



BEDOK SOUTH SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE
NAME

CLASS

REGISTER
NUMBER

--	--

ADDITIONAL MATHEMATICS

Paper 2

4049 / 02

2 hours 15 minutes

90

Answer **all** the questions.

- 1
 - (a) Solve the equation $2(4^x) - 8 = 11(2^x) - 2^{2x+3}$, giving your answer correct to 3 significant figures. [5]
 - (b) Show that the solution from **part (a)** may be written in the form $m - \log_n p$ where m , n and p are integers to be determined. [2]

- 2

The mass, M grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $M = 200e^{-kt}$, where k is a constant. The mass of the substance was 123.7 g after being observed for 2 years.

 - (a)
 - (i) State the initial mass of the substance. [1]
 - (ii) Show that k is approximately 0.240, correct to 3 significant figures. [1]
 - (iii) Find the mass of the substance when $t = 5$, [1]
 - (iv) Find the value of t when the mass of the substance is 15% of its initial mass. [2]

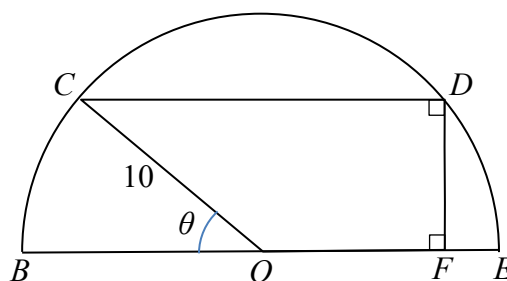
Give your answers correct to three significant figures.
 - (b) Explain, with clear working, why the mass of the substance can never be more than 200 grams. [1]
 - (c) Sketch the graph of M against t . [2]

- 3
 - (a) It is given that $y = x^2\sqrt{2x-5}$, where $x \geq h$.
 Show that $\frac{dy}{dx} = \frac{kx(x-2)}{\sqrt{2x-5}}$, where k is an integer and determine the value of h and of k . [4]
 - (b) Hence evaluate $\int_3^7 \left\{ \frac{10x(x-2)}{\sqrt{2x-5}} + 6 \right\} dx$. [4]

- 4
 - (a)
 - (i) Factorise completely $x^3 + 64$. [2]
 - (ii) Hence, express $\frac{3x^2}{x^3 + 64}$ in partial fractions. [5]
 - (b) Using the results in **part (a)**, or otherwise, find $\int \frac{2x^3 + 3x^2 + 128}{x^3 + 64} dx$. [3]

- 5 A particle moves in a straight line so that, at time t seconds after passing a fixed point O , its velocity is v m/s, where $v = 4 + 8\cos 2t$. Find
- (a) the velocity of the particle at the instant it passes O , [1]
 - (b) the least value of the particle's acceleration, [1]
 - (c) the values of t , in terms of π , when the particle is at rest for $0 \leq t \leq 3$, [4]
 - (d) the distance travelled in the first 2 seconds. [4]

6



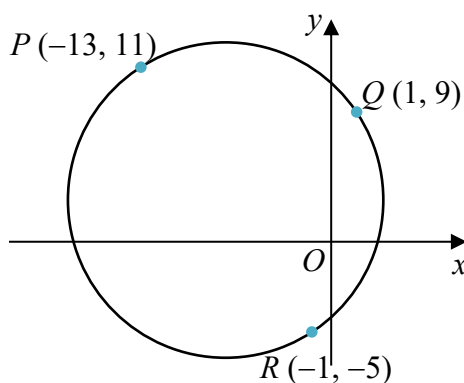
The diagram shows right angled trapezium $OCDF$ inside a semicircle with centre O and radius 10 cm such that angle BOC is θ radians, and angle CDF and angle OFD are right angles.

- (a) Show that the perimeter, P cm, of trapezium $OCDF$ is given by

$$P = 10 + 30 \cos \theta + 10 \sin \theta$$
 [2]
- (b) Find the value of R when $10 \sin \theta + 30 \cos \theta$ is expressed as $R \cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum perimeter of the trapezium. [3]
- (c) Show that the area, A cm², of trapezium $OCDF$ is given by

$$A = 75 \sin 2\theta$$
 [2]
- (d) The area of the trapezium varies with the value of θ . Find the value of θ for which the area has a stationary value and determine whether this area is a maximum or a minimum. [4]

7 Solutions to this question by accurate drawing will not be accepted.

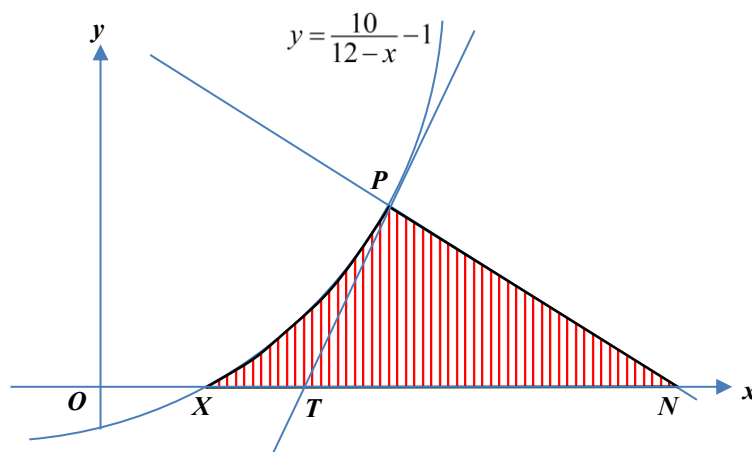


In the diagram, P , Q and R are points on the circle.

- (a) Explain, with geometrical reason, why PR is the diameter of the circle. [3]
- (b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a , b and c are integers. [3]

- (c) Find the equation of the perpendicular bisector of PQ . [3]
- (d) The point S lies on the circle such that it is furthest from the point Q .
Show that the coordinates of S are $(-15, -3)$ and hence calculate the area of the quadrilateral $PQRS$. [3]

8



The diagram shows part of the curve $y = \frac{10}{12-x} - 1$ passing through the point $P(2k, k-1)$, where k is a constant. The curve meets the x -axis at the point X . The tangent and normal at P meet the x -axis at the points T and N respectively.

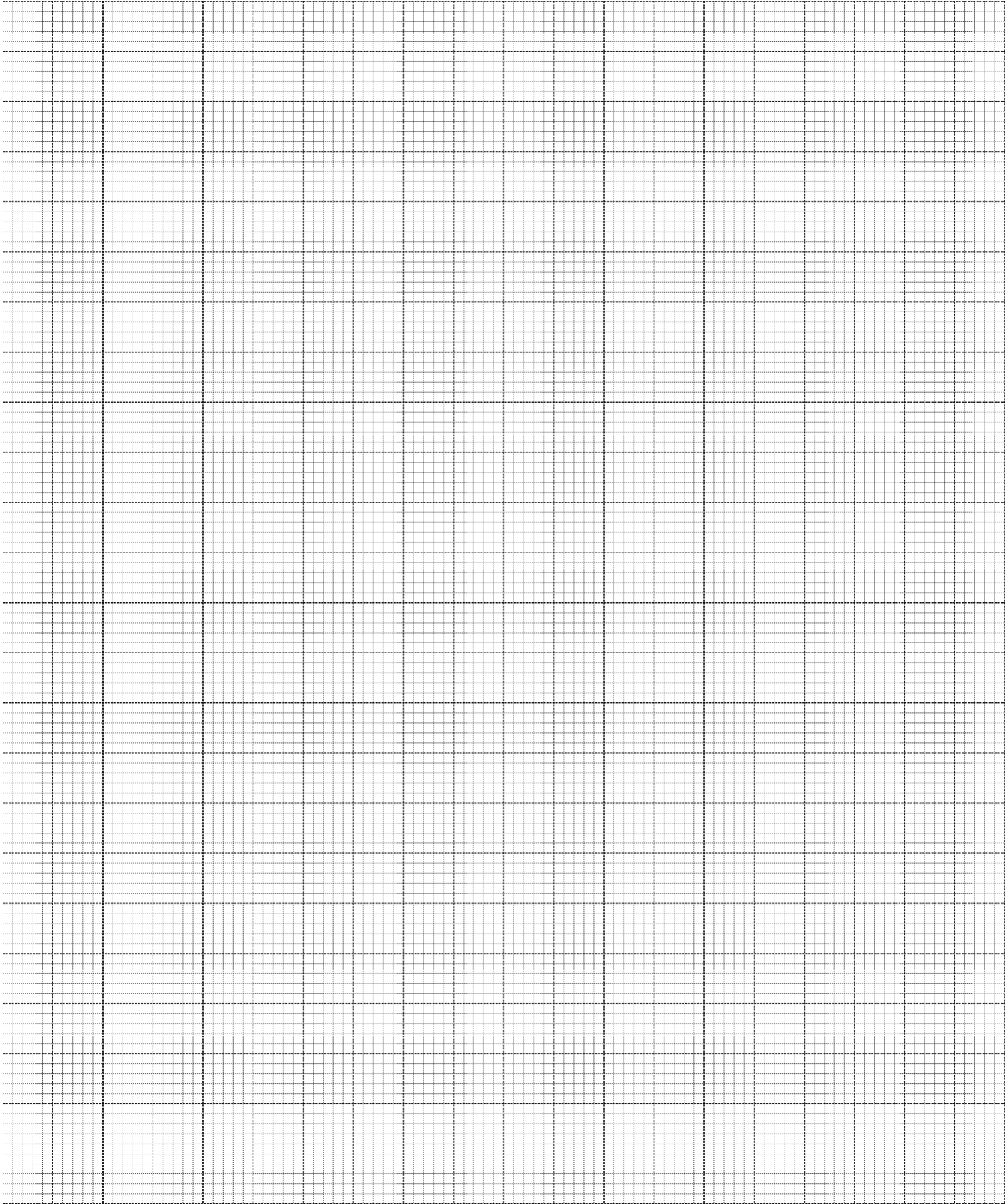
- (a) Find the equation of the normal at P . [6]
- (b) Find the **exact** area of the shaded region. [6]

9 The table below shows experimental values of two variables x and y .

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.83	1.87	1.96

The variables x and y are related by the equation $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$, where a and b are constants. One value of y has been recorded incorrectly.

- (a) Show how $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ is transformed to plot a graph of xy against $x\sqrt{x}$. [1]
- (b) Draw a straight line graph of xy against $x\sqrt{x}$ for the given data. [2]
- (c) Using your graph,
- find an approximate value of y to replace the incorrect value, [2]
 - estimate the value of a and of b , [3]
 - find the value of y when $x = 2.52$. [2]
- (d) A pair of values of x and y is considered acceptable only if the xy value is within 2% vertical difference from the straight line. A student recorded a pair of values such that $x = 4.50$ and $y = 1.86$.
Verify whether the recorded values by the student are acceptable. [2]



Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)! r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

ANSWERS
(Bedok South) 4E5N AM P2 (Prelim 2025) (w Ans)

1. (a) $x = 0.678$ (to 3 sig. fig.)

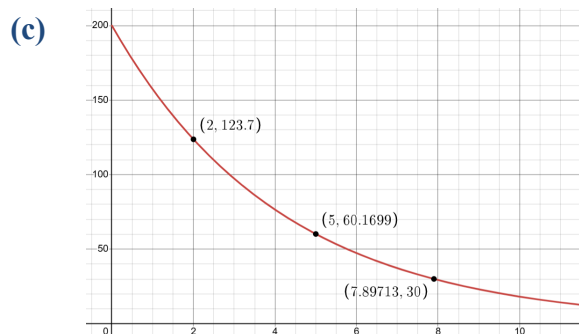
(b) $m = 3, n = 2, p = 5$

2. (a) (i) $M_0 = 200 \text{ g}$

(ii) $k = 0.240$ (to 3 sig. fig.)

(iii) $M = 60.2 \text{ g}$ (to 3 sig. fig.)

(iv) $t = 7.90 \text{ years}$ (to 3 sig. fig.)



3. (a) $h = \frac{5}{2}, k = 5$

(b) 300

4. (a) (i) $(x + 4)(x^2 - 4x + 16)$

(ii) $\frac{1}{x+4} + \frac{2x-4}{x^2-4x+16}$

(b) $2x + \ln(x^3 + 64) + c$

5. (a) $v = 12 \text{ m/s}$

(b) Least $a = -16 \text{ m/s}^2$

(c) $t = \frac{1}{3}\pi, \frac{2}{3}\pi$

(d) 10.3 m (to 3 sig. fig.)

6. (a) **Hint:** Draw a line $CG \perp BE$

(b) $R = 10\sqrt{10}$ or 31.6 (to 3 sig. fig.)
Max $P = 41.6 \text{ cm}$ (to 3 sig. fig.)

(c) **Hint:** Apply Double Angle formula

(d) $\theta = \frac{\pi}{4}$

7. (a) **Hint:** Right angle in semicircle

(b) $x^2 + y^2 + 14x - 6y - 42 = 0$

(c) $y = 7x + 52$

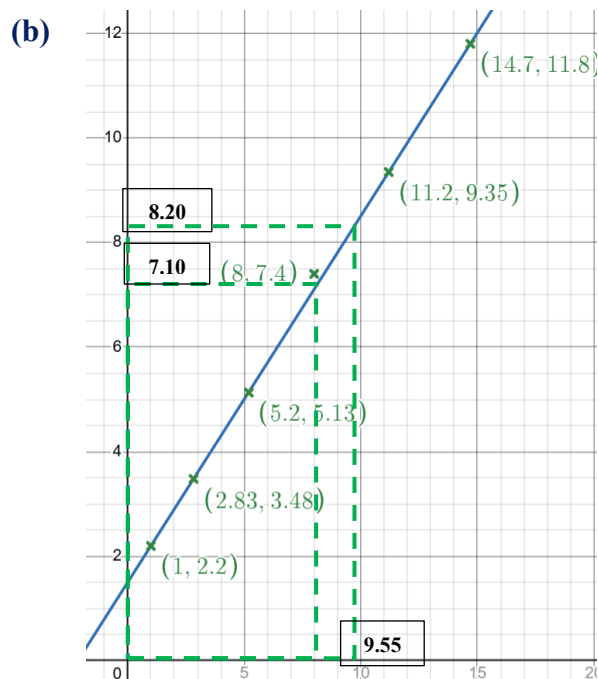
(d) **Hint:** The furthest two points on a circle is its diameter.

$\text{Area of PQRS} = 200 \text{ units}^2$

8. (a) $y = -\frac{2}{5}x + 8$

(b) $10 \ln 5 + 12$

9. (a) **Hint:** Multiply equation with ax to get $xy = ab(x\sqrt{x}) + a$



(c) (i) $y = 1.78$ (to 3 sig. fig.)

(ii) $a = 1.50$ (to 3 sig. fig.)

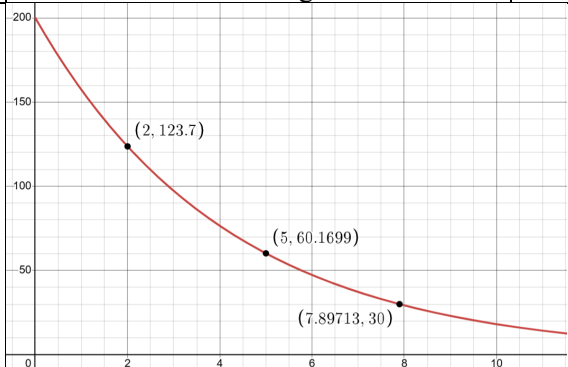
$b = 0.467$ (to 3 sig. fig.)

(iii) $y = 1.71$ (to 3 sig. fig.)

(d) Unacceptable,
Vertical difference = 2.073%

MARKING SCHEME
(Bedok South) 4E5N AM P2 (Prelim 2023) (w Ans)

Qn	Solution	Testing
1.	Exponential Equations	
(a)	$2(4^x) - 8 = 11(2^x) - 2^{2x+3}$ $2(2^{2x}) - 8 = 11(2^x) - (2^{2x})2^3$ $2(2^x)^2 - 8 = 11(2^x) - 8(2^x)^2$ Let $u = 2^x$ $2u^2 - 8 = 11u - 8u^2$ $10u^2 - 11u - 8 = 0$ $(2u + 1)(5u - 8) = 0$ $u = -\frac{1}{2} \text{ or } u = \frac{8}{5}$ $2^x = -\frac{1}{2} \text{ (reject) } 2^x = \frac{8}{5}$ $\lg 2^x = \lg \frac{8}{5}$ $x = \frac{\lg \frac{8}{5}}{\lg 2}$ $= 0.678072$ $= \mathbf{0.678} \text{ (to 3 sig. fig.)}$	Convert to base 2 Apply substitution correctly Solving for u Apply logarithm appropriately
(b)	From (a), $2^x = \frac{8}{5}$ $\log_2 2^x = \log_2 \frac{8}{5}$ $x \log_2 2 = \log_2 8 - \log_2 5$ $x = \log_2 2^3 - \log_2 5$ $= 3 \log_2 2 - \log_2 5$ $= 3 - \log_2 5$ $= m - \log_n p$ $m = 3, n = 2, p = 5$	Apply $\log_2$ onto equation Power law Quotient law Identity property
2.	Exponential Functions (Word Problems)	
(a)	When $t = 0$,	
(i)	$M = 200e^{-kt}$ $M_0 = 200e^{-k(0)}$ $= \mathbf{200 \text{ g}}$	
(a)	When $t = 2, M = 123.7$	
(ii)	$M = 200e^{-kt}$ $123.7 = 200e^{-k(2)}$ $\frac{123.7}{200} = e^{-2k}$ $\ln\left(\frac{123.7}{200}\right) = -2k$	Take natural logarithm on both

Qn	Solution	Testing										
	$k = -\frac{1}{2}\ln\left(\frac{123.7}{200}\right)$ $= 0.240229$ $= \mathbf{0.240} \text{ (to 3 sig. fig.)}$	sides of equation										
(a) (iii)	<table><tr><td>When $t = 5$,</td><td></td></tr><tr><td>For $k = 0.24$</td><td>For $k = 0.240229$</td></tr><tr><td>$M = 200e^{-kt}$</td><td>$M = 200e^{-kt}$</td></tr><tr><td>$= 200e^{-0.24(5)}$</td><td>$= 200e^{-0.240229(5)}$</td></tr><tr><td>$= 60.23884$</td><td>$= 60.16991$</td></tr></table> $= \mathbf{60.2 \text{ g}} \text{ (to 3 sig. fig.)}$	When $t = 5$,		For $k = 0.24$	For $k = 0.240229$	$M = 200e^{-kt}$	$M = 200e^{-kt}$	$= 200e^{-0.24(5)}$	$= 200e^{-0.240229(5)}$	$= 60.23884$	$= 60.16991$	
When $t = 5$,												
For $k = 0.24$	For $k = 0.240229$											
$M = 200e^{-kt}$	$M = 200e^{-kt}$											
$= 200e^{-0.24(5)}$	$= 200e^{-0.240229(5)}$											
$= 60.23884$	$= 60.16991$											
(a) (iv)	$M = M_0e^{-kt}$ $\frac{M}{M_0} = e^{-kt}$ $15\% = e^{-kt}$ $0.15 = e^{-kt}$ <table><tr><td>For $k = 0.24$</td><td>For $k = 0.240229$</td></tr><tr><td>$0.15 = e^{-0.24t}$</td><td>$0.15 = e^{-0.240229t}$</td></tr><tr><td>$\ln 0.15 = -0.24t$</td><td>$\ln 0.15 = -0.240229t$</td></tr><tr><td>$t = -\frac{\ln 0.15}{0.24}$</td><td>$t = -\frac{\ln 0.15}{0.240229}$</td></tr><tr><td>$= 7.90467$</td><td>$= 7.89713$</td></tr></table> $t = \mathbf{7.90 \text{ years}} \text{ (to 3 sig. fig.)}$	For $k = 0.24$	For $k = 0.240229$	$0.15 = e^{-0.24t}$	$0.15 = e^{-0.240229t}$	$\ln 0.15 = -0.24t$	$\ln 0.15 = -0.240229t$	$t = -\frac{\ln 0.15}{0.24}$	$t = -\frac{\ln 0.15}{0.240229}$	$= 7.90467$	$= 7.89713$	Take natural logarithm on both sides of equation
For $k = 0.24$	For $k = 0.240229$											
$0.15 = e^{-0.24t}$	$0.15 = e^{-0.240229t}$											
$\ln 0.15 = -0.24t$	$\ln 0.15 = -0.240229t$											
$t = -\frac{\ln 0.15}{0.24}$	$t = -\frac{\ln 0.15}{0.240229}$											
$= 7.90467$	$= 7.89713$											
(b)	Since for $t \geq 0$, $e^{kt} \geq 1$ $\frac{1}{e^{kt}} \leq 1$ $e^{-kt} \leq 1$ $200e^{-kt} \leq 200$ $M \leq 200$ $\therefore \text{ mass of substance can never be more than 200 g.}$											
(c)												

G1 – Initial state (0, 200)
G1 – Smooth curve decreasing towards asymptote t -axis

G1 – Initial state (0, 200)

G1 – Smooth curve decreasing towards asymptote t -axis

Qn	Solution	Testing
3.	Differentiation (Product Rule)	
(a)	For square root to be defined, $2x - 5 \geq 0 \Rightarrow x \geq \frac{5}{2}$ $\therefore h = \frac{5}{2}$ $y = x^2 \sqrt{2x - 5}$ $y = x^2 (2x - 5)^{\frac{1}{2}}$ $\frac{dy}{dx} = 2x(2x - 5)^{\frac{1}{2}}$ $+ x^2(2) \frac{1}{2} (2x - 5)^{-\frac{1}{2}}$ $\frac{dy}{dx} = 2x\sqrt{2x - 5} + \frac{x^2}{\sqrt{2x - 5}}$ $= \frac{2x(2x - 5) + x^2}{\sqrt{2x - 5}}$ $= \frac{2x(2x - 5) + x^2}{\sqrt{2x - 5}}$ $= \frac{5x^2 - 10x}{\sqrt{2x - 5}}$ $\frac{dy}{dx} = \frac{5x(x - 2)}{\sqrt{2x - 5}}$ $\therefore k = 5$	Square root criterion Apply Product rule Combine to single fraction
	Integration (Reverse of Differentiation)	
(b)	$\int_3^7 \left\{ \frac{10x(x - 2)}{\sqrt{2x - 5}} + 6 \right\} dx$ $= 2 \int_3^7 \frac{5x(x - 2)}{\sqrt{2x - 5}} dx + \int_3^7 6 dx$ $= 2 \left[x^2 \sqrt{2x - 5} \right]_3^7 + 6[x]_3^7$ $= 2 \left\{ 7^2 \sqrt{2(7) - 5} - 3^2 \sqrt{2(3) - 5} \right\}$ $+ 6(7 - 3)$ $= 2 \{ 49\sqrt{9} - 9\sqrt{1} \} + 24$ $= 276 + 24$ $= \mathbf{300}$	Split into 2 integrals Integration as reversed differentiation Substitution of boundaries
	OR $\frac{dy}{dx} = \frac{5x(x - 2)}{\sqrt{2x - 5}}$ $\int_3^7 \frac{5x(x - 2)}{\sqrt{2x - 5}} dx = \left[x^2 \sqrt{2x - 5} \right]_3^7$	Integration as reversed differentiation

Qn	Solution	Testing
	$\int_3^7 \frac{10x(x - 2)}{\sqrt{2x - 5}} dx = 2 \left[x^2 \sqrt{2x - 5} \right]_3^7$ $\int_3^7 \frac{10x(x - 2)}{\sqrt{2x - 5}} dx + \int_3^7 6 dx$ $= 2 \left\{ 7^2 \sqrt{2(7) - 5} - 3^2 \sqrt{2(3) - 5} \right\}$ $+ \int_3^7 6 dx$ $\int_3^7 \left\{ \frac{10x(x - 2)}{\sqrt{2x - 5}} + 6 \right\} dx$ $= 2 \{ 49\sqrt{9} - 9\sqrt{1} \} + 6[x]_3^7$ $= 276 + 6(7 - 3)$ $= \mathbf{300}$	Transform to required integral Substitution of boundaries
4.	Cubic Identities	
(a)	$x^3 + 64$	
(i)	$= (x)^3 + (4)^3$ $= (x + 4)((x)^2 - (x)(4) + (4)^2)$ $= (x + 4)(x^2 - 4x + 16)$	Apply sum of cubes identity
	Partial Fractions	
(a)	$\frac{3x^2}{x^3 + 64}$	
(ii)	$= \frac{x^3 + 64}{3x^2}$ $= \frac{A}{x + 4} + \frac{Bx + C}{x^2 - 4x + 16}$ $A(x^2 - 4x + 16) + (Bx + C)(x + 4) = 3x^2$	Express proper fraction in partial fractions with unknown coefficients
	Let $x = -4$, $A((-4)^2 - 4(-4) + 16) = 3(-4)^2$ $A(16 + 16 + 16) = 3(16)$ $A = 1$ Let $x = 0$, $A(16) + C(4) = 0$ $1(16) + 4C = 0$ $16 + 4C = 0$ $C = -4$ Let $x = 1$, $A(1 - 4 + 16) + (B + C)(1 + 4) = 3$ $1(13) + (B - 4)(5) = 3$ $13 + 5B - 20 = 3$	Use Substitution and/or Equating coefficients methods to find unknowns Award 1 mark for at least 2 correct values

Qn	Solution	Testing						
	$B = 2$							
	OR Equating coefficient of <table><tr><td>$x^2 :$</td><td>$A + B = 3$ $B = 3 - A \cdots (1)$</td></tr><tr><td>$x :$</td><td>$-4A + 4B + C = 0$ $\cdots (2)$</td></tr><tr><td>$x^0 :$</td><td>$16A + 4C = 0$ $C = -4A \cdots (3)$</td></tr></table> Subst (1) & (3) into (2), $-4A + 4(3 - A) + (-4A) = 0$ $-4A + 12 - 4A - 4A = 0$ $12A = 12$ $A = 1$ $B = 3 - 1 = 2$ $C = -4(1) = -4$	$x^2 :$	$A + B = 3$ $B = 3 - A \cdots (1)$	$x :$	$-4A + 4B + C = 0$ $\cdots (2)$	$x^0 :$	$16A + 4C = 0$ $C = -4A \cdots (3)$	
$x^2 :$	$A + B = 3$ $B = 3 - A \cdots (1)$							
$x :$	$-4A + 4B + C = 0$ $\cdots (2)$							
$x^0 :$	$16A + 4C = 0$ $C = -4A \cdots (3)$							
	$\therefore \frac{3x^2}{x^3 + 64}$ $= \frac{1}{x + 4} + \frac{2x - 4}{x^2 - 4x + 16}$							
Indefinite Integral								
(b)	$\int \frac{2x^3 + 3x^2 + 128}{x^3 + 64} dx$ $= \int \frac{2(x^3 + 64)}{x^3 + 64} dx + \int \frac{3x^2}{x^3 + 64} dx$ $= \int 2 dx + \int \frac{3x^2}{x^3 + 64} dx$ $= 2x + \ln(x^3 + 64) + c$	Split into 2 integrals Correct integration						
5. Kinematics (Trigonometric Functions)								
(a)	When particle passes O , $t = 0$, Hence, $v = 4 + 8 \cos 2(0)$ $= 12 \text{ m/s}$							
(b)	$v = 4 + 8 \cos 2t$ $\frac{dv}{dt} = -(2)8 \sin 2t$ $a = -16 \sin 2t$ Least acceleration $= -16 \text{ m/s}^2$							
(c)	$0 \leq t \leq 3$ $0 \leq 2t \leq 6$ When particle is at rest, $v = 0 \text{ m/s}$,							

Qn	Solution	Testing								
	$4 + 8 \cos 2t = 0$ $\cos 2t = -\frac{1}{2}$ Basic angle $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\pi$ For $\cos 2t < 0$, $2t$ lies in Q2, Q3 $2t = \pi - \frac{1}{3}\pi, \pi + \frac{1}{3}\pi$ $= \frac{2}{3}\pi, \frac{4}{3}\pi$ $t = \frac{1}{3}\pi, \frac{2}{3}\pi$	Correct Substitution Basic Angle Angles in correct quadrants								
(d)	$s = \int_0^{\frac{\pi}{3}} (4 + 8 \cos 2t) \, dt$ $s = 4t + 4 \sin 2t + c$ <table border="1"><thead><tr><th>t</th><th>s</th></tr></thead><tbody><tr><td>0</td><td>0</td></tr><tr><td>$\frac{1}{3}\pi$</td><td>7.65289</td></tr><tr><td>2</td><td>4.97279</td></tr></tbody></table> Distance travelled in the first 2 sec = $(7.65289 - 0)$ + $(7.65289 - 4.97279)$ = 10.33299 = 10.3 m (to 3 sig. fig.)	t	s	0	0	$\frac{1}{3}\pi$	7.65289	2	4.97279	Correct Integration
t	s									
0	0									
$\frac{1}{3}\pi$	7.65289									
2	4.97279									
(d)	Distance travelled in the first 2 sec $= \left \int_0^{\frac{\pi}{3}} (4 + 8 \cos 2t) \, dt \right $ $+ \left \int_{\frac{\pi}{3}}^2 (4 + 8 \cos 2t) \, dt \right $ $= \left [4t + 4 \sin 2t]_0^{\frac{\pi}{3}} \right $ $+ \left [4t + 4 \sin 2t]_{\frac{\pi}{3}}^2 \right $ $= \left 4\left(\frac{\pi}{3}\right) + 4\left(\sin \frac{2\pi}{3}\right) \right $ $+ \left 4\left(2 - \frac{\pi}{3}\right) + 4\left(\sin 4 - \sin \frac{2\pi}{3}\right) \right $ $= 7.65289 + 2.6801$ $= 10.33299$ $= \mathbf{10.3 \, m} \quad (\text{to 3 sig. fig.})$	Correct Expressions Correct Integration Correct Substitution								

Qn	Solution	Testing
6.	R-Formulae	
(a)	Draw a line $CG \perp BE$. In $\triangle CGO$, $\angle CGO = \frac{\pi}{2}, \quad OC = 10$ $CG = 10 \sin \theta$ $GO = 10 \cos \theta$ $DF = CG = 10 \sin \theta$ $CD = GF = 2GO$ $= 20 \cos \theta$ $FO = GO = 10 \cos \theta$ $P = OC + CD + DF + FO$ $= 10 + 20 \cos \theta + 10 \sin \theta$ $+ 10 \cos \theta$ $P = 10 + 30 \cos \theta + 10 \sin \theta$	
(b)	$30 \cos \theta + 10 \sin \theta$ $= R \cos(\theta - \alpha)$ $= \sqrt{30^2 + 10^2} \cos(\theta - \alpha)$ $= 10\sqrt{10} \cos(\theta - \alpha)$ where $R = 10\sqrt{10}$ $= 31.6227$ $= \mathbf{31.6} \text{ (to 3 s.f.)}$ $\text{Max } P = 10 + R$ $= \mathbf{41.6 \text{ cm}} \text{ (to 3 s.f.)}$	
(c)	$A = \frac{1}{2} DF(CD + FO)$ $= \frac{1}{2} (10 \sin \theta)(20 \cos \theta + 10 \cos \theta)$ $A = 5 \sin \theta (30 \cos \theta)$ $A = 150 \sin \theta \cos \theta$ $A = 75 \sin 2\theta$	Area = $\frac{1}{2} \times \text{base} \times \text{height}$ Double angle formula
	Stationary Values	
(d)	$\frac{dA}{d\theta} = 150 \cos 2\theta$ $\frac{d^2A}{d\theta^2} = -300 \sin 2\theta$ For stationary value, $\frac{dA}{d\theta} = 0$ $150 \cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$	

Qn	Solution	Testing		
	$\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -300 \sin 2\theta$ $= -300 \sin \frac{\pi}{2}$ $= -300$ $\frac{d^2A}{d\theta^2} < 0$ $A = 75 \sin 2\theta$$= 75 \sin \frac{\pi}{2}$$= 75$ A has a maximum area (of 75 cm^2) when $\theta = \frac{\pi}{4}$.	Value of A is not required		
7.	Circles & Coordinate Geometry			
(a)	$P(-13, 11)$ $Q(1, 9)$ $R(-1, -5)$ <table><tr><td>Gradient of PQ $m_{PQ} = \frac{11 - 9}{-13 - 1}$$= -\frac{1}{7}$</td><td>Gradient of QR $m_{QR} = \frac{9 - (-5)}{1 - (-1)}$$= 7$</td></tr></table> Since $m_{PQ} \times m_{QR} = -\frac{1}{7} \times 7 = -1$ $\Rightarrow PQ$ is perpendicular to QR $\Rightarrow \angle PQR = 90^\circ$ (right angle in semicircle) $\Rightarrow PR$ is the diameter of circle.	Gradient of PQ $m_{PQ} = \frac{11 - 9}{-13 - 1}$ $= -\frac{1}{7}$	Gradient of QR $m_{QR} = \frac{9 - (-5)}{1 - (-1)}$ $= 7$	Applying $m_1 \times m_2 = -1$
Gradient of PQ $m_{PQ} = \frac{11 - 9}{-13 - 1}$ $= -\frac{1}{7}$	Gradient of QR $m_{QR} = \frac{9 - (-5)}{1 - (-1)}$ $= 7$			
	OR $PQ^2 = (-13 - 1)^2 + (11 - 9)^2$ $= 200$ $QR^2 = (1 - (-1))^2 + (9 - (-5))^2$ $= 200$ $PR^2 = (-13 + 1)^2 + (11 + 5)^2$ $= 400$ Since $PR^2 = PQ^2 + QR^2 = 400$, by converse of Pythagoras' Theorem, $\angle PQR = 90^\circ$. (right angle in semicircle) $\Rightarrow PR$ is the diameter of circle.			

Qn	Solution	Testing
	OR Midpoint of PR, $M = \left(\frac{-13-1}{2}, \frac{11-5}{2} \right)$ $= (-7, 3) \dots [1]$ $PM = \sqrt{(-13+1)^2 + (11-3)^2}$ $= 10$ $QM = \sqrt{(1+7)^2 + (9-3)^2}$ $= 10$ $RM = (-1+7)^2 + (-5-3)^2 = 10$ $\therefore PM = QM = RM = 10 \dots [2]$ From [1] and [2], midpoint of chord PR is equidistant to P, Q & R $\Rightarrow PR$ is the diameter of circle.	
(b)	As centre is midpoint of diameter, $\text{centre} = \left(\frac{-13-1}{2}, \frac{11-5}{2} \right)$ $= (-7, 3)$ $\text{radius} = \frac{1}{2}PR$ $= \frac{1}{2}\sqrt{(-13-(-1))^2 + (11-(-5))^2}$ $= \frac{1}{2}\sqrt{400}$ $= 10$ Equation of circle $(x-(-7))^2 + (y-3)^2 = 10^2$ $x^2 + 14x + 49 + y^2 - 6y + 9 = 100$ $\mathbf{x^2 + y^2 + 14x - 6y - 42 = 0}$	Award 1 mark for either centre or radius correct Correct substitution
(c)	Let $T(x, y)$ be a general point on the perpendicular bisector of PQ. $PT = QT$ $\sqrt{(x-(-13))^2 + (y-11)^2}$ $= \sqrt{(x-1)^2 + (y-9)^2}$ $(x+13)^2 + (y-11)^2$ $= (x-1)^2 + (y-9)^2$ $x^2 + 26x + 169 + y^2 - 22y + 121$ $= x^2 - 2x + 1 + y^2 - 18y + 81$ $4y = 28x + 208$ $\mathbf{y = 7x + 52}$	
	OR Perpendicular bisector passes through the centre. Using centre $(-7, 3)$ and $m_{\perp PQ} = 7$	

Qn	Solution	Testing															
	Equation of perpendicular bisector of QR is $y - 3 = 7(x - (-7))$ $\mathbf{y = 7x + 52}$																
	OR Perpendicular bisector passes through the midpoint of chord. Midpoint of PQ $= \left(\frac{-13 + 1}{2}, \frac{11 + 9}{2} \right)$ $= (-6, 10)$ $m_{\perp PQ} = 7$ Equation of perpendicular bisector of QR is $y - 10 = 7(x - (-6))$ $\mathbf{y = 7x + 52}$																
(d)	The furthest two points on a circle is the diameter of the circle. $\therefore$ centre $(-7, 3)$ is midpoint of diameter QS , <table><tr><td>$\frac{x_Q + x_S}{2} = -7$</td><td>$\frac{y_Q + y_S}{2} = 3$</td></tr><tr><td>$1 + x_S = -14$</td><td>$9 + y_S = 6$</td></tr><tr><td>$x_S = -15$</td><td>$y_S = -3$</td></tr></table> $\therefore \mathbf{S(-15, -3)}$	$\frac{x_Q + x_S}{2} = -7$	$\frac{y_Q + y_S}{2} = 3$	$1 + x_S = -14$	$9 + y_S = 6$	$x_S = -15$	$y_S = -3$	Diameter is the longest chord Mid-point									
$\frac{x_Q + x_S}{2} = -7$	$\frac{y_Q + y_S}{2} = 3$																
$1 + x_S = -14$	$9 + y_S = 6$																
$x_S = -15$	$y_S = -3$																
	Area of quadrilateral $PQRS$ <table><tr><td>P</td><td>Q</td><td>R</td><td>S</td><td>P</td></tr><tr><td>-13</td><td>1</td><td>-1</td><td>-15</td><td>-13</td></tr><tr><td>11</td><td>9</td><td>-5</td><td>-3</td><td>11</td></tr></table> $= \frac{1}{2} \left \begin{matrix} -13 & 1 & -1 & -15 & -13 \\ 11 & 9 & -5 & -3 & 11 \end{matrix} \right $ $= \frac{1}{2} \left (-117 - 5 + 3 - 165) \right $ $= \frac{1}{2} \left -(11 - 9 + 75 + 39) \right $ $= \mathbf{200 \text{ units}^2}$	P	Q	R	S	P	-13	1	-1	-15	-13	11	9	-5	-3	11	Shoelace method
P	Q	R	S	P													
-13	1	-1	-15	-13													
11	9	-5	-3	11													
8. Tangent & Normal																	
(a)	$y = \frac{10}{12 - x} - 1 \dots \dots [1]$ Subst $P(2k, 4)$ into [1] $4 = \frac{10}{12 - 2k} - 1$ $5 = \frac{5}{6 - k}$ $6 - k = 1$ $k = 5$ $P(10, 4)$ $\frac{dy}{dx} = (-10)(12 - x)^{-2}(-1)$ $\frac{dy}{dx} = \frac{10}{(12 - x)^2} \dots \dots [2]$	Chain rule															

Qn	Solution	Testing																												
	$= -10 \ln \left(\frac{12-10}{12-2} \right) - \{10 - 2\} -$ $\frac{1}{5} \{20^2 - 10^2\} + 8\{20 - 10\}$ $= 10 \ln 5 + 12$	Substitution of boundaries																												
9.	Linear Law																													
(a)	$\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ Multiply equation with ax, $xy = a + abx\sqrt{x}$ $xy = ab(x\sqrt{x}) + a$	Linearise equation $Y = mX + C$																												
(b)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>y</td><td>2.20</td><td>1.74</td><td>1.71</td><td>1.85</td><td>1.87</td><td>1.96</td></tr><tr><td>$x\sqrt{x}$</td><td>1.00</td><td>2.83</td><td>5.20</td><td>8.00</td><td>11.2</td><td>14.7</td></tr><tr><td>xy</td><td>2.20</td><td>3.48</td><td>5.13</td><td>7.40</td><td>9.35</td><td>11.8</td></tr></table>	x	1	2	3	4	5	6	y	2.20	1.74	1.71	1.85	1.87	1.96	$x\sqrt{x}$	1.00	2.83	5.20	8.00	11.2	14.7	xy	2.20	3.48	5.13	7.40	9.35	11.8	Tabulate values for Y vs. X
x	1	2	3	4	5	6																								
y	2.20	1.74	1.71	1.85	1.87	1.96																								
$x\sqrt{x}$	1.00	2.83	5.20	8.00	11.2	14.7																								
xy	2.20	3.48	5.13	7.40	9.35	11.8																								
G1 – Plotted points G1 – Best-fit line																														
(c)	From the graph,																													
(i)	the incorrect point is (8, 7.40). Corresponding to $x = 4$ (in the table), For $x\sqrt{x} = 8$ Correct $xy = 7.05$ to 7.15 $4y = 7.05$ to 7.15 $y = 1.7625$ to 1.7875 $y = \mathbf{1.76}$ to $\mathbf{1.79}$ (to 3 sig. fig.)	Read from graph																												
(c)	From the graph,																													
(ii)	xy intercept = 1.5 $a = \mathbf{1.50}$ (to 3 sig. fig.) Gradient = $\frac{11.8 - 2.20}{14.7 - 1.00}$ $ab = 0.7007$ $1.50b = 0.7007$ $b = 0.467153$ $b = \mathbf{0.467}$ (to 3 sig. fig.)	Read from graph																												
(c)	When $x = 2.52$,																													
(iii)	for $x\sqrt{x} = 2.52\sqrt{2.52}$ $= 4.00037$ From the graph, for $x\sqrt{x} = 4.00$ $xy = 4.3$ $2.52y = 4.3$ $y = 1.70634$ $y = \mathbf{1.71}$ (to 3 sig. fig.)	Read from graph																												

Qn	Solution	Testing
(d)	From readings of experiment, $x = 4.50$ and $y = 1.86$. $x\sqrt{x} = 9.5459$ $xy = (4.50)(1.86)$ $= 8.37$ From the graph, for $x\sqrt{x} = 9.5459$ $xy = 8.20$ Vertical difference $= \frac{8.37 - 8.20}{8.20} \times 100\%$ $= 2.073\% > 2\%$ Since vertical difference is more than 2%, the experimental readings are unacceptable.	Read from graph

Qn	Solution	Testing
	OR Acceptable range $= 8.20 \times (100 \pm 2)\%$ $= 8.036 \text{ to } 8.364$ Since $xy = 8.37$ is out of the acceptable range, the experimental readings are unacceptable.	

