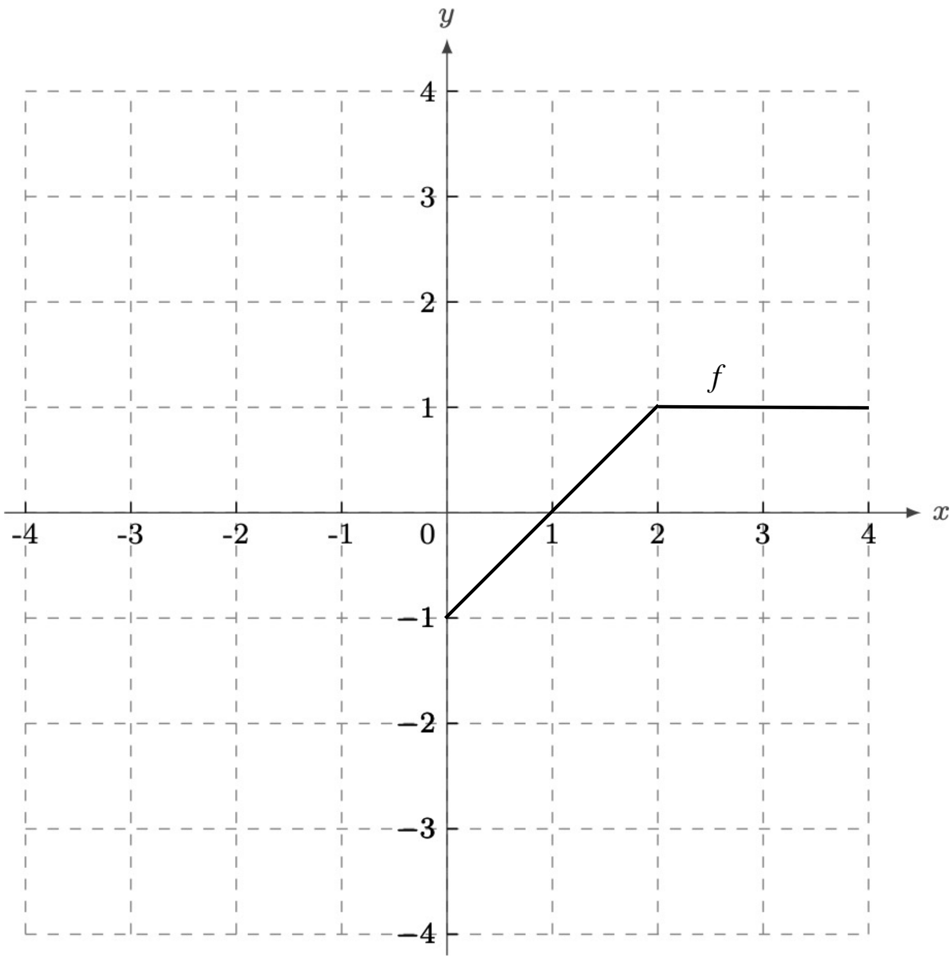
1. [Maximum mark: 4]

(a) Show that $(3n+1)^2-(n+1)^2=8n^2+4n$, where $n \in \mathbb{Z}$. [2]

(b) Hence, prove that $(3n+1)^2-(n+1)^2$ is a multiple of 4. [2]

2. [Maximum mark: 5]

The following diagram shows the graph of a function $y = f(x)$, with domain $0 \leq x \leq 4$.



(a) (i) Write down $f'(3)$;

(ii) Find $(f \circ f)(1)$. [3]

(b) On the same digram, sketch the graph of $y = 3f(-x)$. [2]

....

....

....

....

3. [Maximum mark: 6]

Consider the function $f(x) = 2x^2 - mx + m - 2$, where m is a constant.

(a) Find the value of m for which the x–axis is a tangent to the graph of f. [4]

(b) Find the range of values of m for which the graph of f has a positive y–intercept. [2]

....

....

....

....

....

....

....

....

....

....

....

....

4. [Maximum mark: 5]

The derivative of the function f is given by $f'(x) = \dfrac{6}{(1-x)^4}$.

The graph of $y = f(x)$ has an x–intercept at $x = 3$. Find an expression for $f(x)$.

5. [Maximum mark: 9]

The following table shows the probability distribution of a discrete random variable X.

x	0	1	2
$\mathrm{P}(X=x)$	$\frac{1}{3}$	$\sin\theta$	$\frac{2}{3}-\cos 2\theta$

(a) Explain why $\cos 2\theta < \frac{2}{3}$. [1]

(b) Show that $\sin\theta = \frac{1}{2}$. [5]

(c) Hence, find $\mathrm{E}(X)$. [3]

....

....

....

....

....

....

....

....

....

....

6. [Maximum mark: 6]

A particle moves along a straight line so that after t seconds, its displacement s, in metres, satisfies the equation $s^3 + s - 2t = 0$.

(a) Show that its velocity, $v = \dfrac{2}{1+3s^2}$. [3]

(b) Find its acceleration, in terms of s. [3]

7. [Maximum mark: 7]

Consider $f(x)=\dfrac{1}{x-1}, \quad x \neq 1$.

(a) Sketch the graph of $y=f(x)$, indicating the asymptotes and the axial intercepts. [3]

(b) $g(x)=\left|f(x+p)\right|$ is an even function. Find the value of p. [2]

(c) Find the set of values which satisfy $g\left(x\right)=g\left(\left|x\right|\right)$. [2]

....

....

....

....

....

....

8. [Maximum mark: 8]

Consider the following system of equations where $a, b \in \mathbb{R}$.

$$\begin{aligned} x+2y+z&=5 \\ 2x+y-4z&=b \\ x+y+az&=4 \end{aligned}$$

(a) Find the value of a for which the system of equations does not have a unique solution. [4]

(b) For this value of a, find the value of b for which the system is consistent. [1]

(c) In the case when the system has infinitely many solutions, find the general solution. [3]

9. [Maximum mark: 8]

Two basketball players, Amanda and Clariss, try to shoot a free throw. Amanda shoots first, then Clariss. The probability that Amanda scores is $\frac{2}{3}$ and the probability that Clariss scores is $\frac{1}{2}$. The shots are independent of each other and the first player who scores wins.

(a) Find the probability that Amanda wins with her second shot. [2]

(b) Show that the probability that Amanda wins is $\frac{4}{5}$. [4]

(c) Given that Amanda wins, find the probability that Amanda wins with her second shot. [2]

....

....

....

....

....

....

....

....

....

....

....

....

Do **not** write solutions on this page.

Section B

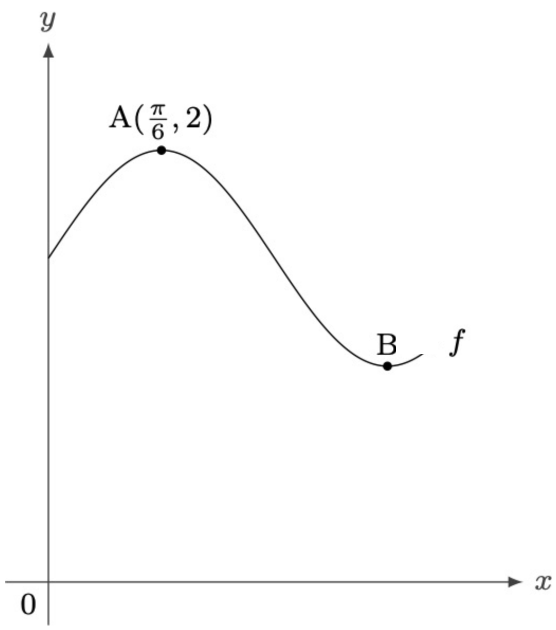
Answer **all** questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 15]

Let $f(x) = 0.5\sin(px) + q$, $0 \le x \le 1.8$, be a periodic function with $f(x) = f\left(x + \frac{2\pi}{3}\right)$.

The graph of f has a maximum point at $\mathrm{A}\left(\frac{\pi}{6}, 2\right)$ and a minimum point at B.

The following diagram shows the graph of f .



(a) (i) Write down the amplitude of f .

(ii) Find the minimum value of f .

(iii) Find the value of q. [5]

(b) (i) Write down the period of f .

(ii) Find the x–coordinate of B.

(iii) Show that $p = 3$. [4]

(c) (i) Find $f'(x)$.

(ii) At point $x = a$, the gradient of the tangent to the curve is -1.5.

Find the value of a. [6]

Turn over

Do **not** write solutions on this page.

11. [Maximum mark: 18]

Points $\mathrm{A}\ (3, 1, -3)$, $\mathrm{B}\ (2, 3, -1)$ and $\mathrm{C}\ (2, 4, 1)$ lie in the plane Π.

(a) Find $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. [2]

(b) Find the area of the triangle ABC. [2]

(c) Find the Cartesian equation of Π. [2]

Point D has coordinates $(-3, -3, 4)$.

(d) Find a vector equation of the line L through D perpendicular to Π. [2]

(e) (i) Find the point of intersection of L and Π.

(ii) Hence, find the distance of D from Π. [6]

Point $\mathrm{D'}$ is the mirror image of D in the plane Π.

(f) Find the coordinates of $\mathrm{D'}$. [2]

(g) Find the volume of the bipyramid $\mathrm{DABCD'}$. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

(a) Show that $\left(z-\mathrm{e}^{i\theta}\right)\left(z-\mathrm{e}^{-i\theta}\right)=z^2-2z\cos\theta+1$. [4]

(b) Find the four 4^{th} roots of -1 in the form $\mathrm{e}^{i\theta}$, where $-\pi<\theta\le\pi$. [5]

(c) Express $z^4+1=\left(z^2-2z\cos\alpha+1\right)\left(z^2-2z\cos\beta+1\right)$ where α and β are to be determined. [3]

(d) (i) Find a similar expression for z^8+1.

(ii) Hence, by substituting $z=i$, show that

$$\cos\left(\frac{\pi}{8}\right)\cos\left(\frac{3\pi}{8}\right)\cos\left(\frac{5\pi}{8}\right)\cos\left(\frac{7\pi}{8}\right)=\frac{1}{8}.$$ [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.