BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME		CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS 4049 / 01
Paper 1 2 hours 15 minutes

90

Answer **all** the questions.

1 Prove that $\sin 2x - \cos 2x \tan x = \tan x$. [4]

2 Show that the solution of $2^{3x+4} \times 5^{2x+1} = 5^{3(x+1)} \times 16^x$ is $2\lg\frac{4}{5}$. [4]

3 Giving your answer in the form $\frac{c+d\sqrt{2}}{3}$, solve, without using a calculator,

$$x\sqrt{18} = 3x + \sqrt{32}.$$ [5]

4 A square of area $(11+\sqrt{120})\ \text{cm}^2$ has a length of $(\sqrt{a}+\sqrt{b})$ cm, where $a < b$.
Without using a calculator, find the values of *a* and *b*. [5]

5 The line $3x + 4y = 13$ intersects the curve $y = \frac{6x-10}{3x-1}$ at points *P* and *Q*.
Calculate the exact length of the line segment *PQ*. [5]

6 The coefficient of x^3 in the expansion of $(10+kx)\left(2-\frac{1}{2}x\right)^8$ is zero.
Find the value of the constant *k*. [5]

7 Solve the equation $\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$ for $0 \le x \le 180°$. [6]

8 **(a)** Show that the derivative of $x(x-3)^4$ with respect to *x* is $(5x-3)(x-3)^3$. [2]

(b) Hence, given that $y = \frac{x(x-3)^4}{5x-3}$ and y is decreasing at a constant rate of 70 units/s, calculate the rate of change of *x* when $x = 1$. [4]

9 **(a)** Express $y = 5 - 12x - 3x^2$ in the form $y = a(x+b)^2 + c$ and hence show that y can never be greater than 20. [3]

(b) Explain why there are no values of k for which the curve
$y = (k-1)x^2 + 2(k+2)x + k + 3$ is always positive. [4]

10 A rectangular field has sides $(3x-5)$ m and $(x-10)$ m. Its area is at most 200 m^2.

(a) Find the range of values of x that satisfies the above sides and area conditions. [4]

(b) Justify whether a fence of 98 m is enough to enclose the field. [3]

11 **(a)** State the range of values of x for which the equation below is valid.

$$2\log_3(4-x) - \log_9(x-4)^2 = 2$$ [1]

(b) Express $2\log_3(4-x) - \log_9(x-4)^2 = 2$ as a quadratic equation $x^2 + bx + c = 0$ and explain why there is only one real solution. [6]

12 A quadratic curve is given by $y = hx^2 - 2x - 9h + 6$, where h is a constant.

(a) Show that the equation $y = 0$ has real roots for all values of h except $h = 0$. [3]

(b) State the value of h in the case where $y = 0$ has two real and equal roots. [1]

(c) Given that the line $y = 2x - h - 12$ meets the curve $y = hx^2 - 2x - 9h + 6$, find the range of values of h. [5]

13 It is given that $f(x) = x^2 e^{x+2}$.

(a) Show that the range of values of x for which $f(x)$ is a decreasing function is $-2 < x < 0$. [4]

(b) The gradient with the least value is in the range $-2 < x < 0$.
Find the value of this gradient, giving your answer in exact form. [5]

14 It is given that $f(x) = 3x^3 + 2x^2 + 16$. The remainder when $f(x)$ is divided by $x - 3a$, where a is a constant, is the same as the remainder when it is divided by $3x + 2$.

(a) Find the possible values of a. [3]

(b) Show, with clear working, that $x = -2$ is a solution of $f(x) = 0$. [1]

(c) Explain why $f(x) = 0$ has only one real root. [5]

(d) Hence use your answers to parts (b) and (c) to solve the equation

$$16y^3 + 2y - 3 = 0.$$ [2]

END OF PAPER

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

ANSWERS
(Bedok South) AM4 P1 (2025) Students

1. **Hint:** Convert to $\sin x$ and $\cos x$ by apply double angle formulae.

2. $2\lg\left(\frac{4}{5}\right)$

3. $\frac{8+4\sqrt{2}}{3}$

4. $a = 5,\ \ b = 6$

5. $PQ = 5$ units

6. $k = 5$

7. $30°,\ \ 90°,\ \ 150°$

8. (a) **Hint:** Apply Product Rule
$\frac{\mathrm{d}(uv)}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}v + u\frac{\mathrm{d}v}{\mathrm{d}x}$

(b) 2.5 units/s

9. (a) $y = -3(x+2)^2 + 17$

(b) **Hint:** Apply the two conditions:
(i) coefficient of $\underline{x^2 > 0}$
(ii) discriminant < 0

10.(a) $10 < x \leq 15$

(b) It is enough to enclose the field.

11.(a) $x < 4$

(b) $x = -5$

12.(a) **Hint:** Apply completing the square to the discriminant.

(b) $h = \frac{1}{3}$

(c) $h \leq \frac{1}{4}$ or $h \geq 2$ and $h \neq 0$

13.(a) **Hint:** For decreasing function, $\mathrm{f}'(x) < 0$

(b) $\left(2 - 2\sqrt{2}\right)e^{\sqrt{2}}$

14.(a) $a = 0$ or $a = -\frac{2}{9}$

(b) **Hint:** Apply Factor Theorem

(c) **Hint:** Show that discriminant < 0

(d) $y = \frac{1}{2}$

MARKING SCHEME

(Bedok South) 4E5N AM P1 (Prelim 2025) (w Ans)

Qn	Solution	Marks	Testing
1	**Trigonometric Identities**		
	LHS		
	$= \sin 2x - \cos 2x \tan x$		
	$= 2\sin x\cos x - (2\cos^2 x - 1)\dfrac{\sin x}{\cos x}$	M2	Apply double angle for sine & cosine
	$= 2\sin x\cos x - 2\sin x\cos x + \dfrac{\sin x}{\cos x}$	M1	Expand & simplify
	$= \tan x$	A1	
	$= RHS$		
2	**Exponential Equations**		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^4(2^{2x}) \times 5 = 5^3(5^x) \times 2^{3x}$	M1	Split the powers
	$\dfrac{2^{2x}}{(2^{3x})(5^x)} = \dfrac{5^3}{2^4 \times 5}$	M1	Separate factors with x and non-x
	$\dfrac{1}{(2^x)\,(5^x)} = \dfrac{25}{16}$		
	$\dfrac{1}{10^x} = \left(\dfrac{5}{4}\right)^2$		Combine common index
	$10^x = \left(\dfrac{4}{5}\right)^2$		
	$x = \lg\left(\dfrac{4}{5}\right)^2$	M1	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	A1	Power law
	OR		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^{2x+4} \times 5 = 5^{x+3} \times 2^{3x}$	M1	Convert to prime bases
	$\dfrac{2^{2x+4}}{2^{3x}} = \dfrac{5^{x+3}}{5}$	M1	Separate factors of different bases
	$\dfrac{2^4}{2^x} = 5^x 5^2$		
	$10^x = \dfrac{2^4}{5^2}$		
	$10^x = \dfrac{4^2}{5^2}$		Combine common index
	$x = \lg\left(\dfrac{4}{5}\right)^2$	M1	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	A1	Power law
3	**Surds Expressions**		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3x\sqrt{2} - 3x = 4\sqrt{2}$		
	$3x(\sqrt{2} - 1) = 4\sqrt{2}$	M1	Grouping
	$3x = \dfrac{4\sqrt{2}}{\sqrt{2} - 1}$		
	$= \dfrac{4\sqrt{2}\ (\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$	M1	Rationalise denominator with conjugate
	$= \dfrac{8 + 4\sqrt{2}}{(\sqrt{2})^2 - (1)^2}$	M1	Expansion
		M1	Difference of squares
	$\boldsymbol{x = \dfrac{8 + 4\sqrt{2}}{3}}$	A1	
	OR		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3\sqrt{2}x = 3x + 4\sqrt{2}$		
	$(3\sqrt{2} - 3)x = 4\sqrt{2}$	M1	Grouping
	$x = \dfrac{4\sqrt{2}}{3\sqrt{2} - 3}$		
	$= \dfrac{4\sqrt{2}\ (3\sqrt{2}+3)}{(3\sqrt{2}-3)(3\sqrt{2}+3)}$	M1	Rationalise denominator with conjugate
	$= \dfrac{24 + 12\sqrt{2}}{(3\sqrt{2})^2 - (3)^2}$	M1	Expansion
		M1	Difference of squares
	$= \dfrac{24 + 12\sqrt{2}}{9}$		
	$\boldsymbol{= \dfrac{8 + 4\sqrt{2}}{3}}$	A1	
4	**Surds (Application)**		
(a)	Area of square $= 11 + \sqrt{120}$		
	$(\sqrt{a} + \sqrt{b})^2 = 11 + 2\sqrt{30}$		
	$a + b + 2\sqrt{ab} = 11 + 2\sqrt{30}$	M1	Expand
	Equating the <u>rational</u> part,	M1	Equating rational & irrational parts separately
	$a + b = 11$		
	$b = 11 - a$......(1)		
	Equating the <u>irrational</u> part,		
	$2\sqrt{ab} = 2\sqrt{30}$		
	$ab = 30$ ......(2)		
	Subst. (1) into (2),		
	$a(11 - a) = 30$	M1	Substitution method
	$11a - a^2 = 30$		
	$a^2 - 11a + 30 = 0$		
	$(a - 5)(a - 6) = 0$		
	$a = 5$ or $a = 6$	M1	Solve quadratic equation
	Corresponding b values:		
	$b = 6$ or $b = 5$		
	Since $a < b$		
	$\boldsymbol{a = 5,\ \ b = 6}$	A1	Correct set of values

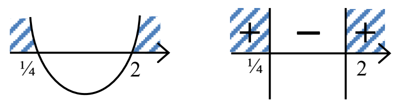
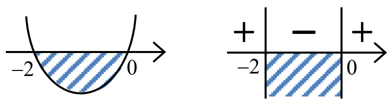
Qn	Solution	Marks	Testing
5	**Simultaneous Equations**		
	$4x - y = 5 \quad \ldots\ldots [1]$		
	$y = 2x^2 - 6x + 7 \quad \ldots\ldots [2]$		
	Subst (2) into (1)		
	$4x - (2x^2 - 6x + 7) = 5$	M1	Substitution or Elimination Method
	$4x - 2x^2 + 6x - 7 = 5$		
	$2x^2 - 10x + 12 = 0$		
	$x^2 - 5x + 6 = 0$		
	$(x-2)(x-3) = 0$	M1	Factorisation or Quadratic Formula
	$x = 2 \quad or \quad x = 3$		
	Subst respective x values into [1],		
	$y = 4(2) - 5$, $y = 3$; $y = 4(3) - 5$, $y = 7$	M1	Find values of 2nd variable
	$P(2,3)$ and $Q(3,7)$		
	$PQ = \sqrt{(3-2)^2 + (7-3)^2}$	M1	Apply Distance formula
	$PQ = \sqrt{\mathbf{17}}$	A1	
6	**Binomial Theorem**		
	General term of expansion $\left(2 - \frac{1}{2}x\right)^8$ is		
	$T_{r+1} = \binom{8}{r}(2)^{8-r}\left(-\frac{x}{2}\right)^r$	M1	General term of binomial expansion
	$= \binom{8}{r}(-1)^r(2)^{8-2r}\,x^r$		
	$T_3 = \binom{8}{2}(-1)^2(2)^{8-2(2)}\,x^2$		
	$= 448x^2$	A1	Extract specific terms
	$T_4 = \binom{8}{3}(-1)^3(2)^{8-2(3)}\,x^3$		
	$= -224x^3$	A1	
	$(10 + kx)\left(2 - \frac{1}{2}x\right)^8$		
	$= (10 + kx)(\ldots 448x^2 - 224x^3 \ldots)$		
	Coeff. of $x^3 = 0$		
	$(10)(-224) + k(448) = 0$	M1	Equating coefficient
	$448k = 2240$		
	$\boldsymbol{k = 5}$	A1	
7	**Trigonometric Equations**		
	$\dfrac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$		
	$\dfrac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \dfrac{\cos x}{\sin x}$	M1	Convert to $\sin x$ & $\cos x$
	$\sin x\,(5\cos x + \sin 2x)$		
	$= \cos x\,(\cos 2x + 5\sin x)$		

Qn	Solution	Marks	Testing
	$\sin x\,(5\cos x + 2\sin x\cos x)$	M1	Apply double angle formula $\sin 2x$ & $\cos 2x$
	$= \cos x\,(\cos^2 x - \sin^2 x + 5\sin x)$		
	$5\sin x\cos x + 2\sin^2 x\cos x$		
	$= \cos^3 x - \sin^2 x\cos x + 5\sin x\cos x$		
	$2\sin^2 x\cos x = \cos^3 x - \sin^2 x\cos x$		
	$3\sin^2 x\cos x - \cos^3 x = 0$	M1	Expand & simplify
	$\cos x\,(3\sin^2 x - \cos^2 x) = 0$		
	$\cos x = 0$; $3\sin^2 x - \cos^2 x = 0$	M1	Zero product rule
	$x = 90°$; $3\sin^2 x = \cos^2 x$		
	$3\tan^2 x = 1$		
	$\tan^2 x = \dfrac{1}{3}$		
	$\tan x = \pm\dfrac{1}{\sqrt{3}}$		Either solution correct
	$x = 30°,\ 150°$	A1	
	$\therefore x = \mathbf{30°,\ 90°,\ 150°}$	A1	Final solution
8	**Differentiation (Product Rule)**		
(a)	$\dfrac{d}{dx}[x(x-3)^4]$		
	$= (x-3)^4 + 4x(x-3)^3$	M1	
	$= (x-3)^3(x - 3 + 4x)$		Apply Product Rule
	$= \mathbf{(5x-3)(x-3)^3}$	A1	
	Differentiation (Quotient Rule)		
(b)	$y = \dfrac{x(x-3)^4}{5x-3}$		
	$\dfrac{dy}{dx} = \dfrac{(5x-3)(x-3)^3(5x-3) - 5x(x-3)^4}{(5x-3)^2}$	M1	Apply Quotient Rule
	$= \dfrac{(x-3)^3\left((5x-3)^2 - 5x(x-3)\right)}{(5x-3)^2}$		
	$= \dfrac{(x-3)^3(25x^2 - 30x + 9 - 5x^2 + 15x)}{(5x-3)^2}$		
	$= \dfrac{\mathbf{(x-3)^3(20x^2 - 15x + 9)}}{\mathbf{(5x-3)^2}}$	M1	Factorise & simplify
	When $x = 1$,		
	$\dfrac{dy}{dt} = \dfrac{dy}{dx} \times \dfrac{dx}{dt}$		
	$-70 = \dfrac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2} \times \dfrac{dx}{dt}$		
	$= \dfrac{(1-3)^3(20 - 15 + 9)}{(5-3)^2} \times \dfrac{dx}{dt}$	M1	Rate of change
	$-70 = -28 \times \dfrac{dx}{dt}$		
	$= \mathbf{2.5\ units/s}$	A1	

Qn	Solution	Marks	Testing
9	**Quadratic Functions**		
(a)	$y = 5 - 12x - 3x^2$ $= -3x^2 - 12x + 5$ $= -3(x^2 + 4x) + 5$		
	$= -3\{(x+2)^2 - (2)^2\} + 5$	**M1**	Completing the Square
	$= -3(x+2)^2 + 12 + 5$ $\boldsymbol{y = -3(x+2)^2 + 17}$	**A1**	
	Since $(x+2)^2 \geq 0$ $-3(x+2)^2 \leq 0$ $-3(x+2)^2 + 17 \leq 17$ max $y = 17 < 20$, $\therefore$ for all values of y, it can never be greater than 20.	**B1**	
(b)	Given $y = (k-1)x^2 + 2(k+2)x + k + 3$ For y to be always positive, (i) Coeff of $x^2 > 0$, and (ii) Discriminant < 0.		2 conditions for $y > 0$
	From (i), $k - 1 > 0$ $k > 1$	**D1**	
	From (ii), $\{2(k+2)\}^2 - 4(k-1)(k+3) < 0$	**M1**	
	$4(k+2)^2 - 4(k-1)(k+3) < 0$ $(k+2)^2 - (k-1)(k+3) < 0$ $(k^2 + 4k + 4) - (k^2 + 2k - 3) < 0$ $2k + 7 < 0$ $k < -\frac{7}{2}$	**D1**	$D = b^2 - 4ac$
	From the two inequalities, $k > 1$ and $k < -\frac{7}{2}$, there are no values of k for which $y > 0$.	**A1**	
10	**Quadratic Inequalities (Word Problem)**		
(a)	Since $Side > 0$, $3x - 5 > 0$ and $x - 10 > 0$ $x > \frac{5}{3}$ and $x > 10$ ......(1)	**M1**	Form Side inequalities
	$Area\ of\ rectangle \leq 200$ $(3x-5)(x-10) \leq 200$	**M1**	Form Area inequality
	$3x^2 - 35x + 50 \leq 200$ $3x^2 - 35x - 150 \leq 0$ $(3x+10)(x-15) \leq 0$	**M1**	Factorisation
	$-\frac{10}{3} \leq x \leq 15$ ......(2) From (1) & (2), $\boldsymbol{10 < x \leq 15}$	**A1**	

Qn	Solution	Marks	Testing
(b)	To enclose the field, $Perimeter \leq 98$ $2(3x-5) + 2(x-10) \leq 98$	**M1**	Form Perimeter inequality
	$2(4x - 15) \leq 98$ $4x - 15 \leq 49$ $x \leq 16$ ......(3)	**A1**	
	From part (a) $10 < x \leq 15$ and (3) $x \leq 16$, it is **enough** to enclose the field.	**B1**	
11	**Logarihemic Equations**		
(a)	For a valid logarithm, $4 - x > 0$ $\therefore\ x < 4$	**B1**	
(b)	$2\log_3(4-x) - \log_9(x-4)^2 = 2$ $2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		Identity property
	$2\log_3(4-x)^2 - \log_3(x-4)^2 = 4$ $\log_3(4-x)^4 - \log_3(x-4)^2 = 4$ $\log_3 \dfrac{(4-x)^4}{(x-4)^2} = 4$	**M1**	Quotient Law
	$\dfrac{(4-x)^4}{(x-4)^2} = 3^4$	**M1**	Convert Logarithmic form to Exponential form
	$(4-x)^4 = 81(x-4)^2$ $(x-4)^4 - 81(x-4)^2 = 0$ $(x-4)^2((x-4)^2 - 81) = 0$ $(x-4)^2 = 0$ or $(x-4)^2 = 81$ $x - 4 = 0$ or $x - 4 = \pm 9$ $x = 4$ or $x = -5$ or $x = 13$	**A1**	Solve factorised equation
	From (a), since $x < 4$, $\therefore x = -5$ is the only one real solution	**B1**	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$ $2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		Identity property
	$\dfrac{\log_3(4-x)^2}{2} = 2$ $\log_3(4-x)^2 = 4$ $(4-x)^2 = 3^4$	**M1**	Convert Logarithmic form to

Qn	Solution	Marks	Testing
	$x^2 - 8x + 16 = 81$		Exponential form
	$x^2 - 8x - 65 = 0$	M1	
	$(x+5)(x-13) = 0$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4 - x > 0$, i.e. $x - 4 < 0$
	$x = -5$ or $x = 13$	A1	
	From (a), since $x < 4$,		
	$\therefore x = -5$ is the only one real solution	B1	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$	M1	Change-base law
	$2\log_3(4-x) - \dfrac{\log_3(x-4)^2}{\log_3 9} = 2$	M1	Power law
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{\log_3 3^2} = 2$		Identity property
	$\log_3(4-x)^2 - \dfrac{\log_3(x-4)^2}{2} = 2$		
	$\log_3(4-x)^2 - \log_3(4-x) = 2$		
	$\log_3 \dfrac{(4-x)^2}{(4-x)} = 2$	M1	Quotient law
	$\dfrac{(4-x)^2}{(4-x)} = 3^2$	M1	Convert Logarithmic form to Exponential form
	$x^2 - 8x + 16 = 9(4-x)$		
	$x^2 - 8x + 16 = 36 - 9x$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4 - x > 0$, i.e. $x - 4 < 0$
	$x^2 + x - 20 = 0$		
	$(x+5)(x-4) = 0$		
	$x = -5$ or $x = 4$	A1	
	From (a), since $x < 4$,		
	$\therefore x = -5$ is the only one real solution	B1	
12	**Quadratic Equations**		
(a)	When $y = 0$		
	$hx^2 - 2x - 9h + 6 = 0$		
	$hx^2 - 2x + (6 - 9h) = 0$		
	For a quadratic curve,		
	coeff of $x^2 \neq 0$,		
	$h \neq 0$ … [1]	M1	
	$D = (-2)^2 - 4h(6-9h)$	M1	$D = b^2 - 4ac$
	$= 4 - 4h(6-9h)$		
	$= 4(1 - 3h(2-3h))$		
	$= 4(9h^2 - 6h + 1)$		
	$= 4(3h-1)^2$ or $(6h-2)^2$	M1	
	$D \geq 0$ …[2]		
	$\therefore$ from [1] & [2], $y = 0$ has real roots for all values of h except $h = 0$.		

Qn	Solution	Marks	Testing
(b)	For 2 real and equal roots,		
	$D = 0$		
	$4(3h-1)^2 = 0$		
	$h = \dfrac{\mathbf{1}}{\mathbf{3}}$	A1	
(c)	For line meeting the curve,		
	$hx^2 - 2x - 9h + 6 = 2x - h - 12$		
	$hx^2 - 4x + 18 - 8h = 0$	M1	
	$D \geq 0$		
	$(-4)^2 - 4h(18 - 8h) \geq 0$	M1	
	$16 - 8h(9 - 4h) \geq 0$		
	$2 - h(9 - 4h) \geq 0$		
	$4h^2 - 9h + 2 \geq 0$		
	$(4h-1)(h-2) \geq 0$	M1	
			
	$h \leq \frac{1}{4}$ or $h \geq 2$ ……[3]	A1	
	From [1] & [3],		
	$\boldsymbol{h \leq \frac{1}{4}}$ **or** $\boldsymbol{h \geq 2}$ **and** $\boldsymbol{h \neq 0}$	A1	
13	**Increasing / Decreasing Fns**		
(a)	$f(x) = x^2 e^{(x+2)}$		
	$f'(x) = 2xe^{(x+2)} + x^2 e^{(x+2)}$	M1	Apply Product Rule
	$= xe^{(x+2)}(2 + x)$		
	$= x(x+2)e^{(x+2)}$	A1	
	For decreasing function,		
	$f'(x) < 0$		
	$x(x+2)e^{(x+2)} < 0$	M1	Decreasing function
	since $e^{(x+2)} > 0$		Must explain exponential function is <u>positive</u> for all values of x
	$x(x+2) < 0$	M1	
			
	$\boldsymbol{-2 < x < 0}$		
	Maxima & Minima		
(b)	$f'(x) = (x^2 + 2x)e^{(x+2)}$		
	$f''(x)$ $= (2x+2)e^{(x+2)} + (x^2+2x)e^{(x+2)}$	M1	Apply Product Rule
	$f''(x) = (x^2 + 4x + 2)e^{(x+2)}$		
	For the least gradient,		
	$f''(x) = 0$		

Qn	Solution	Marks	Testing
	$(x^2+4x+2)e^{(x+2)}=0$	M1	
	$x^2+4x+2=0$		Quadratic formula or Completing the Square
	$x=\dfrac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}$	M1	
	$=\dfrac{-4\pm\sqrt{8}}{2}$		
	$=\dfrac{-4\pm 2\sqrt{2}}{2}$		
	$=-2\pm\sqrt{2}$	A1	
	Since the least gradient lies in $-2<x<0$, $x=-2+\sqrt{2}$		
	Least gradient $f'(x)=x(x+2)e^{(x+2)}$		
	$f'(-2+\sqrt{2})$ $=(-2+\sqrt{2})(-2+\sqrt{2}+2)e^{(-2+\sqrt{2}+2)}$ $=(-2+\sqrt{2})\sqrt{2}e^{\sqrt{2}}$		
	$=(2-2\sqrt{2})e^{\sqrt{2}}$	A1	
14	**Remainder & Factor Thm**		
(a)	Given $f(x)=3x^3+2x^2+16$.		
	$f(3a)=f\left(-\dfrac{2}{3}\right)$		
	$3(3a)^3+2(3a)^2+16$ $=3\left(-\dfrac{2}{3}\right)^3+2\left(-\dfrac{2}{3}\right)^2+16$	M1	Substitute values correctly
	$3(27a^3)+2(9a^2)$ $=3\left(-\dfrac{8}{27}\right)+2\left(\dfrac{4}{9}\right)$		
	$81a^3+18a^2=-\dfrac{8}{9}+\dfrac{8}{9}$		
	$9a^2(9a+2)=0$	M1	Factorisation
	$\boldsymbol{a=0}$ **or** $\boldsymbol{a=-\dfrac{2}{9}}$	A1	
(b)	Subst $x=-2$ into $f(x)$,		
	$f(-2)=3(-2)^3+2(-2)^2+16$		Apply Factor Theorem
	$=-24+8+16$		
	$=0$	A1	
	$\therefore\ x=-2$ is a solution of $f(x)$.		

Qn	Solution	Marks	Testing
(c)	From (b), $x+2$ is a factor of $f(x)$.		
	By inspection, $f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2+kx+8)$	M1	
	Equating coefficient of		
	x^2 : $k+6=2$, $k=-4$	M1	Comparing coefficients or Long Division
	or x : $2k+8=0$, $k=-4$		
	$f(x)=3x^3+2x^2+16$ $=(x+2)(3x^2-4x+8)$	M1	Factorisation
	For $f(x)=0$ $(x+2)(3x^2-4x+8)=0$ $x+2=0$ $x=-2$		Solution for linear equation
	or $3x^2-4x+8=0$		
	$D=(4)^2-4(3)(8)$ $D=-80$	M1	Use of discriminant or Quadratic formula
	Since $D<0$, there is no real roots for the quadratic equation	B1	
	$\therefore\ f(x)=0$ has only one real root $x=-2$.		
(d)	$16y^3+2y-3=0$		
	$16+\dfrac{2}{y^2}-\dfrac{3}{y^3}=0$		
	$3\left(-\dfrac{1}{y}\right)^3+2\left(-\dfrac{1}{y}\right)^2+16=0$	M1	Transform into the required format
	Let $x=-\frac{1}{y}$		
	$3x^3+2x^2+16=0$		
	$x=-2$		
	$-\dfrac{1}{y}=-2$		
	$\boldsymbol{y=\dfrac{1}{2}}$	A1	