RAFFLES INSTITUTION
2025 YEAR 6 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9758/02

Paper 2

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **9** printed pages and **1** blank page.

RAFFLES INSTITUTION

Mathematics Department

Section A: Pure Mathematics [40 marks]

1 **(a)** Without using a calculator, solve the inequality $\dfrac{x^2-5x+6}{x^2-4}<\dfrac{2x-3}{x+2}$. [4]

(b) **(i)** Sketch on the same diagram the graphs of $y=\ln x$ and $y=x-5$, giving the equations of any asymptotes and the x-coordinates of the points of intersection between the two graphs. [2]

(ii) Hence solve the inequality $\ln|x|<|x|-5$. [2]

2 **(a)** In a triangle ABC, $AB=2$, angle $CAB=x$ radians and angle $CBA=\dfrac{\pi}{6}$ radians.

(i) Show that $AC=\dfrac{2}{\cos x+\sqrt{3}\sin x}$. [2]

(ii) Given that x is a sufficiently small angle, show that

$$AC\approx a+bx+cx^2,$$

where a, b and c are constants to be determined. [3]

(b) It is given that $2xy+\ln y=\ln 3$. Show that

$$(2xy^2+y)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4y^2\frac{\mathrm{d}y}{\mathrm{d}x}-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0.$$

Hence find the Maclaurin series for y, up to and including the term in x^2. [5]

3 The terms of the sequence U are given by

$$u_1 = k \quad \text{and} \quad u_{n+1} = \frac{8u_n - 14}{u_n - 1}, \qquad n \geq 1.$$

(a) For the following values of k, describe the behaviour of the sequence U.

(i) $k = 3$ [1]

(ii) $k = 10$ [1]

(b) Find the possible value(s) of k if the sequence U is a constant sequence. [2]

The nth term of the sequence V is given by $v_n = \frac{a^n}{b} + \frac{b}{a(1-a^n)} - \frac{a}{n+1}$, where a and b are non-zero real constants and $a \neq \pm 1$.

(c) For some values of a, $v_n \to L$ as $n \to \infty$. Find, with justification, the range of values of a for L to exist, and state the value of L in terms of a and b. [3]

The nth term of the sequence W is given by

$$w_n = \begin{cases} u_n & \text{when } n \text{ is even,} \\ v_n & \text{when } n \text{ is odd.} \end{cases}$$

It is given that the sequence W converges when the sequences U and V converge to the same limit. The sequence W diverges otherwise.

(d) For $k = 10$, by using part **(a)(ii)** and part **(c)**, find the range of values of b for the sequence W to converge. Hence explain whether $\sum_{r=1}^{\infty} w_r$ is a convergent series. [3]

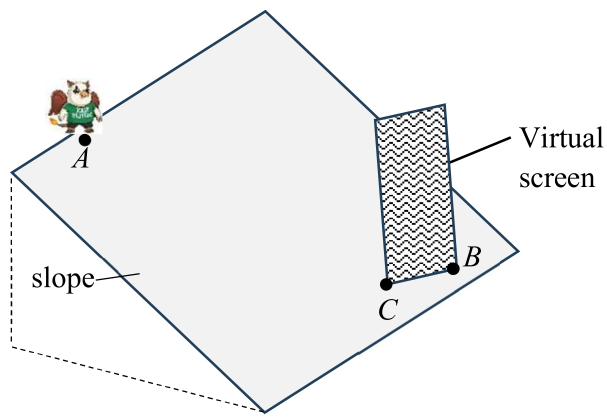
4 In a computer game, a slope can be modelled as a plane p containing three points, $A(1,\ 0,\ -3)$, $B(1,\ 4,\ -15)$ and $C(2,\ 3,\ -14)$.

(a) Show that p has equation $\mathbf{r}.\begin{pmatrix}2\\3\\1\end{pmatrix}=-1$. [2]

In Stage 1 of the game, Griffles travels on the slope from A along a path with equation $\mathbf{r}=\begin{pmatrix}1\\0\\-3\end{pmatrix}+\lambda\begin{pmatrix}1\\m\\n\end{pmatrix}$, where m and n are constants and λ is a real parameter.

(b) Find an equation relating m and n. [1]

In the rest of the question, it is given that $m=\frac{1}{3}$, $n=-3$ and Griffles is modelled as a point travelling on the slope. An obstacle in the form of a rectangular virtual screen stands on the slope with its base modelled by the **line segment** BC (see diagram).



(c) By considering the **line segment** BC, show that Griffles is able to successfully navigate this obstacle without colliding into it. [3]

A laser gun with a sensor is mounted at the point $E(5,\ 2,\ 1)$ above the slope. The sensor is activated when an object is within a range of 5 units.

(d) Show that Griffles does not activate the sensor. [3]

After completing Stage 1, Griffles is teleported to another slope. This slope can be modelled as a plane q parallel to p such that E is equidistant from p and q.

(e) Find the cartesian equation of q. [3]

Section B: Probability and Statistics [60 marks]

5 A discrete random variable X has the following probability distribution:

$$\mathrm{P}(X = x) = k\left(x^2 + x\right) \text{ for } x = 1,\ 2,\ 3,\ 4,\ 5, \text{ where } k \text{ is a constant.}$$

(a) Show that $k = \dfrac{1}{70}$. [1]

(b) Find the exact values of $\mathrm{E}\left(X\right)$ and $\mathrm{Var}\left(X\right)$. [3]

(c) Two independent observations X_1 and X_2 are taken of X. Find the probability that the difference between the two observations is at least 3. [3]

6 Kitty has 5 small, 3 medium and 2 large spherical charms and each of the 10 charms is uniquely designed.

(a) Kitty arranges all the charms in a circle on a corkboard with charms of the same sizes next to each other. How many ways are there for her to do so? [2]

(b) On another occasion, Kitty arranges all the charms in a line at the base of a photo frame with none of the small charms next to each other. How many ways are there for her to do so? [2]

The diameters of all the small, medium and large charms are 0.5 cm, 1 cm and 2 cm respectively. Each of the spherical charms has a hole through its centre that allows it to be threaded through a chain.

(c) Kitty makes a keychain that includes a 6 cm chain with one end attached to a keyring, as shown in **Fig. 1**. It is given that the 6 cm chain is fully threaded with charms, with no gaps between them, and is stretched taut in a straight line. For example, **Fig. 2** shows the 6 cm chain threaded with 2 large and 2 medium charms.

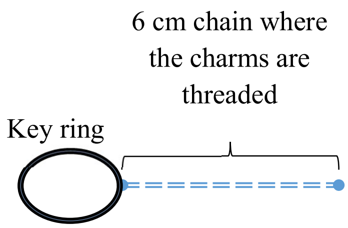


Fig. 1

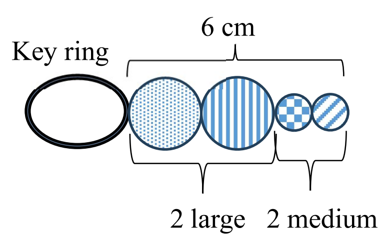


Fig. 2

How many different ways can she make a keychain with at least one charm of each of the three sizes? [4]

7 **In this question you should state the parameters of any distributions you use.**

A snack company produces two types of potato chips, Rays and Luffles. The mass, in grams, of a regular packet of Rays follows the distribution $\mathrm{N}\left(100,\ \sigma^2\right)$ and the mass, in grams, of a regular packet of Luffles follows the distribution $\mathrm{N}\left(120,\ 16\right)$. The masses of all regular packets of potato chips are independent of one another.

(a) If the probability of the mass of a randomly chosen packet of Rays not exceeding $\left(100-\sigma^2\right)$ grams is less than 0.2, find the possible range of values of σ. [2]

It is given that $\sigma = 3$ for the rest of this question.

(b) Find the probability that the total mass of 2 randomly chosen regular packets of Rays and 3 randomly chosen packets of Luffles is greater than 0.55 kg. [3]

(c) The snack company decides to launch a new product called Mega Jumbo Pack, which consists of $\left(24-n\right)$ regular packets of Rays and n regular packets of Luffles. It is given that the probability of the mass of a Mega Jumbo Pack exceeding 20 times the mass of a regular packet of Luffles by more than 500 g is at least 0.1. Find the minimum value of n. [4]

8 A small cafe sells one type of coffee. The selling price of a cup of coffee is reviewed and adjusted at the beginning of every year depending on market conditions. Based on sales figures collected over 7 years, the cafe owner, Mr Tay, studied the effect of the selling price of a cup of coffee, $\$x$, on the average number of cups, y cups, sold per day within the year. The data is shown in the table below.

x	2.0	2.2	2.5	3.0	4.0	4.4	4.5
y	280	250	190	150	90	100	70

(a) On the grid in the Printed Answer Booklet, draw a scatter diagram of the data. [2]

Mr Tay realised that one of the values of y has been wrongly stated in the data table.

(b) Indicate the corresponding point on your diagram by labelling it P. Explain why the scatter diagram for the remaining points may be consistent with a model of the form $y = a + \frac{b}{x}$, where a and b are constants. [2]

(c) Omitting P, calculate the product moment correlation coefficient and the least squares estimates of a and b for the model $y = a + \frac{b}{x}$. [2]

(d) Use the model $y = a + \frac{b}{x}$ with the values of a and b found in part **(c)** to estimate the value of y that has been wrongly stated in the data table. Give two reasons why you would expect this estimate to be reliable. [3]

9 Based on observations over a long period, the mean time taken by male students in Griffles Junior College (GJC) to complete a 2.4 km run is known to be 11.3 minutes with a standard deviation of 2.2 minutes. As part of a review of the current physical training programme, the Physical Education (PE) department in GJC decided to test, at the 2.5% level of significance, whether the mean time taken by male students in GJC to complete a 2.4 km run has changed. The time taken, x minutes, by a random sample of 8 Year 5 male students from GJC to complete a 2.4 km run, are as follows.

11 11.5 10.8 11.2 11.4 11 11.8 12.5

(a) Write down the null and alternative hypotheses for this test, defining any symbols you use. [2]

(b) Stating a necessary assumption, find the critical region for this test. Hence state the conclusion of the test in the context of the question. [6]

The PE department in GJC also conducted a second test, at the 3% level of significance, to determine whether the mean time taken by female students in GJC to complete a 2.4 km run is less than 14.5 minutes. A random sample of n female students from GJC is taken, where n is large. The mean and standard deviation of the time taken by this sample to complete a 2.4 km run are found to be 14.2 minutes and 1.5 minutes respectively.

(c) Given that the PE department concludes that the mean time taken by female students to complete a 2.4 km run is less than 14.5 minutes, find the range of values that n can take. [5]

10 A bag contains 4 identical red balls and 6 identical blue balls.

(a) In the first game, the balls are randomly picked by a player from the bag, one at a time, without replacement, until there are no balls left in the bag.

(i) Find the probability that the last ball picked is red. [1]

(ii) Find the probability that in the first five picks, exactly 2 blue balls are picked given that at least 3 red balls are picked. [3]

(b) In the second game, each ball picked by a player will be placed back into the bag before the next ball is picked. The player makes a total of 20 random picks from the bag and the colour of each pick is recorded.

(i) Find the probability that the colour red is recorded exactly 4 times. [2]

(ii) Find the probability that the colour red is recorded more than 4 times but not more than 8 times. [2]

(iii) Find the probability that the 8^{th} pick is the 6^{th} time the colour blue is recorded. [2]

(iv) The second game is played by 50 randomly chosen players. Estimate the probability that the average number of times the colour red is recorded is more than 8.5. [4]

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