DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		**INDEX NUMBER**	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS — 4049/02

Paper 2

25 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Calculator Model

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

Total Marks

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** A ball is thrown vertically upwards. Its height h m, above the ground at time t seconds after being thrown is given by the formula $h = 7 + 20t - 5t^2$.
Find the maximum height attained by the ball and the time at which this occurs. [4]

(b) Find the set of values of the constant k for which the curve $y = x^2 + 6x + k$ lies entirely above the line $y = k(x-1)$. [4]

2 A and B are acute angles, such that $\cos A = \frac{3}{5}$ and $\tan B = \frac{12}{5}$.

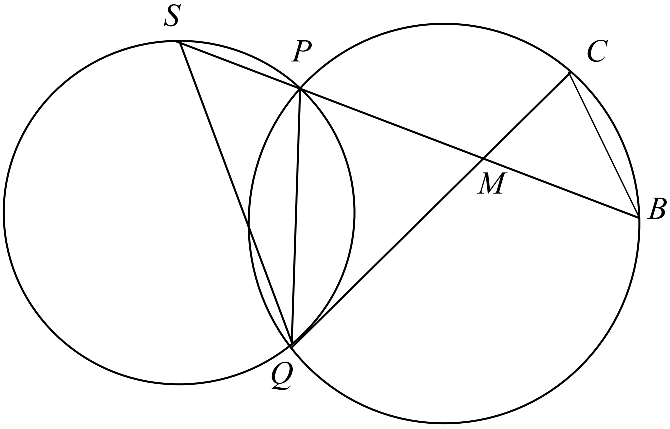
Find the exact values of

(a) $\tan 2A$, [2]

(b) $\sin\left(B + \frac{3\pi}{2}\right)$, [3]

(c) $\cos\frac{B}{2}$. [3]

3 In the diagram, *PQS* lies on a circle and *PCBQ* lies on another circle.
PQ is a common chord of the two circles. *BPS* is a straight line.
QC is a tangent to the circle *PQS* at *Q*. *QC* and *BP* intersect at *M*.



(a) Prove that *BC* is parallel to *QS*. [3]

(b) Prove that triangle *PQM* is similar to triangle *QSM*. [2]

(c) Hence show that $QM^2 = PM \times SM$. [2]

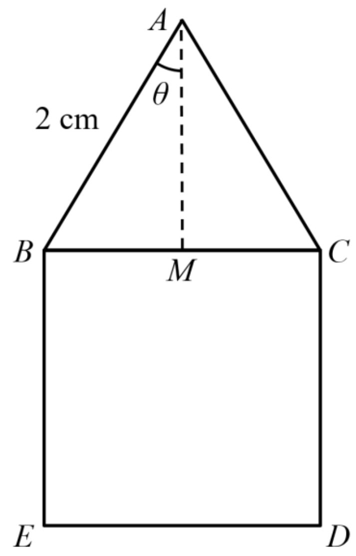
4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [4]

(b) Hence find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x \, \mathrm{d}x$. [4]

5 **(a)** Given that the constant term in the binomial expansion of $\left(x-\frac{k}{x^3}\right)^8$ is 7, find the value of the positive constant k. [4]

(b) Using the value of k found in part **(a)**, show that there is no constant term in the expansion of $\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8$. [4]

6 The diagram below shows an isosceles triangle ABC, where $AB = AC = 2$ cm, and a square $BCDE$.
M is the midpoint of BC.
Angle $BAM = \theta$ and can vary.



(a) Show that the total area of $ABEDC$ is $16 \sin^2 \theta + 4 \sin \theta \cos \theta$ cm^2. [3]

(b) Show that $16 \sin^2 \theta + 4 \sin \theta \cos \theta = 2 \sin 2\theta - 8 \cos 2\theta + 8$. [2]

(c) Express $2 \sin 2\theta - 8 \cos 2\theta$ in the form $R \sin (2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

7 **(a)** Prove the identity $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{1 - \tan x}{1 + \tan x}$. [4]

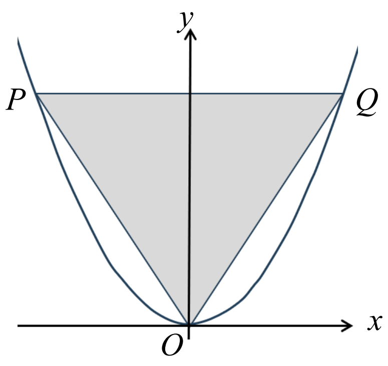
(b) Solve the equation $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{2}{3}\tan x$ for $0 \le x \le 2\pi$. [4]

8 **(a)** Using long division, divide $2x^3 - 7x^2 + 2x + 3$ by $2x - 4$. [2]

(b) Solve $2x^3 - 7x^2 + 2x + 3 = 0$. [4]

(c) Kun Ye claims that the solutions to the equation $2x^3 - 7x^2 + 2x + 3 = 0$ can be used to solve $3y^6 + 2y^4 - 7y^2 + 2 = 0$. Explain how he is correct. [2]

9 The diagram shows part of the curve $y = 3x^2$.
The points P and Q lie on the curve, and have the same y-coordinates.



(a) Show that the area of the triangle OPQ, A cm^2, is given by $A = 3x^3$. [2]

The points P and Q move along the curve such that their y-coordinates are changing at a rate of 3 units per second.

Find, at the point where $x = 4$,

(b) the rate of change of PQ, [4]

(c) the rate of change of A. [3]

10 The lines $y = 9$, $y = -1$ and $3y = 4x + 9$ are tangents to a circle.

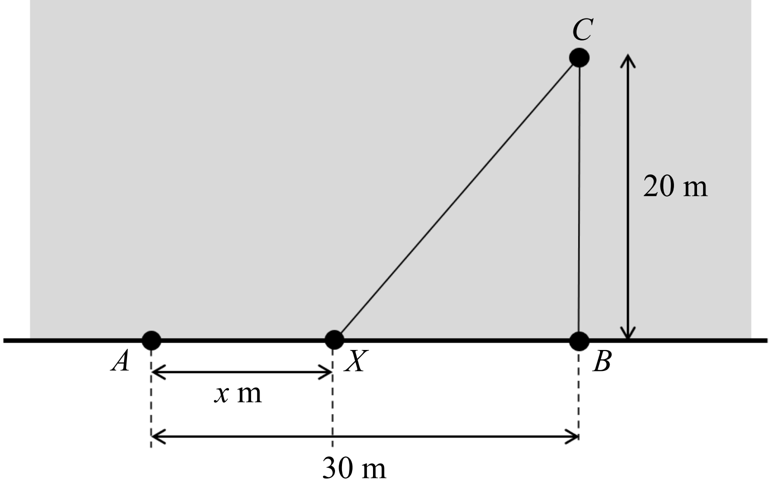
The x-coordinate of the centre of the circle is positive.

(a) Explain why the y-coordinate of the centre of the circle is 4. [1]

(b) By considering the discriminant, or otherwise, show that the x-coordinate of the centre of the circle is 7. [7]

(c) Determine if the point $(3,0)$ lies within the circle. [2]

11 The diagram shows a lake (shaded) bordered by a straight level road *AXB*.
Point *C* is on the surface of the lake and *B* is the point closest to *C*.
The distance *AB* is 30 m and *BC* is 20 m.



Natalie is at point *A* on the road and wants to get to point *C* as quickly as possible. She can run at a speed of 4 m/s and swim at a speed of 2 m/s.
She realizes that in order to minimize the total time to get to *C*, it is necessary to leave the road at some point *X* between *A* and *B*, and then swim directly to *C*.

Let *AX* be *x* m and *T* be the total time, in seconds, taken by Natalie to get from *A* to *X* and then from *X* to *C*.

(a) Show that $T = \frac{x}{4} + \frac{1}{2}\sqrt{x^2 - 60x + 1300}$. [2]

(b) Find an expression for $\frac{\mathrm{d}T}{\mathrm{d}x}$. [2]

(c) Find the value of x for which the total time taken is minimum. You are **not** required to justify that this value of x leads to the minimum total time. [4]

END OF PAPER

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