DUNMAN SECONDARY SCHOOL

CANDIDATE NAME	

CLASS		INDEX NUMBER	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/02**

Paper 2

25 August 2025

2 hours 15 minutes

Marking Scheme

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1a	$h = 7 + 20t - 5t^2$
	$= -5\left(t^2 - 4t\right) + 7$
	$= -5\left[\left(t-2\right)^2 - 4\right] + 7$
	$= -5\left(t-2\right)^2 + 27$
	maximum height = 27
	OR
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = -10t + 20$
	$-10t + 20 = 0$
	$t = 2$
	When $t = 2$, $h = 7 + 20\left(2\right) - 5\left(2\right)^2 = 27\,\mathrm{m}$
1b	$x^2 + 6x + k = k\left(x-1\right)$
	$x^2 + \left(6-k\right)x + 2k = 0$
	$\left(6-k\right)^2 - 4\left(1\right)\left(2k\right) < 0$
	$36 - 12k + k^2 - 8k < 0$
	$k^2 - 20k + 32 < 0$
	$\left(k-2\right)\left(k-18\right) < 0$
	$\therefore 2 < x < 18$
2a	$\tan A = \dfrac{4}{3}$
	$\tan 2A = \dfrac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$
	$= -\dfrac{24}{7}$
Question	**Answer**
2b	$\sin B = \dfrac{12}{13}$, $\cos B = \dfrac{5}{13}$
	$\sin\left(B + \dfrac{3\pi}{2}\right) = \sin B \cos\dfrac{3\pi}{2} + \sin\dfrac{3\pi}{2}\cos B$
	$= \dfrac{12}{13} \times 0 + \left(-1\right) \times \dfrac{5}{13}$
	$= -\dfrac{5}{13}$
	OR
	$\sin\left(B + \dfrac{3\pi}{2}\right) = \sin\left(2\pi - \dfrac{\pi}{2} + B\right)$

	$=\sin\left(-\frac{\pi}{2}+B\right)=-\sin\left(\frac{\pi}{2}-B\right)$
	$=-\cos B=-\frac{5}{13}$
2c	$\cos B=2\cos^2\frac{B}{2}-1$
	$\frac{5}{13}=2\cos^2\frac{B}{2}-1$
	$2\cos^2\frac{B}{2}=\frac{18}{13}$
	$\cos^2\frac{B}{2}=\frac{9}{13}$
	$\cos\frac{B}{2}=\frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ (rej)

Question	**Answer**
3a	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PQC = \angle PBC$ (angles in same segment)
	Hence $\angle PBC = \angle PSQ$
	By alternate angles of parallel lines, BC is parallel to QS
3b	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PMQ = \angle QMS$ (Common angle)
	ΔPQM is similar to ΔQSM (AA Similarity Test)
3c	Since ΔPQM is similar to ΔQSM, $\dfrac{QM}{SM} = \dfrac{PM}{QM}$
	$(QM)^2 = PM \times SM$

4a	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cos x}{\sin x}\right)$
	$=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$
	$=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$
	$=-\frac{1}{\sin^2 x}$
	$=-\operatorname{cosec}^2 x$ (Shown)
	OR
	$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x)=\frac{\mathrm{d}}{\mathrm{d}x}(\tan x)^{-1}$
	$=-(\tan x)^{-2}(\sec^2 x)$
	$=-\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$
	$=-\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
	$=-\frac{1}{\sin^2 x}$
	$=-\operatorname{cosec}^2 x$ (Shown)
4b	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x \, \mathrm{d}x=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1-\operatorname{cosec}^2 x \, \mathrm{d}x$
	$=\left[x+\cot x\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
	$=\left[\frac{\pi}{3}+\cot\frac{\pi}{3}\right]-\left[\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$
	$=\frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{4}-1$
	$=\frac{\pi}{12}+\frac{1}{\sqrt{3}}-1$ or -0.161

Question	Answer
5a	General term $=\binom{8}{r}x^{8-r}\left(-\dfrac{k}{x^3}\right)^r$
	$=\binom{8}{r}x^{8-r}\left(-1\right)^r k^r\left(x^{-3}\right)^r$
	$=\binom{8}{r}\left(-1\right)^r k^r x^{8-4r}$
	When $8-4r=0$, $r=2$
	$\binom{8}{2}\left(-1\right)^2 k^2 x^{8-4(2)}=7$
	$28k^2=7$
	$k^2=\dfrac{1}{4}$
	$k=\dfrac{1}{2}$
5b	When $8-4r=-4$, $r=3$
	$T_4=\binom{8}{3}\left(-1\right)^3\left(\dfrac{1}{2}\right)^3 x^{-4}$
	$=-7x^{-4}$
	$\left(1+x^4\right)\left(x-\dfrac{k}{x^3}\right)^8=\left(1+x^4\right)\left(...+7-7x^{-4}+...\right)$
	$=...+7-7+...=...+0+...$
	There is no constant term in the expansion. (Shown)

Question	Answer
6a	$AM = 2\cos\theta$
	$BM = 2\sin\theta$
	$BC = 4\sin\theta$
	Total area $= (4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$
	$= 16\sin^2\theta + 4\sin\theta\cos\theta$
6b	$16\sin^2\theta + 4\sin\theta\cos\theta$
	$= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$
	$= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$
	$= 2\sin 2\theta - 8\cos 2\theta + 8$
6c	$R = \sqrt{2^2 + 8^2} = \sqrt{68}$
	$\tan\alpha = \frac{8}{2}$
	$\alpha = 76.0°$
	$2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$
7a	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$
	$= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$
	$= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$
	$= \dfrac{\cos x - \sin x}{\cos x + \sin x}$
	$= \dfrac{1 - \tan x}{1 + \tan x} = RHS$

Question	Answer
7b	$\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$
	$3\left(1-\tan x\right)=2\tan x+\tan^2 x$
	$2\tan^2 x+5\tan x-3=0$
	$\left(2\tan x-1\right)\left(\tan x+3\right)=0$
	$\tan x=\dfrac{1}{2}$ or -3
	$x=0.464$ or 3.61
	$x=1.89$ or 5.03
8a	$x^2-\frac{3x}{2}-2$ $2x-4\ \big)\ 2x^3-7x^2+2x+3$ $-$ $2x^3-4x^2$ $-3x^2+2x+3$ $-$ $-3x^2+6x$ $3-4x$ $-$ $8-4x$ -5

Question	Answer
8b	$2x^3-7x^2+2x+3=0$
	Let $\mathrm{f}(x)=2x^3-7x^2+2x+3$
	When $x=3$, $\mathrm{f}(3)=2(3)^3-7(3)^2+2(3)+3=0$
	By factor theorem, $(x-3)$ is a factor of f(x).
	$(x-3)(2x^2-x-1)=0$
	$(x-3)(2x+1)(x-1)=0$
	$x=-\frac{1}{2}$, 1 or 3
8c	$3y^6+2y^4-7y^2+2=0$
	$3+\frac{2}{y^2}-\frac{7}{y^4}+\frac{2}{y^6}=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^4}\right)+2\left(\frac{1}{y^6}\right)=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^2}\right)^2+2\left(\frac{1}{y^2}\right)^3=0$
	$\therefore x=\frac{1}{y^2}$, Kun Ye is correct

Question	**Answer**
9a	Height = $3x^2$
	Base = $2x$
	Area = $\frac{1}{2}(2x)(3x^2) = 3x^3$
9b	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x$
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$3 = 6x \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$3 = 6x \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2x}$
	When $x = 4$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{8}$
	Rate of change of $PQ = \frac{1}{4}$ unit/s
9c	$\frac{\mathrm{d}A}{\mathrm{d}x} = 9x^2$
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$\frac{\mathrm{d}A}{\mathrm{d}t} = 9x^2 \times \frac{1}{8}$
	When $x = 4$, $\frac{\mathrm{d}A}{\mathrm{d}t} = 18$ unit^2/s

Question	**Answer**
10a	y-coordinate of centre = $\dfrac{-1+9}{2}=4$
10b	Let the x-coordinate of the centre be a,
	$(x-a)^2+(y-4)^2=25$
	From the line, $y=\dfrac{4x+9}{3}$
	$(x-a)^2+\left(\dfrac{4x+9}{3}-4\right)^2=25$
	$(x-a)^2+\left(\dfrac{4}{3}x-1\right)^2=25$
	$x^2-2ax+a^2+\dfrac{16}{9}x^2-\dfrac{8}{3}x+1=25$
	$\dfrac{25}{9}x^2+\left(-2a-\dfrac{8}{3}\right)x+a^2-24=0$
	Since the line is a tangent, $b^2-4ac=0$
	$\left(-2a-\dfrac{8}{3}\right)^2-4\left(\dfrac{25}{9}\right)\left(a^2-24\right)=0$
	$4a^2+\dfrac{32}{3}a+\dfrac{64}{9}-\dfrac{100}{9}a^2+\dfrac{2400}{9}=0$
	$36a^2+96a+64-100a^2+2400=0$
	$-64a^2+96a+2464=0$
	$2a^2-3a-77=0$
	$(2a+11)(a-7)=0$
	$a=7$ or -5.5 (rej)

Question	Answer
10c	Distance between point and centre $=\sqrt{(7-3)^2+(4-0)^2}$
	$=5.66$ unit
	Since the distance is longer than the radius, the point lies outside.
11a	$CX=\sqrt{(30-x)^2+20^2}$
	$=\sqrt{x^2-60x+1300}$
	$T=\frac{x}{4}+\frac{1}{2}\sqrt{x^2-60x+1300}$
11b	$\frac{\mathrm{d}T}{\mathrm{d}x}=\frac{1}{4}+\frac{1}{2}\left[\frac{1}{2}\left(x^2-60x+1300\right)^{-\frac{1}{2}}(2x-60)\right]$
	$=\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}$
11c	$\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}=0$
	$\sqrt{x^2-60x+1300}+2x-60=0$
	$\sqrt{x^2-60x+1300}=60-2x$
	$x^2-60x+1300=3600-240x+4x^2$
	$3x^2-180x+2300=0$
	$x=\frac{180\pm\sqrt{180^2-4(3)(2300)}}{2(3)}$
	$x=41.5$ (rej) or 18.5