Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2025 Tuesday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use												
Qn	1	2	3	4	5	6	7	8	9	10	11		*Marks Deducted*
Marks													

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
/ 90

Setter: Ms Sabrina Tan
Vetter: Mr Tan Lip Sing

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

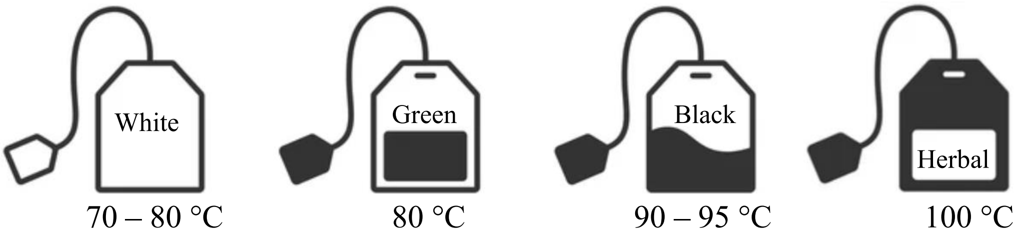
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a curve is $y = \dfrac{5x}{6 - 2x}$. Find the x-coordinates of the points at which the gradient of the curve is 7.5. [4]

2 The temperature of water in a tea cup, T °C, t minutes after boiling water is poured into the empty cup can be modelled by the formula $T = 25 + 75e^{-kt}$.



The diagram above shows the ideal brewing temperatures for various teas at a particular teahouse. At this teahouse, green tea leaves are placed into the tea cup for brewing at the ideal temperature when $t = 6$.

Determine whether the teahouse's standards for brewing tea would be adhered to if black tea leaves are placed into the teacup when $t = 2$. [4]

3 It is given that $f'(x) = 6\cos 3x - 12\sin 3x$ and $f\left(\frac{\pi}{3}\right) = 0$.

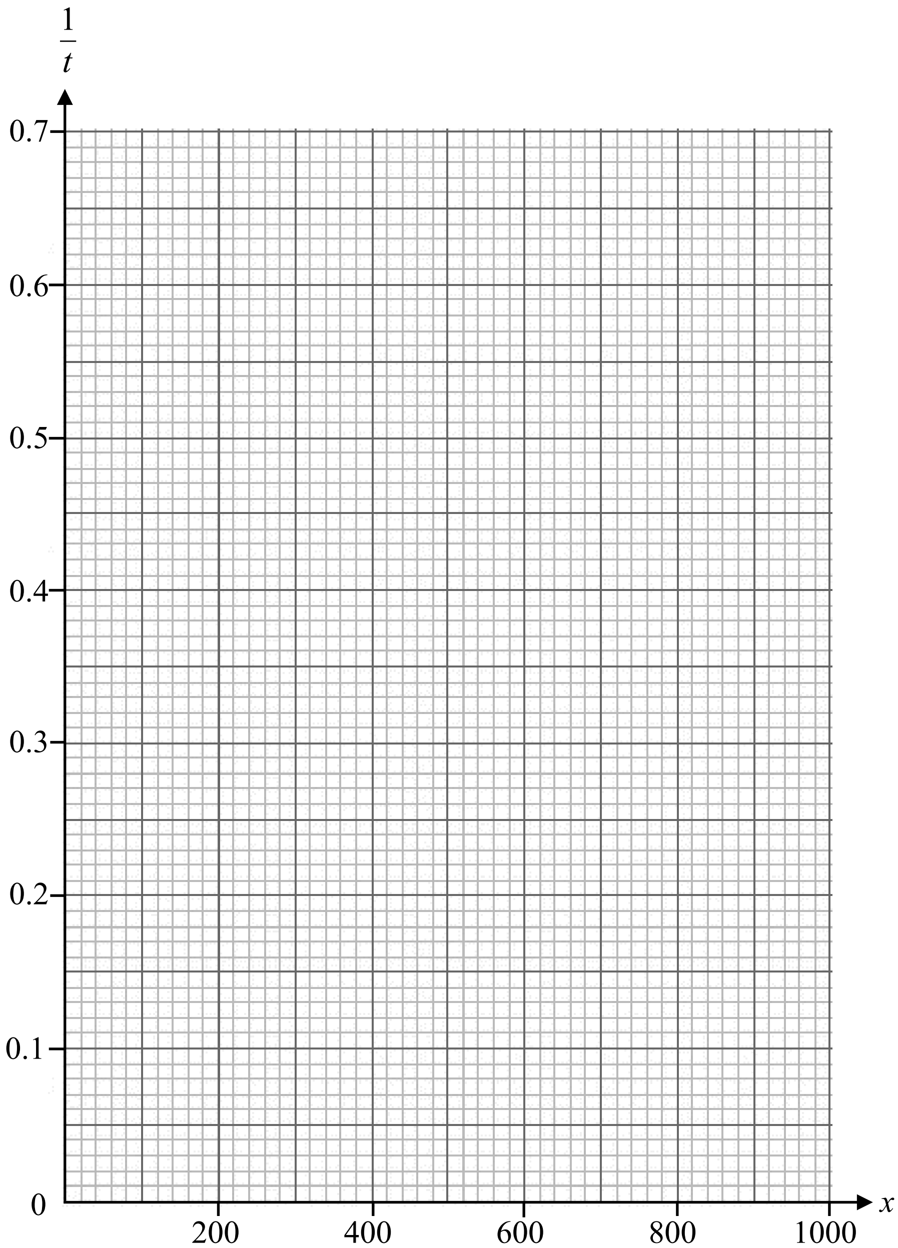
(a) Find f(x). [4]

(b) Find the exact value of $f''\left(\frac{\pi}{12}\right)$. [2]

4 **(a)** The time taken, t in minutes, to download a particular gaming software is related to internet speed, x Mbps. The variables x and t are related by the formula $t = \frac{a}{b+x}$, where a and b are positive constants. The data below shows some measured values of x and t.

x (Mbps)	50	100	200	500	1000
t (minutes)	23	13	7.3	3.2	1.6

(i) Plot $\frac{1}{t}$ against x and draw a straight line graph. [2]



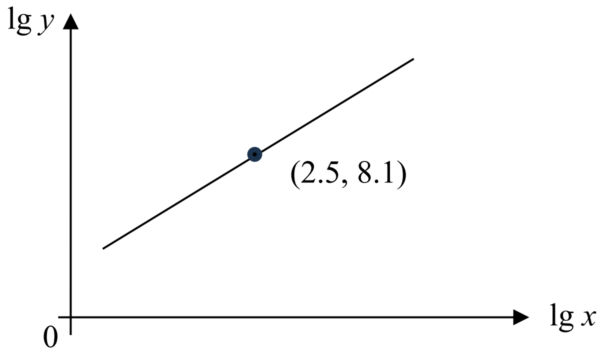
(ii) Use the graph to estimate the value of a and of b, correct to 2 significant figures. [4]

(iii) Use the graph to estimate the time taken for John to download the gaming software if the speed of his internet is 240 Mbps. [2]

(iv) Suggest an alternative set of variables to be plotted on each axis to draw a straight line to represent the formula. [1]

(b) The diagram shows part of a straight line graph drawn to represent the equation $y = px^q$.

Given that the line passes through the point (2.5, 8.1) and has a gradient of 3, find the value of p correct to the nearest integer. [3]



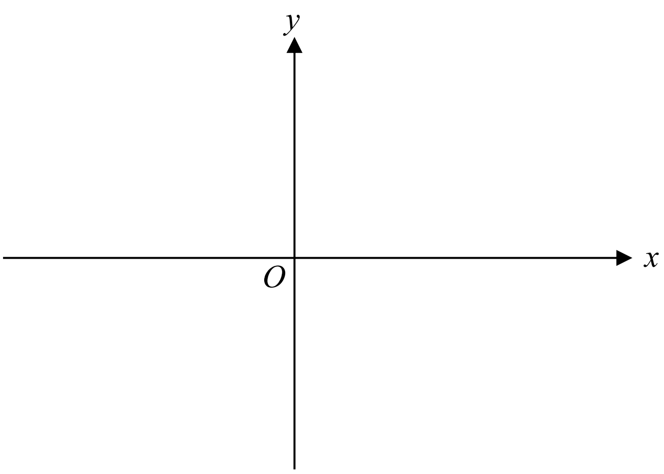
5 **(a)** Find the range of values of the constant k for which $k\left(x^2+x\right)+2\left(x^2+x+1\right)=0$ has no real roots. [4]

(b) Hence, explain why $-\left(x^2+x\right)+2\left(x^2+x+1\right)$ is always positive. [2]

6 **(a)** By using a suitable substitution, or otherwise, solve the equation

$$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25\,.$$ [4]

(b) Sketch the graph of $y = \log_6 x$ on the given axes. Label any axial intercepts. [1]



(c) **(i)** It is given that the equation of a curve is $y = e^{2x+1} + 2e^{x+1} - 3e$.

Explain why the curve has no stationary point. [3]

(ii) Find the x-intercept of the curve $y = e^{2x+1} + 2e^{x+1} - 3e$. [3]

7 **(a)** Factorise $x^3 - 125$. [1]

(b) The expression $-2x^3 + 7x^2 + ax + b$, where a and b are integers, has a factor of $x - 5$ and a remainder of -11 when divided by $2x + 1$. Find the value of a and of b. [4]

(c) Hence, or otherwise, show that the equation $x^3 - 125 = -2x^3 + 7x^2 + ax + b$ has only one real root. [4]

8 The equation of a curve is $y = e^{-2x} \tan x$.

(a) Show that $\dfrac{dy}{dx} = e^{-2x}(1 - \tan x)^2$. [3]

(b) Find, in terms of π, the x-coordinate of the stationary point for $0 < x < \dfrac{\pi}{2}$. [2]

(c) Explain why y is never decreasing. [2]

(d) What does your answer to **part (c)** imply about the stationary point found in **part (b)**? Explain your answer. [2]

9 **(a)** State

(i) the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

(ii) the values between which the principal value of $\tan^{-1}x$ must lie, [1]

(iii) the range of values of k for which the equation $4\sin^2 x = k$ has a solution. [1]

(b) **(i)** Prove the identity $\cot x - \cot x \tan^2 x + \tan x = \dfrac{1 + \cos 2x}{\sin 2x}$. [4]

(ii) Hence solve the equation $\cot x - \cot x \tan^2 x + \tan x = \operatorname{cosec} 2x - 3$ for $0° \leq x \leq 180°$.

[5]

10 **(a)** Points $P(-2, 0)$, $Q(5, 1)$, $R(6, -6)$ and $S(-1, -7)$ lie on a circle.

PQ and SR are parallel chords of the same length.

Show that the centre of the circle is at $(2, -3)$. [2]

(b) Find the equation of the circle. [3]

(c) A second circle has centre C. It is given that the y-axis is the perpendicular bisector of CR and is also a tangent to the second circle.

(i) State the coordinates of C. [1]

(ii) Hence explain why the x-axis is a tangent to the second circle. [2]

11 A particle moves from point P towards point O. The speed of the particle, t seconds after passing point P, can be modelled by the formula $v = (k - 2t)^n$, where k and n are constants.

(a) Find an expression for the acceleration of the particle in terms of k, t and n. [1]

The particle has a speed of 27 m/s when $t = 3.5$ and comes to instantaneous rest 4.8 metres to the right of point O when $t = 8$.

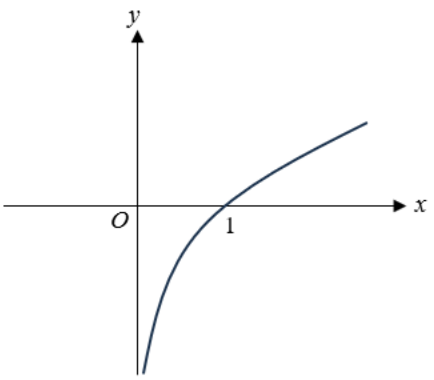
(b) Find the value of k and of n. [3]

(c) Using the values of k and n found in **part (b)**, find the total distance travelled by the particle in the first 8 seconds. [4]

(d) Explain whether the given model can continue to describe the motion of the particle after 8 seconds. [1]

END OF PAPER

Answer Key

1	$x = 2$ or $x = 4$
2	When $t = 2$, $T = 92.6$°C Since the temperature of the water would be within the ideal range of 90 – 95 °C, the standards for brewing black tea **<u>would</u>** be adhered to.
3a	$\mathrm{f}(x) = 2\sin 3x + 4\cos 3x + 4$
3b	$-27\sqrt{2}$
4a(ii)	grad=0.0006-0.00062 and a=1600 or 1700 Y-intercept= 0.01, 0.015 or 0.02 and $b = 16 - 33$
4a(iii)	Accept $t = 6.06$, 6.25 or 6.45
4a(iv)	xt against t **OR** t against xt
4b	4
5a	$-2 < k < 6$
5b	Since the **discriminant < 0 when k = –1** and the **coefficient of x^2**, i.e. 1, is **positive**, $-(x^2 + x) + 2(x^2 + x + 1)$ is always positive
6a	$x = 0.550$, 6
6b	
6c(i)	$\frac{dy}{dx} = 2e^{2x+1} + 2e^{x+1}$ Since $e^{2x+1} > 0$ and $e^{x+1} > 0$ for all real values of x, $2e^{2x+1} + 2e^{x+1} > 0$, $\frac{dy}{dx} \neq 0$ Therefore the curve has no stationary points
6c(ii)	$x = 0$
7a	$x^3 - 125 = (x - 5)(x^2 + 5x + 25)$
7b	$a = 16$, $b = -5$

8b	$x=\frac{\pi}{4}$
8c	Since $e^{-2x} > 0$ and $(\tan x - 1)^2 \geq 0$ for all real values of x, $\frac{dy}{dx} \geqslant 0$ therefore y is never decreasing.
8d	The stationary point at $x=\frac{\pi}{4}$ is a **point of inflexion.** Since y is never decreasing, the **gradient of the curve must be positive (or dy/dx > 0) slightly to the left and right** of the stationary point.
9a(i)	$-60°$ or $-\frac{\pi}{3}$
9a(ii)	$-90° < \tan^{-1}x < 90°$
9a(iii)	$0 \leq k \leq 4$
9b(ii)	x = 80.8°, 170.8°
10b	$(x-2)^2+(y+3)^2=25$ OR $x^2-4x+y^2+6y-12=0$
10c(i)	(–6, –6)
10c(ii)	Since the y-axis is a tangent to the second circle, the radius of the circle is 6 units. The highest point, H, of the second circle is (–6, 0). Since the circle touches the x-axis at exactly one point at (–6, 0), the x-axis is a tangent to the second circle.
11a	$a = -2n(k-2t)^{n-1}$
11b	k = 16, n = 1.5
11c	204.8 m
11d	No since when t > 8, v and s are undefined.