2025 PRELIM
CHUNG CHENG HIGH (YISHUN)
ADDITIONAL MATH (G3)
PAPER 1

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

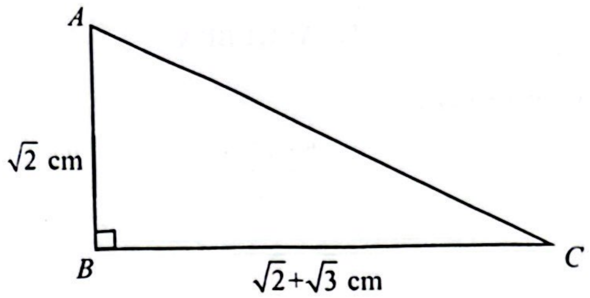
Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram above shows a right-angled triangle ABC, where $AB = \sqrt{2}$ cm, and $BC = \sqrt{2} + \sqrt{3}$ cm.

(a) Find the exact value of $\tan \angle ACB$ in the form of $a + \sqrt{b}$, where a and b are integers. [2]

(b) Hence, find the exact value of $\sec^2 \angle ACB$ in the form $m + n\sqrt{k}$, where m and n are integers. [2]

2 A container has a capacity of $840\ \text{cm}^3$ and is initially filled completely with water. The volume, $V\ \text{cm}^3$, of water in the container is given by $V = h^2 + 2h$, where h cm is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of $\frac{5t}{3}$ cm/s, where t is the time in seconds.

(a) Find the initial height of the water level in the container. [2]

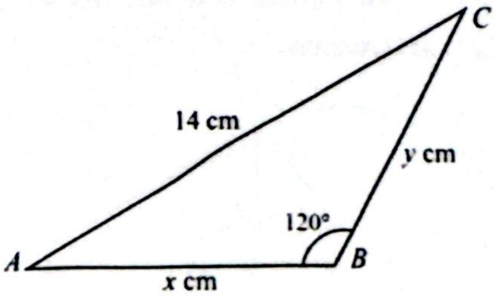
(b) Show that the height of the water level h can be expressed as $h = -\frac{5t^2}{6} + 28$. [2]

(c) Find the rate of decrease of volume when $t = 3$. [5]

3 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{\frac{1}{2}x}\right)$. [2]

(b) Hence evaluate $\int_0^4 x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$, leaving your answer in the form $k\left(\mathrm{e}^2+1\right)$, where k is a constant to be found. [5]

4 In triangle ABC below, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120°$.



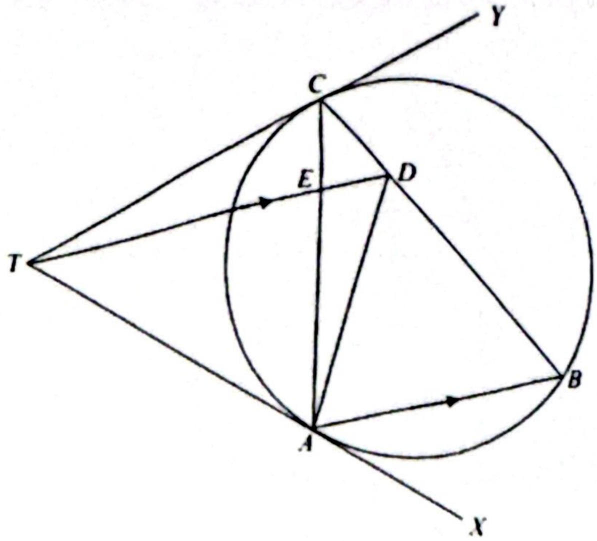
(a) Using cosine rule, form an equation involving x and y. [1]

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [4]

5 (a) By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [2]

(b) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [4]

6 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E. D is on BC such that TD is parallel to AB.

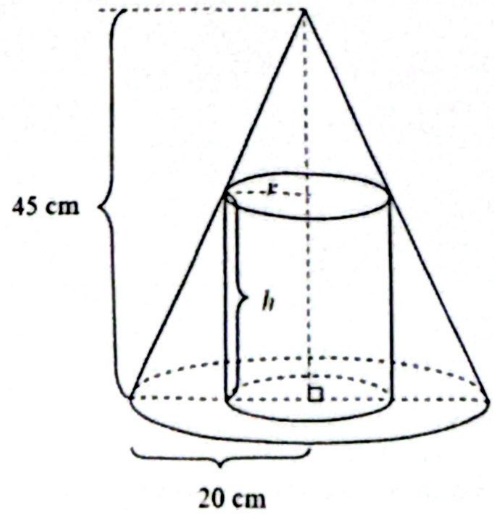


(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

(c) Hence prove that $CE \times AE = DE \times TE$. [3]

7 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(a) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \frac{9}{4}\pi r^3$. [3]

(b) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [5]

(c) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

8 (a) Find the value of a and of b for which $\{x : x < -3.5 \text{ or } x > 2\}$ is the solution set of $b < x^2 + ax$. [3]

(b) Find the range of values of p for which $y = x^2 - px - 3x - p$ is always positive. [3]

(c) Explain whether the line $y = -5x - 2$ intersects the curve $y = kx^2 + 3$ where $k < 1$. [3]

9 **(a)** Express $5\sin^2 x - \cos^2 x - 1$ in the form $1 - k\cos 2x$. [2]

(b) State the amplitude and period, in radians, of $5\sin^2 x - \cos^2 x - 1$. [2]

(c) Sketch the graph of $y = 5\sin^2 x - \cos^2 x - 1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [3]

(d) By drawing the line $y = 1 - \frac{3x}{2\pi}$ on the same axes, state the number of solutions to the equation $2\pi - 3x = 2\pi\left(5\sin^2 x - \cos^2 x - 1\right)$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

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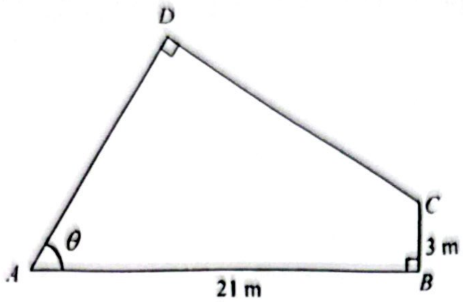


The diagram shows a quadrilateral field $ABCD$, where $AB = 21$ m and $BC = 3$ m.
Angle ABC = angle $ADC = 90°$. Angle $BAD = \theta$, for $0° < \theta < 90°$, and can vary.
The perimeter of the fencing around the quadrilateral field $ABCD$ is P m.

(a) Show that $P = 24 + 18\cos\theta + 24\sin\theta$. [3]

(b) Express P in the form $R\sin(\theta + \alpha) + 24$, where $R > 0$ and $0° < \alpha < 90°$. [3]

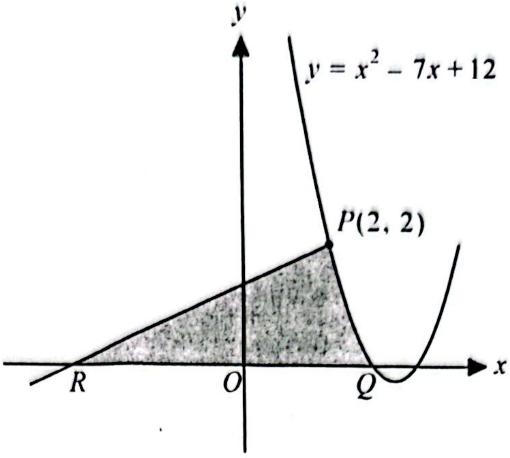
(c) Given that the total perimeter of the fencing is 53 m, find the value(s) of θ. [2]

(d) Explain why the total length of the fencing will never exceed a certain value and state this value. [2]

11 (a) Express $\dfrac{11x+12}{x^2(x+4)}$ in partial fractions. [5]

(b) Hence find $\displaystyle\int \frac{11x+12}{x^2(x+4)}\,\mathrm{d}x$. [2]

12 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the x-axis at Q.
The normal to the curve at $P(2, 2)$ meets the x-axis at R.



Show that the area of the shaded region bounded by the x-axis, the line PR and the curve is $6\frac{5}{6}$ units^2. [8]

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CHUNG CHENG HIGH SCHOOL (YISHUN)

2025 Preliminary Examination

Secondary Four Express / Five Normal Academic

CANDIDATE NAME	

FORM CLASS / SUBJECT GROUP	/	INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1

21 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	9	
3	7	
4	5	
5	6	
6	6	
7	10	
8	9	
9	9	
10	10	
11	7	
12	8	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **17** printed pages and **1** blank page.

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For the equation $ax^2+bx+c=0$,

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Binomial Expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
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$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
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$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
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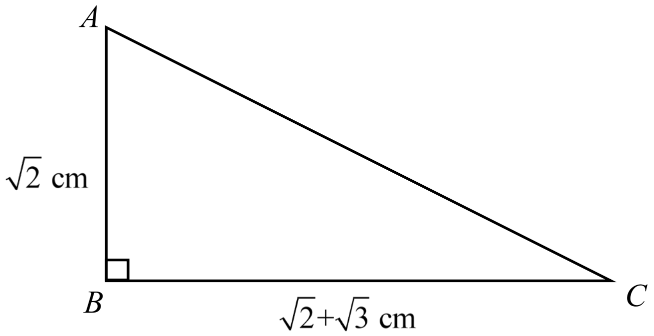
1



The diagram above shows a right-angled triangle ABC, where $AB=\sqrt{2}$ cm, and $BC=\sqrt{2}+\sqrt{3}$ cm.

(a) Find the exact value of $\tan\angle ACB$ in the form of $a+\sqrt{b}$, where a and b are integers. [2]

$$\tan\angle ACB=\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$
$$=\frac{2-\sqrt{6}}{2-3}$$
$$=-2+\sqrt{6}$$

(b) Hence, find the exact value of $\sec^2\angle ACB$ in the form $m+n\sqrt{k}$, where m and n are integers. [2]

$$\sec^2 A=1+\tan^2 A$$
$$=1+(\sqrt{6}-2)^2$$
$$=1+6-4\sqrt{6}+4$$
$$=11-4\sqrt{6}$$

2 A container has a capacity of 840 cm^3 and is initially filled completely with water. The volume, V cm^3, of water in the container is given by $V = h^2 + 2h$ where h cm is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water leve in the container decreases at a rate of $\frac{5t}{3}$ cm/s, where t is the time in seconds.

(a) Find the initial height of the water level in the container. [2]

at $t = 0,\ V = 840$

$h^2 + 2h = 840$

$h^2 + 2h - 840 = 0$

$(h-28)(h+30) = 0$

$h = 28$ or $h = -30$ (rejected)

$\therefore$ Initial Height = 28 cm

(b) Show that the height of the water level h can be expressed as $h = -\frac{5t^2}{6} + 28$. [2]

$$h = \int -\frac{5t}{3}\,\mathrm{d}t$$

$$= -\frac{5t^2}{6} + c$$

when $t = 0$, $h = 28$,

$28 = 0 + c$

$c = 28$

$$\therefore h = -\frac{5t^2}{6} + 28 \text{ (shown)}$$

(c) Find the rate of decrease of volume when $t = 3$. [5]

$$\frac{\mathrm{d}V}{\mathrm{d}h} = 2h + 2$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-5(3)}{3}$$

$$= -5$$

when $t = 3$,

$$h = \frac{-5(3)^2}{6} + 28$$

$$= 20.5$$

$$\frac{\mathrm{d}V}{\mathrm{d}h} = 2(20.5) + 2$$

$$= 43$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$$

$= 43 \times (-5)$

$= -215\ \text{cm}^3/\text{s}$

Rate of decrease $= 215\text{cm}^3/\text{s}$

3 **(a)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{\frac{1}{2}x}\right)$. [2]

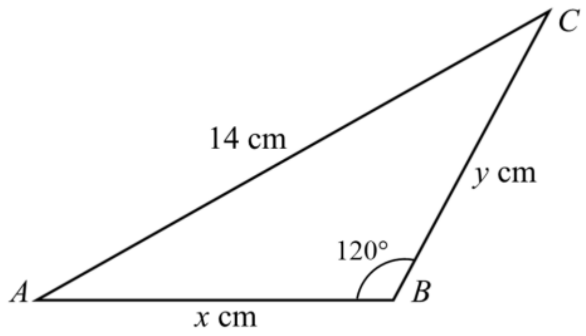
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x\mathrm{e}^{\frac{1}{2}x}\right)=x\mathrm{e}^{\frac{1}{2}x}\left(\frac{1}{2}\right)+\mathrm{e}^{\frac{1}{2}x}$$
$$=\frac{1}{2}x\mathrm{e}^{\frac{1}{2}x}+\mathrm{e}^{\frac{1}{2}x}$$

Accept $\mathrm{e}^{\frac{1}{2}x}\left(\frac{1}{2}x+1\right)$

(b) Hence evaluate $\int_0^4 x\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$, leaving your answer in the form $k\left(\mathrm{e}^2+1\right)$, where k is a constant to be found. [5]

$$\int_0^4 \frac{1}{2}x\mathrm{e}^{\frac{1}{2}x}+\mathrm{e}^{\frac{1}{2}x}\ \mathrm{d}x=\left[x\mathrm{e}^{\frac{1}{2}x}\right]_0^4$$
$$\int_0^4 x\mathrm{e}^{\frac{1}{2}x}+2\mathrm{e}^{\frac{1}{2}x}\ \mathrm{d}x=2\left[x\mathrm{e}^{\frac{1}{2}x}\right]_0^4$$
$$\int_0^4 x\mathrm{e}^{\frac{1}{2}x}\ \mathrm{d}x=2\left[x\mathrm{e}^{\frac{1}{2}x}\right]_0^4-\int_0^4 2\mathrm{e}^{\frac{1}{2}x}\mathrm{d}x$$
$$=2\left[x\mathrm{e}^{\frac{1}{2}x}\right]_0^4-\left[4\mathrm{e}^{\frac{1}{2}x}\right]_0^4$$
$$=2\left[4\mathrm{e}^2-0\right]-\left[4\mathrm{e}^2-4\right]$$
$$=8\mathrm{e}^2-4\mathrm{e}^2+4$$
$$=4\mathrm{e}^2+4$$
$$=4(\mathrm{e}^2+1)$$

4 In triangle ABC below, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120^{\circ}$.



(a) Using cosine rule, form an equation involving x and y. [1]

$14^2 = x^2 + y^2 - 2xy\cos 120^{\circ}$

$196 = x^2 + y^2 + xy$

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [4]

$x + y + 14 = 30$

$x = 16 - y$ ---------- [1]

$196 = x^2 + y^2 + xy$ ------[2]

sub [1] into [2]

$196 = (16-y)^2 + y^2 + (16-y)y$

$196 = 256 - 32y + y^2 + y^2 + 16y - y^2$

$y^2 - 16y + 60 = 0$

$(y-6)(y-10) = 0$

$y = 6$ or $y = 10$

$x = 10$ or $x = 6$

Area of $\Delta ABC = \frac{1}{2}(10)(6)\sin 120^{\circ}$

$= 30\left(\frac{\sqrt{3}}{2}\right)$

$= 15\sqrt{3}$ cm^2

5 **(i)** By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [2]

$$T_{r+1}=\binom{7}{r}\left(x^5\right)^{7-r}\left(\frac{1}{x}\right)^r$$
$$=\binom{7}{r}(x)^{35-5r}x^{-r}$$
$$=\binom{7}{r}x^{35-6r}$$

$6r$ is always even by all non-integers of r.

$35-6r$ is always odd.

$\therefore$ There are no even powers of x in its expansion.

(ii) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [4]

$$T_{r+1}=\binom{7}{r}(kx)^{7-r}3^r$$
$$=\binom{7}{r}(k)^{7-r}3^r x^{7-r}$$

$7-r=6$

$r=1$

$$T_2=\binom{7}{1}(k)^6 3^1 x^6$$

$21k^6=1344$

$k=2$ or $k=-2$ (rejected)

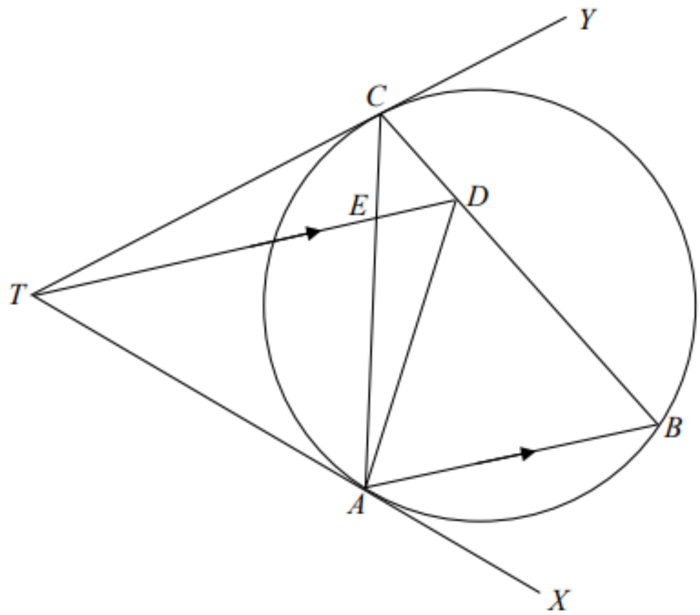
$7-r=4$

$r=3$

$$T_4=\binom{7}{3}(2)^4 3^3 x^4$$

$\therefore$ coefficient of $x^4=15120$

6 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively.
AC meets TD at E. D is on BC such that TD is parallel to AB.



(i) Prove that angle ACB is equal to angle ATD. [2]

$\angle ATD = \angle XAB$ (corr angles, $AB // TD$)
$\quad\quad\quad = \angle ACB$ (alternate segment theorem)

(ii) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

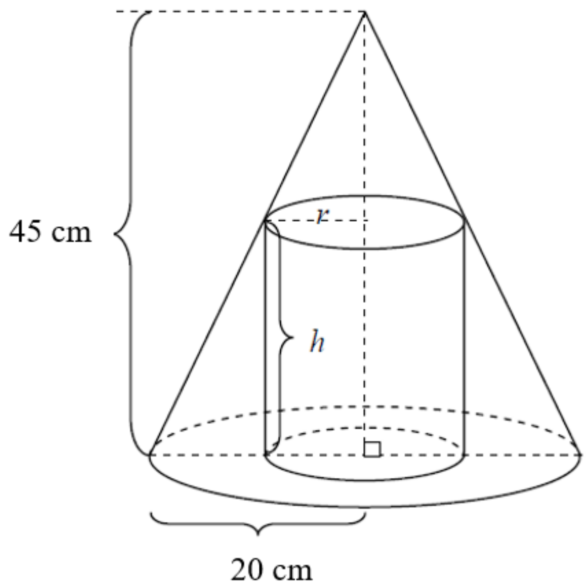
Since $\angle ATD = \angle ACB$, they fulfil the property of angles in the same segment.
$\therefore$ A circle can be drawn passing through the points T, A, D and C.

(iii) Hence prove that $CE \times AE = DE \times TE$. [3]

$\angle ATE = \angle DCE$ [from part (a)]
$\angle TEA = \angle CED$ [vert opp $\angle$]
$\therefore \Delta ATE$ is similar to ΔDCE (AA similarity test)

$\frac{TE}{CE} = \frac{EA}{ED}$ (Ratio of corr sides of similar triangles are equal)
$CE \times AE = DE \times TE$ (proven)

7 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(i) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \frac{9}{4}\pi r^3$. [3]

$$\frac{45-h}{45} = \frac{r}{20}$$
$$20(45-h) = 45r$$
$$900-20h = 45r$$
$$h = \frac{900-45r}{20}$$
$$= 45 - \frac{9}{4}r$$
$$V = \pi r^2 h$$
$$= \pi r^2 (45 - \frac{9}{4}r)$$
$$= 45\pi r^2 - \frac{9}{4}\pi r^3 \text{ (shown)}$$

(ii) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [5]

$$\frac{dV}{dr} = 90\pi r - \frac{27}{4}\pi r^2$$

when V is max,

$$90\pi r - \frac{27}{4}\pi r^2 = 0$$

$$\pi r\left(90 - \frac{27}{4}r\right) = 0$$

$r = 0$ or $r = 13\frac{1}{3}$

(rejected)

$$\frac{d^2V}{dr^2} = 90\pi - \frac{27}{2}\pi r$$

when $r = \frac{40}{3}$,

$$\frac{d^2V}{dr^2} = 90\pi - \frac{27}{2}\pi\left(\frac{40}{3}\right)$$
$$= -282.74$$
$$< 0$$

Since $\frac{d^2V}{dr^2} < 0$, $V = \frac{8000\pi}{3}\text{cm}^3$ is a maximum volume.

(iii) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

$$\text{Volume of cone} = \frac{1}{3}\pi(20)^2(45)$$
$$= 6000\pi$$

$$\therefore \text{ratio} = \frac{\frac{8000\pi}{3}}{6000\pi}$$
$$= \frac{4}{9} \text{ (shown)}$$

8 **(a)** Find the value of a and of b for which $\{x : x < -3.5 \text{ or } x > 2\}$ is the solution set of $b < x^2 + ax$. [3]

$$(2x+7)(x-2) > 0$$
$$2x^2 + 3x - 14 > 0$$
$$x^2 + ax - b > 0$$
$$2x^2 + 2ax - 2b > 0$$
$$2a = 3$$
$$a = 1.5$$
$$2b = 14$$
$$b = 7$$

$$x^2 + 1.5x > 7$$

(b) Find the range of values of p for which $y = x^2 - px - 3x - p$ is always positive. [3]

$$b^2 - 4ac < 0$$
$$(-p-3)^2 - 4(1)(-p) < 0$$
$$p^2 + 6p + 9 + 4p < 0$$
$$p^2 + 10p + 9 < 0$$
$$(p+1)(p+9) < 0$$
$$-9 < p < -1$$

(c) Explain whether the line $y = -5x - 2$ intersects the curve $y = kx^2 + 3$ where $k < 1$. [3]

$$-5x - 2 = kx^2 + 3$$
$$kx^2 + 5x + 5 = 0$$
$$b^2 - 4ac = 25 - 4k(5)$$
$$= 25 - 20k$$
$$k < 1$$
$$-20k > -20$$
$$25 - 20k > 5$$
$$> 0$$

Since $b^2 - 4ac > 0$, for $k < 1$, the line intersects the curve.

9 **(i)** Express $5\sin^2 x - \cos^2 x - 1$ in the form $1 - k\cos 2x$. [2]

$$5\sin^2 x - \cos^2 x - 1 = 5\left(\frac{1-\cos 2x}{2}\right) - \left(\frac{\cos 2x + 1}{2}\right) - 1$$
$$= \frac{5}{2} - \frac{5\cos 2x}{2} - \frac{1}{2}\cos 2x - \frac{1}{2} - 1$$
$$= 1 - 3\cos 2x$$

(ii) State the amplitude and period, in radians, of $5\sin^2 x - \cos^2 x - 1$. [2]

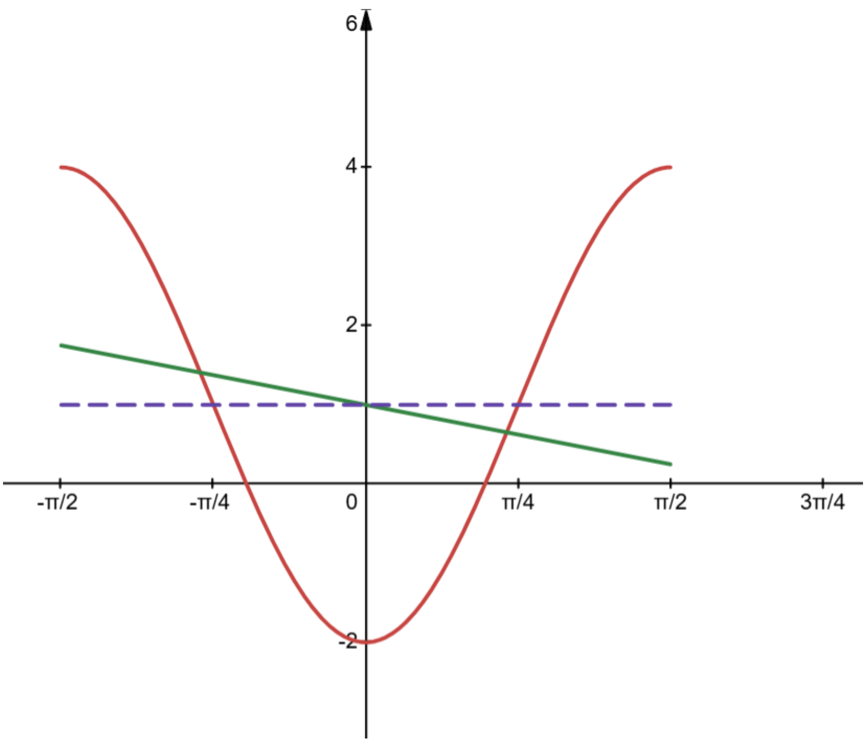
$5\sin^2 x - \cos^2 x - 1 = 1 - 3\cos 2x$

Amplitude $= 3$

Period $= \dfrac{2\pi}{2}$

$= \pi$ radian

(iii) Sketch the graph of $y = 5\sin^2 x - \cos^2 x - 1$ for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$. [3]

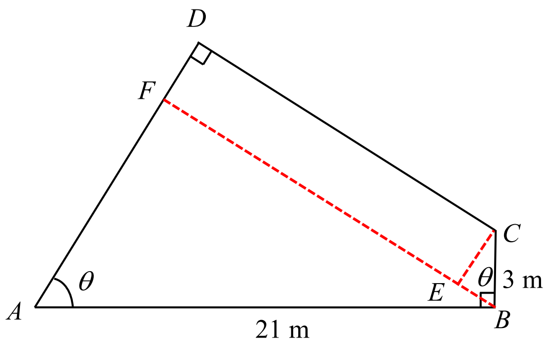


(iv) By drawing the line $y = 1 - \dfrac{3x}{2\pi}$ on the same axes, state the number of solutions to the equation $2\pi - 3x = 2\pi\left(5\sin^2 x - \cos^2 x - 1\right)$ in the range $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$. [2]

$y = -\dfrac{3}{2\pi}x + 1$

$\therefore$ Number of solutions = 2.

10



The diagram shows a quadrilateral field $ABCD$, where AB = 21 m, BC = 3 m and angle $ABC =$ angle $ADC = 90°$. Angle $BAD = \theta$ and can vary. The perimeter of the fencing around the quadrilateral field $ABCD$ is P m.

(i) Show that $P = 24 + 18\cos\theta + 24\sin\theta$. [3]

$$\sin\theta = \frac{BF}{21}$$
$$BF = 21\sin\theta$$
$$\cos\theta = \frac{EB}{3}$$
$$EB = 3\cos\theta$$
$$DC = 21\sin\theta - 3\cos\theta$$
$$\cos\theta = \frac{AF}{21}$$
$$AF = 21\cos\theta$$
$$\sin\theta = \frac{EC}{3}$$
$$EC = 3\sin\theta$$
$$AD = 21\cos\theta + 3\sin\theta$$

Hence, $P = 21 + 3 + 21\sin\theta - 3\cos\theta + 21\cos\theta + 3\sin\theta$
$= 24 + 18\cos\theta + 24\sin\theta$ (shown)

(ii) Express P in the form $R\sin(\theta+\alpha)+24$, where $R>0$ and $0°<\alpha<90°$. [3]

$$R=\sqrt{18^2+24^2}$$
$$=\sqrt{900}$$
$$=30$$
$$\tan\alpha=\frac{18}{24}$$
$$=36.8699°$$
$$=36.9°$$

$$\therefore P=30\sin(\theta+36.9°)+24$$

(iii) Given that the total perimeter of the fencing is 53 m, find the value(s) of θ. [2]

$$30\sin(\theta+36.8699°)+24=53$$
$$\sin(\theta+36.8699°)=\frac{29}{30}$$
$$\text{Basic angle}=\sin^{-1}\left(\frac{29}{30}\right)$$
$$=75.1649°$$

$$\theta+36.8699°=75.1649°,104.8351°$$
$$\theta=38.3°,\ 68.0°$$

(iv) Explain why the total length of the fencing will never exceed a certain value and state this value. [2]

$$-1\le\sin(\theta+36.9°)\le1$$
$$-30\le30\sin(\theta+36.9°)\le30$$
$$-6\le30\sin(\theta+36.9°)+24\le54$$

The total length will not exceed 54m as the max length is 54m.

11 **(i)** Express $\dfrac{11x+12}{x^2(x+4)}$ in partial fractions. [5]

$$\frac{11x+12}{x^2(x+4)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+4}$$

$$11x+12=Ax(x+4)+B(x+4)+Cx^2$$

when $x=0$,

$$12=0+4B+0$$
$$B=3$$

when $x=-4$,

$$-32=0+0+16C$$
$$C=-2$$

when $x=1$,

$$23=5A+15-2$$
$$A=2$$

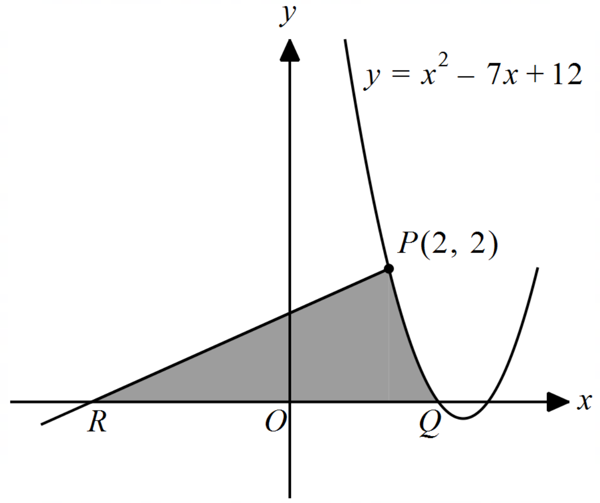
$$\therefore \frac{11x+12}{x^2(x+4)}=\frac{2}{x}+\frac{3}{x^2}-\frac{2}{x+4}$$

(ii) Hence find $\displaystyle\int \frac{11x+12}{x^2(x+4)}\,\mathrm{d}x$.. [2]

$$\int \frac{11x+12}{x^2(x+4)}\,\mathrm{d}x=\int \frac{2}{x}+\frac{3}{x^2}-\frac{2}{x+4}\,\mathrm{d}x$$

$$=2\ln x-\frac{3}{x}-2\ln(x+4)+c$$

12 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the x-axis at Q.
The normal to the curve at $P(2, 2)$ meets the x-axis at R.



Show that the area of the shaded region bounded by the x-axis, the line PR and the curve is $6\frac{5}{6}$ units^2. [8]

$$\frac{dy}{dx} = 2x - 7$$

when $x = 2$,

$$\frac{dy}{dx} = 2(2) - 7$$
$$= -3$$

Gradient of normal = $\frac{1}{3}$

$$y = \frac{1}{3}x + c$$

at (2, 2)

$$2 = \frac{2}{3} + c$$

$$c = \frac{4}{3}$$

Equation of normal: $y = \frac{1}{3}x + \frac{4}{3}$

when $y = 0$,

$$-\frac{1}{3}x = \frac{4}{3}$$
$$x = -4$$
$$R = (-4, 0)$$

when $y = 0$,

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$x = 3 \quad \text{or} \quad x = 4$$

$$Q = (3, 0)$$

$$\text{Area of shaded region} = \frac{1}{2}(6)(2) + \int_2^3 x^2 - 7x + 12 \, \mathrm{d}x$$

$$= 6 + \left[\frac{x^3}{3} - \frac{7}{2}x^2 + 12x \right]_2^3$$

$$= 6 + \left[(9 - \frac{63}{2} + 36) - (\frac{8}{3} - 14 + 24) \right]$$

$$= 6\frac{5}{6} \text{ units}^2 \quad \text{(shown)}$$