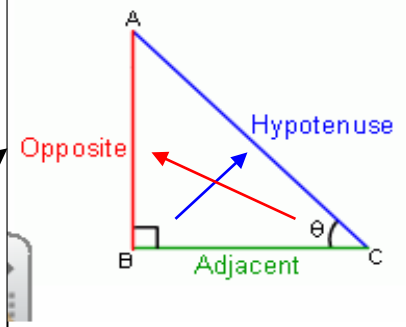
Trigonometry Concept Map

Is it a right-angled triangle?

Yes

ANGLE INVOLVED



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

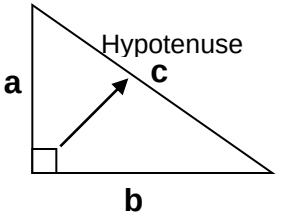
$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}}$$

where
hypotenuse – length opposite the right angle
opposite – length opposite the angle, θ
adjacent – length next to the angle.θ

NO ANGLES INVOLVED

Pythagoras' Theorem



$c^2 = a^2 + b^2$

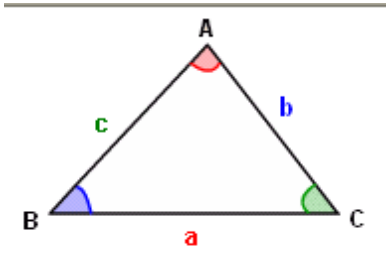
Find

- *Longest side* = $\sqrt{a^2 + b^2}$
- *Not longest side* = $\sqrt{c^2 - a^2}$

Steps:
1) Name the sides (hyp, opp, adj)
2) Decide which ratio to use (TOA, CAH, SOH)
3) Form the ratio
4a) Solve unknown lengths
- do cross multiplication

4b) Solve unknown angles
- use inverse functions ($\tan^{-1}$, $\cos^{-1}$, $\sin^{-1}$)

No

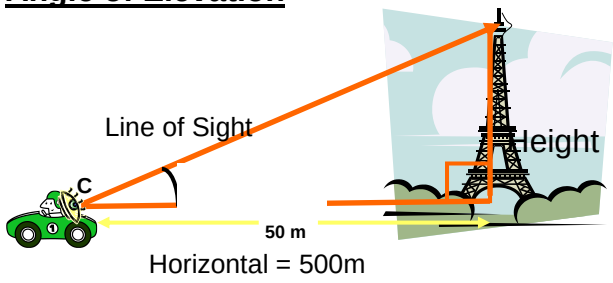


Sine Rule or Cosine Rule

- 3 sides given ➔find angle ➔use Cosine Rule: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- 2 sides & 1 **included angle** given ➔find length ➔use Cosine Rule: $a = \sqrt{b^2 + c^2 - 2bc\cos A}$
- 2 sides & 1 **non-included angle** given ➔ find angle ➔ use Sine Rule: $\frac{\sin A}{a} = \frac{\sin B}{b}$
- 1 side & 2 angles given ➔ find length ➔use Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

Area of Triangle = $\frac{1}{2}ab\sin C$, where angle *C* is the **included angle** of sides *a* and *b*.

Angle of Elevation



Eg. 1: Find the height of the tower.

Solution:

$$\tan 25° = \frac{h}{500}$$

h = 500tan25°
= 233m (3sf)

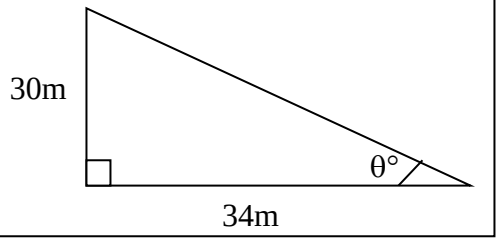
When the observer looks **up**, the angle formed between the **line of sight** and the **horizontal** is called the "**angle of elevation**".

Eg. 2: A 34 m shadow is cast when the sun is slightly above a 30 m high tower. What is the angle of elevation from the sun.

$$\tan\theta = \frac{30}{34}$$

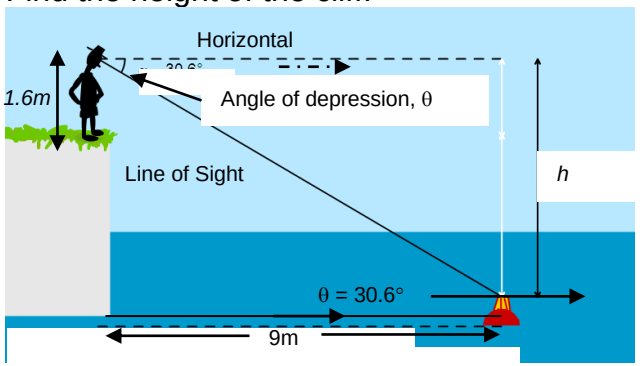
$$\theta = \tan^{-1}\left(\frac{30}{34}\right)$$

$$= 41.4°\,(1dp)$$



Angle of Depression

Find the height of the cliff.



When the observer looks **down**, the angle formed between the **line of sight** and the **horizontal** is called the "**angle of depression**".

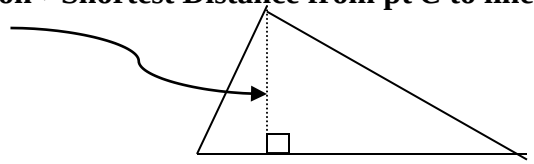
Solution:

$$\tan 30.6 = \frac{h}{9}$$

h = 9 tan 30.6°
= 5.323m (4sf)

∴Height of cliff = 5.323 – 1.6
= 3.72m (3sf)

Greatest angle of elevation/Depression->Shortest Distance from pt *C* to line *AB* = Perpendicular Distance



Method 1 – Using Area of Triangle

$$\frac{1}{2}ab\sin C = \frac{1}{2}\times base \times ht(shortest\ dist)$$

$$Ht(shortest\ dist) = \frac{Area\ of\ \Delta}{\frac{1}{2}\times base}$$

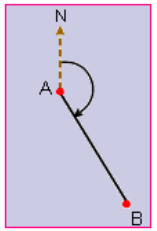
Method 2 – Using TOA or SOH

- **Opposite is normally the shortest distance**

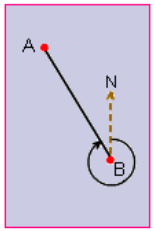
$$shortest\ dist = hyp \times \sin\theta$$

Bearings:

Bearings are always measured **clockwise from the North line.**



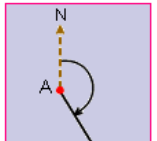
Bearing of **B** from **A**

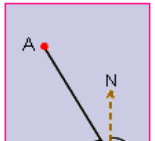


Bearing of **A** from **B**

Remember!
A bearing must always be written with 3 figures
eg., 005°, 013°, 140°

Be familiar with: (parallel lines)
- Interior ∠s
- Alternate ∠s
- Corresponding ∠s





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