

Sec 1-2 Math notes

Thursday, 26 September 2024 7:27 AM

Numbers

- Natural numbers $\rightarrow 1, 2, 3, 4, 5, \dots$
- Whole numbers $\rightarrow 0, 1, 2, 3, 4, \dots$
- Prime numbers $\rightarrow 2, 3, 5, 7, 11, 13, 17, 19, 23, \dots$
- Rational numbers $\rightarrow$ numbers that can be expressed in fraction including terminating (e.g. 0.2, 0.35, 0.6) and recurring decimals (e.g. 0.33333333..., 0.2133333333..., 0.213213213213...)
- Integer $\rightarrow$ numbers without fraction (0 is neutral)
- Find HCF $\rightarrow$ take the power that is included in all numbers
- Find LCM $\rightarrow$ take the power that is the high must be from all the numbers
- Perfect square $\rightarrow$ indices of prime factorisation must be multiples of 2
- Perfect cube $\rightarrow$ indices of prime factorisation must be multiples of 3
- $(-)(-)=+$
- $(+)(-)= -$
- $(-)(+)= -$
- $++=+$
- $--=+$
- $+=+$
- $-=-$
- $x \div x = 1$
- $x^2 = \pm \sqrt{\quad}$
- $x^3 = \pm \sqrt[3]{\quad}$

Rounding significant figures

- All non 0 are significant
- All 0 in between digits are significant

Expansion

- $(X + Y)(a + b) = Xa + Xb + Ya + Yb$
- $a(x + y) = ax + ay$
- $\frac{a}{3} = \frac{1}{3}a$
- #1 $(a + b)^2 = a^2 + 2ab + b^2$
- #2 $(a - b)^2 = a^2 - 2ab + b^2$
- #3 $(a + b)(a - b) = a^2 - b^2$

Factorisation

- Take out common factors $\rightarrow x - y = -(x + y)$
- four terms $\rightarrow$ grouping $\rightarrow (a - b) = -(b - a)$
- three terms $\rightarrow$ cross method
- two terms $\rightarrow$ #3 $(a + b)(a - b) = a^2 - b^2$

Linear graph

- straight line graph
- Linear functions: $y = mx + c$
- e.g. 1) $y = 2x + 3$ 2) $3 = 2x + c$ 3) $3 = x + 3$
- $m = \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$
- $c = y\text{-intercept}$ (where line cuts y-axis)
- $m > 0$: positive
- $m < 0$: negative
- Parallel lines have the same gradient

Horizontal line

- Gradient = 0
- Equation $\rightarrow y = \text{constant}$

Vertical line

- Gradient = undefined
- Equation $\rightarrow x = \text{constant}$

*cuts x-axis, sub $y = 0$

*cuts y-axis, sub $x = 0$

Number pattern

- Even number $T_n = 2n$
- Odd number $T_n = 2n - 1$
- Perfect square $T_n = n^2$
- Perfect cube $T_n = n^3$
- if add or subtract a number to get the next term
- $T_n = a + (n - 1)(d)$
- $a \rightarrow$ first term of the sequence
- $d \rightarrow$ the consistent increase or decrease in the sequence

Percentage

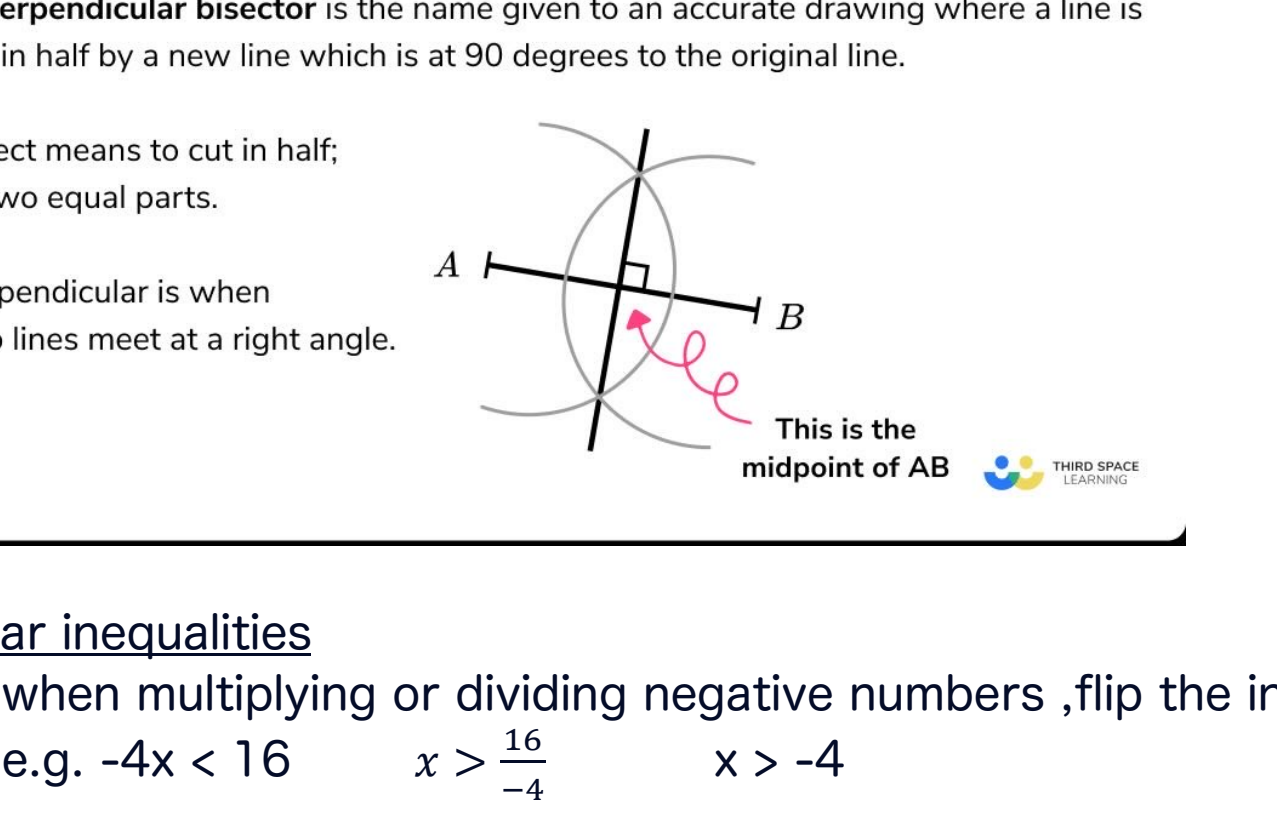
- $x\% = \frac{x}{100}$
- $x\%$ of A $\rightarrow \frac{x}{100} \times A$
- to convert to $\rightarrow x \times 100\%$
- express A as a percentage of B $\rightarrow \frac{a}{b} \times 100\%$
- percentage change $\rightarrow \frac{\text{increase or decrease}}{\text{original}} \times 100\%$
- increased by x percent $\rightarrow (100 + x)\%$
- decreased by x percent $\rightarrow (100 - x)\%$
- increase/decrease to x percent $\rightarrow x\%$
- profit $\rightarrow \frac{\text{profit}}{\text{selling price} - \text{cost price}} \times 100$
- loss $\rightarrow \frac{\text{loss}}{\text{cost price} - \text{selling price}} \times 100$
- discount percentage $\rightarrow \frac{\text{discount}}{\text{marked price}} \times 100\%$

Polygons

- triangle = 180°
- quadrilateral = 360°
- Sum of angles in an n-sided polygon $\rightarrow 180(n - 2)$
- One interior angle $\rightarrow \frac{(n - 2) \times 180^\circ}{n}$
- One exterior angle $\rightarrow \frac{360^\circ}{n}$
- One interior angle + One exterior angle = 180°
- Triangle: 3 sides, 180 degrees.
- Quadrilateral: 4 sides, 360 degrees.
- Pentagon: 5 sides, 540 degrees
- Hexagon: 6 sides, 720 degrees
- Heptagon: 7 sides, 900 degrees
- Octagon: 8 sides, 1080 degrees
- Nonagon: 9 sides, 1260 degrees
- Decagon: 10 sides, 1440 degrees
- 11 sides, 1620 degrees

Triangles

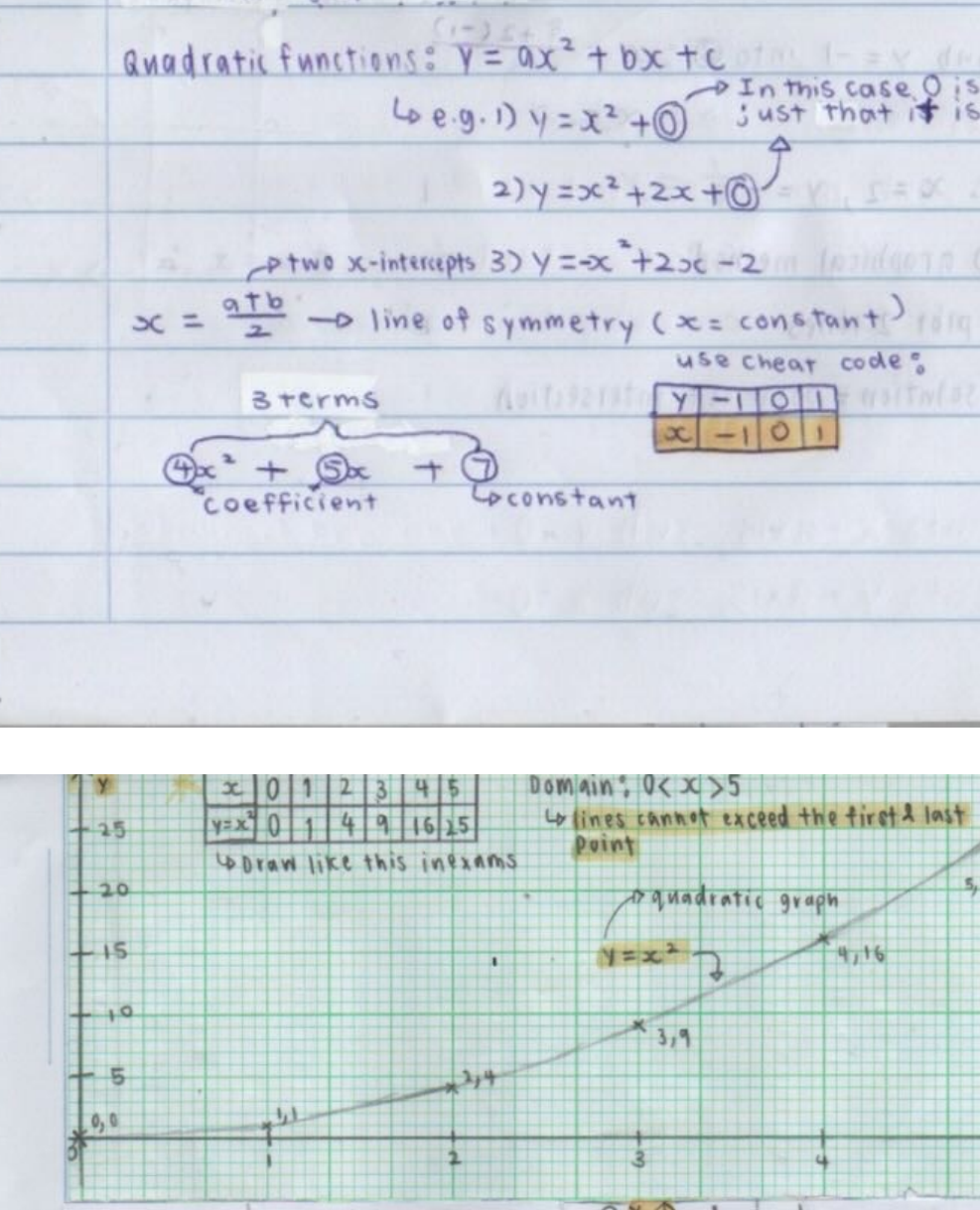
- 3 sides same $\rightarrow$ equilateral
- 2 sides same $\rightarrow$ isosceles
- alternate angles (alt. $\angle$ s)



- corresponding angles (corr. $\angle$ s)
- $a + b = 180^\circ \rightarrow$ interior angles (int. $\angle$ s)

Geometry

- $\angle$ less than $90^\circ \rightarrow$ acute angle
- $\angle$ $\rightarrow$ right angle
- $\angle$ more than 90° but less than $180^\circ \rightarrow$ obtuse angle
- $\angle$ more than 180° but less than $360^\circ \rightarrow$ reflex angle



Angle bisector

Angle Bisector

An **angle bisector** is the name given to an accurate drawing where an angle is cut in half by a straight line. Bisector means to cut in half; in two equal pieces.

To do this we need to use a pencil, a ruler (a straight-edge) and compasses.

E.g. Here is an angle bisector of angle ABC.

Perpendicular bisector

Perpendicular Bisector

A **perpendicular bisector** is the name given to an accurate drawing where a line is cut in half by a new line which is at 90 degrees to the original line.

Bisect means to cut in half; in two equal parts.

Perpendicular is when two lines meet at a right angle.

Linear inequalities

- when multiplying or dividing negative numbers, flip the inequality sign

e.g. $-4x < 16$ $x > \frac{16}{-4}$ $x > -4$

Which of the following diagrams represents the inequality $-3 < x \leq 0$?

- greater than / less than
- greater than / equal to / less than / equal to

① $-3 < x < 0$

② $-3 \leq x < 0$

③ $-3 < x \leq 0$

④ $-3 \leq x \leq 0$

Quadratic graph

Quadratic graph

Quadratic function: $y = ax^2 + bx + c$

e.g. $y = x^2 + 2x + 3$

$2y = x^2 + 2x + 6$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(3)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 - 12}}{2}$

$x = \frac{-2 \pm \sqrt{-8}}{2}$

$x = \frac{-2 \pm 2\sqrt{-2}}{2}$

$x = -1 \pm \sqrt{-2}$

$x = -1 \pm i\sqrt{2}$

$x = -1 + i\sqrt{2}$

$x = -1 - i\sqrt{2}$

Quadratic graph

Graphical method: $y = x^2 + 2x + 3$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(3)}}{2(1)}$

$x = \frac{-2 \pm \sqrt{4 - 12}}{2}$

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- factorisation $\rightarrow (0, 0) = 0$
- graphical method $\rightarrow$ solution = x intercept

Simultaneous equations

1) elimination method

$\rightarrow$ make one alphabet the same

$\rightarrow$ if the same sign, subtract equations

$\rightarrow$ if different sign, add equations

e.g. $4x - 5y = 17$ (1)

$2x - 5y = 8$ (2)

(1) - (2): $(4x - 5y) - (2x - 5y) = 17 - 8$

$4x - 5y - 2x + 5y = 17 - 8$

$2x = 9$

$x = \frac{9}{2} = 4.5$

sub $x = 4.5$ into (2): $2(4.5) - 5y = 8$

$9 - 5y = 8$

$-5y = -1$

$y = \frac{1}{5}$

$x = 4.5, y = \frac{1}{5}$

- 2) substitution method

- $\rightarrow$ make one alphabet the subject

- $\rightarrow$ sub this alphabet into the other equations

- e.g. $4x - 2y = 8$ (1)

- $4x + 3y = 5$ (2)

- From (1): $3x = 8 + 2y$

- $x = \frac{8 + 2y}{3}$ (3)

- Sub (3) into (2): $4(\frac{8 + 2y}{3}) + 3y = 5$

- $\frac{32 + 8y}{3} + 3y = 5$

- $32 + 8y + 9y = 15$

- $17y = -17$

- $y = -1$

- Sub $y = -1$ into (3): $x = \frac{8 + 2(-1)}{3}$

- $x = 2$

- $x = 2, y = -1$

- 3) graphical method

- $\rightarrow$ plot two lines

- $\rightarrow$ solution = point of intersection

Algebraic fraction

- 1) for $\times$ and $\div$, factorise both numerator

- $\rightarrow$ cancel out like terms

- 2) for $+$ and $-$, factorise only the denominator, find LCM

- $\rightarrow$ finalise LCM

Direct & Inverse Proportion

- 1) direct proportion

- Formula: $y = kx$ $*x = \text{constant}$

- e.g. If y is directly proportional to x

- $y = kx$

- 2) inverse proportion

- Formula: $y = \frac{k}{x}$

- e.g. If y is inversely proportional to x

- $y = \frac{k}{x}$

- *Particular value an

- 1) Write: Let particular of m be m

- 2) Sub the number and m in (add brackets)

- 3) Write ... formula

- 4) When this value is doubled ($\times 2$), tripled ($\times 3$), quadrupled ($\times 4$)

- Sub m into the place of $?$ and times by the an ask

- 5) cancel the like terms (get your ans)

- e.g. 1

- y is directly proportional to the square of x and $y = a$ for a particular value of x . Find an expression for y in terms of x , when this value of x is doubled.

- Let particular value of x be $m, y = a$

- $y = kx^2$

- (a) $= k(m)^2$

- $a = km^2$

- $km^2 = a$

- $k = \frac{a}{m^2}$

- $\dots y = \frac{a}{m^2} x^2$

- $y = \frac{a}{m^2} (2m)^2$

- $y = 4a$

- e.g. 2

- The frequency of a radio wave is inversely proportional to its wavelength W .

- Given that the frequency is 100 kHz at a certain wavelength, find the frequency when the wavelength is doubled.

- Let the certain wavelength W be $m, f = 100$

- $f = \frac{k}{w}$

- $(100) = \frac{k}{m}$

- $100 = \frac{k}{m}$

- $100m = k$

- $k = 100m$

- $\dots f = \frac{100m}{w}$

- $f = 50 \text{ kHz}$

Congruence & Similarity

Congruent triangles

- 3 corresponding sides are equal
- 2 corresponding sides and 1 included angle are equal
- 2 corresponding angles and 1 side are equal
- right angle, hypotenuse, 1 side are equal

Similar triangles

- 3 corresponding angles are equal
- 2 ratios of corresponding sides are equal
- ratios of 2 corresponding sides and 1 included angle are equal

Scale factor $\frac{\text{new}}{\text{original}}$

- Scale factor $> 1 \rightarrow$ enlargement
- Scale factor $< 1 \rightarrow$ reduction
- Scale factor $= 1 \rightarrow$ exactly same

Pythagoras Theorem

hypotenuse $\rightarrow$ longest side of triangle

Formula: $a^2 + b^2 = c^2$

Formula: $a^2 + b^2 = AB^2$

$\rightarrow$ determine whether it is right angle triangle

whole no = yes decimal = no

Trigonometry

Formula: $\frac{\text{opposite}}{\text{adjacent}} = \tan \theta$

$\frac{\text{opposite}}{\text{hypotenuse}} = \sin \theta$

$\frac{\text{adjacent}}{\text{hypotenuse}} = \cos \theta$

hypotenuse stays the same only adjacent and opposite changes

hypotenuse is always connected to the adjacent

Formula: $\sin^{-1} \frac{\text{opposite}}{\text{hypotenuse}}$

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