2014 A-Level H2 Mathematics Paper 2

Section A: Pure Mathematics

1 (i)	$x=3t^2,\ y=6t$ $\frac{dx}{dt}=6t,\ \frac{dy}{dt}=6$ $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}=\frac{1}{t}$ When $\frac{dy}{dx}=\frac{1}{t}=0.4$, $t=2.5$.
(ii)	Tangent at *P*: $y-6p=\frac{1}{p}\left(x-3p^2\right)$ This line meets the *y*-axis at *D*. $\Rightarrow x_D=0,\ y_D-6p=\frac{1}{p}\left(0-3p^2\right)$ $y_D=-3p+6p=3p$ $P\left(3p^2,\ 6p\right)$ and $D(0,\ 3p)$ Let $M\left(\frac{3p^2}{2},\frac{9p}{2}\right)$ be the midpoint of *PD*. $x_M=\frac{3p^2}{2}\ ---(1)$, $y_M=\frac{9p}{2}\Rightarrow p=\frac{2}{9}y_M\ ---(2)$ | Aim: Eliminate *p*. Sub (2) into (1): $x_M=\frac{3}{2}\left(\frac{2}{9}y_M\right)^2=\frac{2}{27}{y_M}^2$ $\therefore$ The locus of the midpoint has equation $x=\frac{2}{27}y^2$.
2	Let $\frac{9x^2+x-13}{(2x-5)\left(x^2+9\right)}\equiv\frac{A}{2x-5}+\frac{Bx+C}{x^2+9}$ $9x^2+x-13\equiv A\left(x^2+9\right)+(Bx+C)(2x-5)$ Comparing coefficients of $x^2: A+2B=9\ ---(1)$ $x: -5B+2C=1\ ---(2)$ $x^0: 9A-5C=-13\ ---(3)$ Solving simultaneously, $A=3,\ B=3,\ C=8$ $\int_0^2\frac{9x^2+x-13}{(2x-5)\left(x^2+9\right)}\,dx=\int_0^2\left(\frac{3}{2x-5}+\frac{3x+8}{x^2+9}\right)dx$ $=\frac{3}{2}\int_0^2\frac{2}{2x-5}\,dx+\frac{3}{2}\int_0^2\frac{2x}{x^2+9}\,dx+\int_0^2\frac{8}{x^2+9}\,dx$ $=\frac{3}{2}\Big[\ln\lvert 2x-5\rvert\Big]_0^2+\frac{3}{2}\Big[\ln\left(x^2+9\right)\Big]_0^2+8\Big[\frac{1}{3}\tan^{-1}\frac{x}{3}\Big]_0^2$ $=\frac{3}{2}(\ln 1-\ln 5)+\frac{3}{2}(\ln 13-\ln 9)+\frac{8}{3}\left(\tan^{-1}\frac{1}{3}-\tan^{-1}0\right)$ $=\frac{3}{2}(-\ln 5+\ln 13-\ln 9)+\frac{8}{3}\tan^{-1}\frac{2}{3}$ $=\frac{3}{2}\ln\frac{13}{45}+\frac{8}{3}\tan^{-1}\frac{2}{3}$

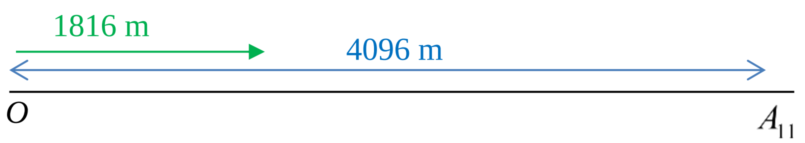
<table>
<tr><td>3 (i)</td><td>The distances of the stages: 8, 16, 24, … form an A.P. with first term 8 and common difference 8.

(a) Total distance after 10 stages,
$S_{10} = \frac{10}{2}\left[2(8)+9(8)\right]$
$= 440$ m

(b) Total distance after n stages,
$S_n = \frac{n}{2}\left[2(8)+(n-1)(8)\right]$
$= 4n+4n^2$ metres

To run at least 5 km (5000 m), $S_n \geq 5000$ (i.e. $4n^2+4n \geq 5000$)
By GC, when $n=34$, $S_n = 4760 < 5000$
$n=35$, $S_n = 5040 > 5000$
$\therefore$ The least number of stages is 35.</td></tr>
<tr><td>(ii)</td><td>The distances of the stages: 8, 16, 32, … form a G.P. with first term 8 and common ratio 2.

Total distance after n stages,
$s_n = \dfrac{8\left(2^n-1\right)}{2-1}$
$=8\left(2^n-1\right)$
By GC, when $n=10$, $s_n = 8184 < 10000$
$n=11$, $s_n = 16376 > 10000$
$\therefore$ After running 10 km, the athlete has completed 10 stages and $10000-8184=1816$ m of the 11th stage.
$OA_{11} = \frac{1}{2}\left(8\times 2^{10}\right) = 4096 \Rightarrow$ The athlete has not reached A_{11}.



He is 1816m away from O and running towards A_{11}.</td></tr>
<tr><td>4(a)</td><td>Out of syllabus</td></tr>
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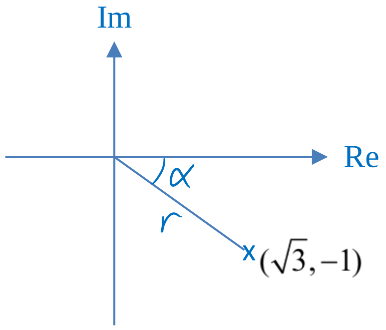
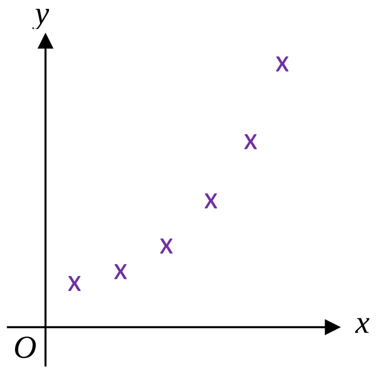
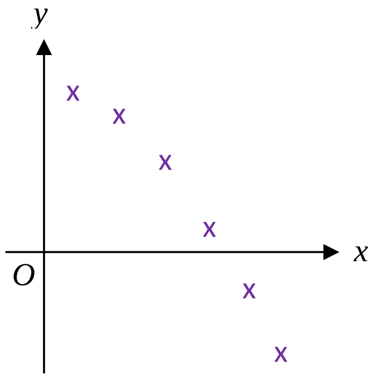
4(b)	(i) $r=\sqrt{3+1}=2$, $\alpha=\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$  $w=2e^{i\left(-\frac{\pi}{6}\right)}$ $w^6=64e^{i(-\pi)}=64e^{i\pi}$ (ii) $\frac{w^n}{w*}=\frac{2^n e^{i\left(-\frac{n\pi}{6}\right)}}{2e^{i\left(\frac{\pi}{6}\right)}}$ $=2^{n-1}e^{i\left(-\frac{n\pi}{6}-\frac{\pi}{6}\right)}$ $=2^{n-1}e^{-\left(\frac{n+1}{6}\pi\right)i}$ For $\frac{w^n}{w*}$ to be real, $\frac{n+1}{6}\pi=k\pi,\ k\in\mathbb{Z}$ $n=6k-1$ $\therefore$ The three smallest positive whole number values of n are 5, 11 and 17. <u>Alternative</u> $\frac{w^n}{w*}=\ldots=2^{n-1}e^{-\left(\frac{n+1}{6}\pi\right)i}$ $=2^{n-1}\left[\cos\left(-\frac{n+1}{6}\pi\right)+i\sin\left(-\frac{n+1}{6}\pi\right)\right]$ For $\frac{w^n}{w*}$ to be real, $\text{Im}\left(\frac{w^n}{w*}\right)=0$ $\sin\left(-\frac{n+1}{6}\pi\right)=0$ $\sin\left(\frac{n+1}{6}\pi\right)=0$ $\frac{n+1}{6}\pi=k\pi,\quad k\in\mathbb{Z}$ $n=6k-1$
6 (i)	Number of teams $={}^3C_1\times{}^8C_4\times{}^5C_2\times{}^6C_4=31500$
(ii)	Case 1: brother as midfielder No. of teams $={}^3C_1\times{}^8C_4\times{}^4C_1\times{}^5C_4=4200$ Case 2: brother as attacker No. of teams $={}^3C_1\times{}^8C_4\times{}^4C_2\times{}^5C_3=12600$ Total number of possible teams $=4200+12600=16800$ Notice the subscripts add up to 10 in each case. That's because we only need to choose 10 players in addition to the brother.
(iii)	Case 1: The particular midfielder is in the team as midfielder. No. of teams $={}^3C_1\times{}^8C_4\times{}^3C_1\times{}^5C_4=3150$ Case 2: The particular midfielder is in the team as defender. No. of teams $={}^3C_1\times{}^8C_3\times{}^3C_2\times{}^5C_4=2520$ Case 3: The particular midfielder is not in the team. No. of teams $={}^3C_1\times{}^8C_4\times{}^3C_2\times{}^5C_4=3150$ By the same reasoning, we only need to choose 10 players in addition to the particular midfielder in Cases 1 & 2.

	Total number of possible teams $=3150+2520+3150=8820$
7(i)	Let X be the number of rolls, out of 10, where the fair die shows a 6. Then $X \sim \mathrm{B}\left(10,\ \frac{1}{6}\right)$ $\mathrm{P}(X=3) \approx 0.15506$ $=0.155 \quad (3\text{ s.f.})$
(ii) (iii)	Out of syllabus
8(a)	(i)  (ii) 
(b)(i)	(A) $r_1 \approx -0.9470$ (B) $r_2 \approx -0.9749$
(b)(ii)	$P = c\ln m + d$ is the better model as the price of the used cars decreases at a decreasing rate according to the table. Thus, the decrease in price is not linear, which is what $P = am+b$ suggests. Furthermore, the value of $\lvert r_2 \rvert > \lvert r_1 \rvert$, so there is a stronger linear correlation between ln m and P. From GC, $P = -33700\ln m + 196000 \quad (3\text{ s.f.})$
(b)(iii)	When $m=50$, $P \approx 64000$ The price of a car that is 50 months old is estimated to be $64000.
9	Out of syllabus
10 (i)	(a) $\mathrm{P}(***) = \frac{1}{10}\times\frac{2}{10}\times\frac{1}{10} = \frac{2}{1000} = \frac{1}{500}$ (b) $\mathrm{P}(\text{at least 1 } *) = 1-\mathrm{P}(\text{no } *)$ $=1-\left(\frac{9}{10}\times\frac{8}{10}\times\frac{9}{10}\right)$ $=\frac{44}{125}$ (c) $\mathrm{P}(\times,\times,+\text{ in any order}) = \mathrm{P}(\times\times+)+\mathrm{P}(\times+\times)+\mathrm{P}(+\times\times)$ $=\left(\frac{3}{10}\times\frac{1}{10}\times\frac{2}{10}\right)+\left(\frac{3}{10}\times\frac{3}{10}\times\frac{4}{10}\right)+\left(\frac{4}{10}\times\frac{1}{10}\times\frac{4}{10}\right)$ $=\frac{29}{500}$

(ii)	$\mathrm{P}(+,\circ,* \text{ in any order} \mid \text{exactly } 1\ *)$ $=\dfrac{\mathrm{P}(+,\circ,* \text{ in any order})}{\mathrm{P}(\text{exactly } 1\ *)}$ $=\dfrac{\mathrm{P}(+\circ*)+\mathrm{P}(+*\circ)+\mathrm{P}(\circ+*)+\mathrm{P}(\circ*+)+\mathrm{P}(*+\circ)+\mathrm{P}(*\circ+)}{\mathrm{P}(*\_\ \ \_)+\mathrm{P}(\_*\_)+\mathrm{P}(\_\ \ \_*)}$ (where "_"means symbol is not "*") $=\dfrac{\frac{4}{10}\times\frac{4}{10}\times\frac{1}{10}+\frac{4}{10}\times\frac{2}{10}\times\frac{3}{10}+\frac{2}{10}\times\frac{3}{10}\times\frac{1}{10}+\frac{2}{10}\times\frac{2}{10}\times\frac{2}{10}+\frac{1}{10}\times\frac{3}{10}\times\frac{3}{10}+\frac{1}{10}\times\frac{4}{10}\times\frac{2}{10}}{\left(\frac{1}{10}\times\frac{8}{10}\times\frac{9}{10}\right)+\left(\frac{9}{10}\times\frac{2}{10}\times\frac{9}{10}\right)+\left(\frac{9}{10}\times\frac{8}{10}\times\frac{1}{10}\right)}$ $=\dfrac{\frac{71}{1000}}{\frac{306}{1000}}$ $=\dfrac{71}{306}$
11	Out of syllabus