2024 PRELIM
SINGAPORE SPORTS SCH
ADDITIONAL MATH (G3)
PAPER 1

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

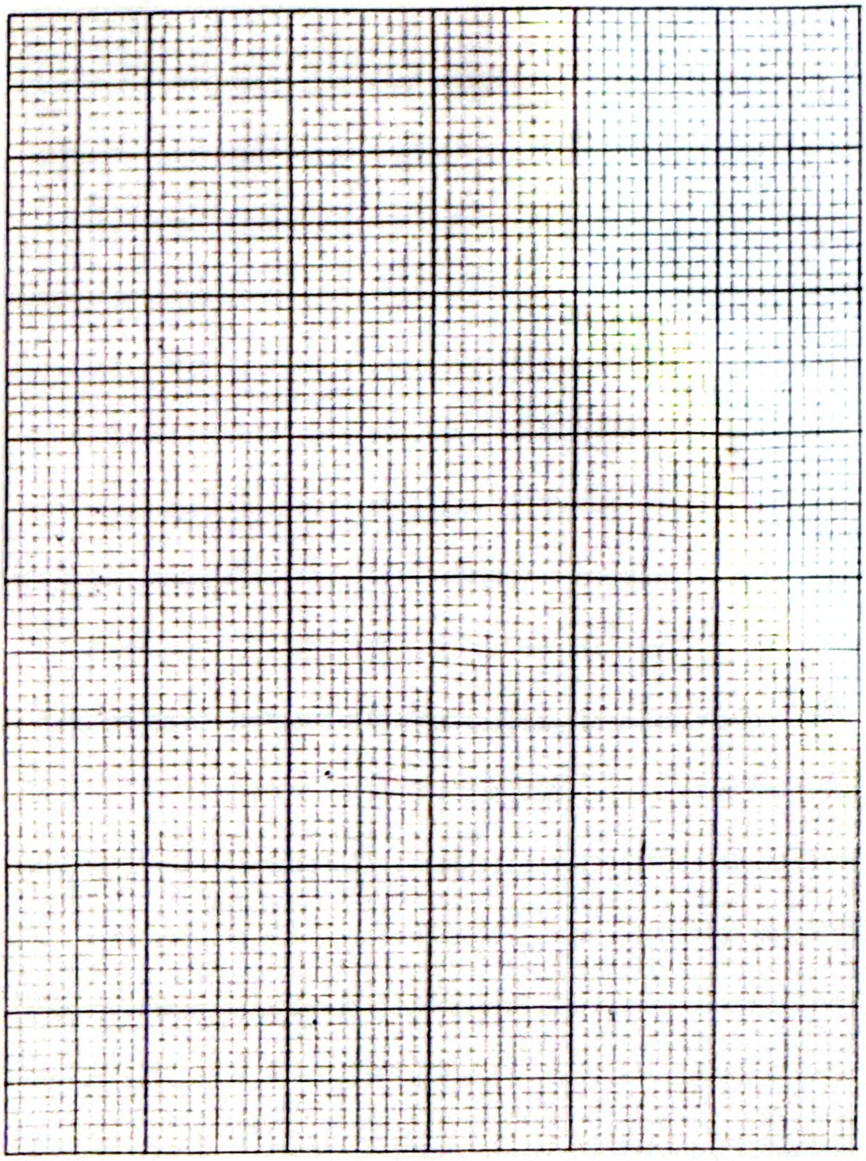
$$\Delta = \frac{1}{2}ab \sin C$$

1 Given $10+2\sqrt{21}=(\sqrt{a}+\sqrt{b})^2$, where a and b are integers. Find the values of a and b. [4]

2 Over a period of 5 years from 1 January 2000 to 1 January 2005, the number of bacteria decreases exponentially. The number of bacteria, P, at time t years from 2000, is given by the formula $P = Ae^{-kt}$, where A and k are constants. The table below shows the population, in thousands, for $1 < t < 5$. One value of P has been recorded incorrectly.

t (years)	1	2	3	4	5
P (thousands)	15.6	12.1	9.5	6.0	5.7

(a) Draw a straight line graph to illustrate the information. [2]



(b) Use your graph to estimate

(i) the correct value of P that is recorded incorrectly, [2]

(ii) the value of A and k, [3]

(iii) the number of years for 50% of the bacteria to be present. [2]

3 (a) Divide $x^3 - 1$ by $x^3 + x^2$. [1]

(b) Express $\dfrac{x^3 - 1}{x^3 + x^2}$ in partial fractions. [4]

4 Find the range of values of x for which the gradient of $y=\frac{2}{3}x^3+\frac{3}{2}x^2-12x+5$ is greater than the gradient of $y=2x^2+16x+5$. [4]

5 (a) Find the first three terms in the expansion of $(1+\frac{x}{4})^n$ in ascending powers of x, simplifying each term. [3]

(b) Given that the first two terms in the expansion of $(2-5x)(1+\frac{x}{4})^n$, in ascending powers of x are $a+bx^2$, where a and b are constants.

(i) Find the value of n. [2]

(ii) Hence, find the value of a and of b. **[3]**

6 A curve is such that $\frac{d^2y}{dx^2} = 9e^{-3x}$. The curve passes through the point $P(0, -9)$ and has a stationary point at the point $x = -\frac{1}{3}$. Find the equation of the curve.

[6]

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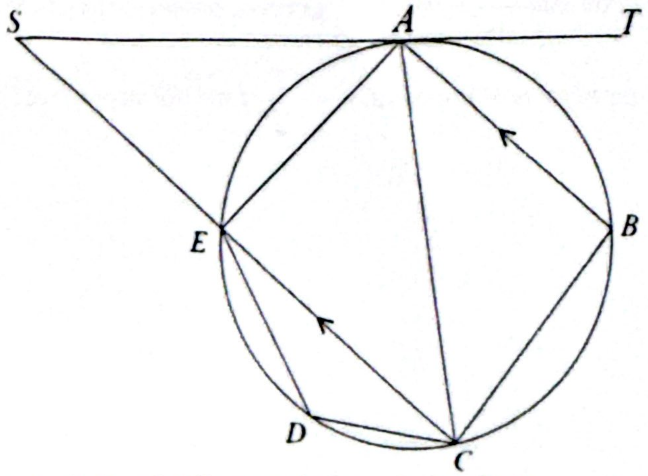


In the diagram, A, B, C, D and E lie on a circle such that CA bisects angle BCE and CE is parallel to BA. The line ST is a tangent to the circle at A and meets CE produced at S. Let angle $EAS = \theta$.

(a) Show that triangle ABC is an isosceles triangle. [3]

(b) Show that angle $CDE = 3\theta$. [3]

8 (a) Find the range of values of c for which the line $y = x + c$ intersects the curve $y = -\frac{2}{x} - x$. [4]

(b) Given that the line $y = x + c$ is a tangent to the curve $y = -\frac{2}{x} - x$ at point P, find the possible x-coordinates of point P. [4]

9 The equation of a curve is $y = 2x^3 + ax^2 + bx + 3$, where k is a constant.

(a) Show that if y is always increasing, then $a^2 < 6b$. [4]

(b) In the case where $a = 3$ and $b = 4$, explain why the curve has exactly 1 root. [4]

10 **(a)** Find the amplitude and period of [3]

(i) $2\cos 3x + 2$,

(ii) $3\sin\frac{x}{2}$

(b) Sketch, on the same diagram, the curves $y = 2\cos 3x + 2$ and $y = 3\sin\frac{x}{2}$ for $0 \le x \le \pi$. [4]

11. A curve has an equation $y=(k-8)x^2+6x+k$.

(a) In the case where $k=5$, explain why $y \leq 8$. [3]

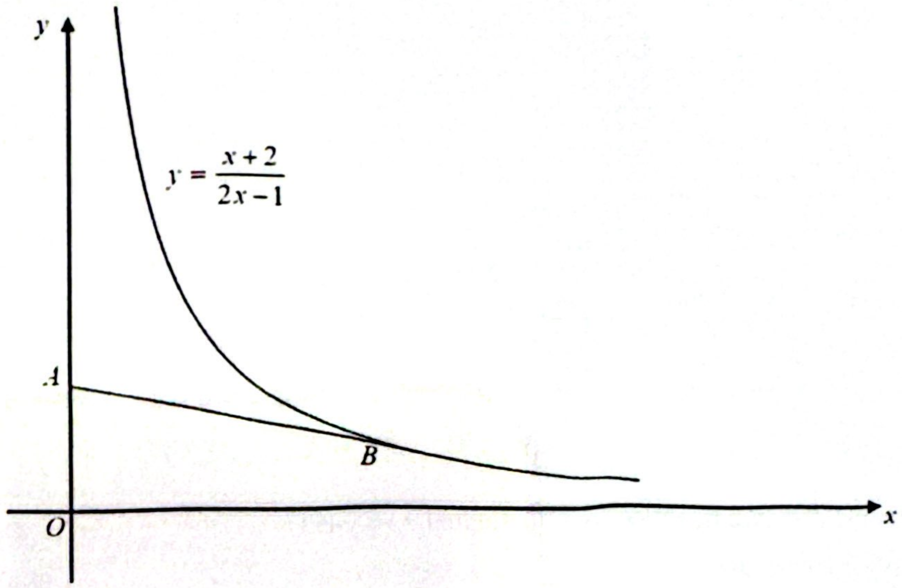
(b) In the case where the curve $y=(k-8)x^2+6x+k$ has two distinct real roots and a minimum point, find the range of values of k. [5]

12 A particle P starting from rest from a fixed point A and moves in a straight line so that, t s after leaving A, its velocity, $v\ \mathrm{m\,s^{-1}}$, is given by $v = \dfrac{5\pi}{3}\sin\dfrac{\pi}{3}t$. Find

(a) the time when the particle is at its first instantaneous rest for $0 \le t \le 4$, [2]

(b) the total distance travelled by P in the first 4 seconds. [5]

13 The diagram shows part of the curve $y = \dfrac{x+2}{2x-1}$ for $x > \dfrac{1}{2}$.



(a) Explain why the curve does not have a stationary point. [3]

(b) The point B lies on the curve and the gradient of the curve at B is $-\frac{1}{5}$. The tangent to the curve at B meets the y-axis at A. Find the area of triangle AOB. [5]

(c) The line $y = k$ intersects the curve. By expressing $\frac{x+2}{2x-1}$ in the form $a + \frac{b}{2x-1}$, where a, b and k are constants, explain why $k > \frac{1}{2}$. [2]

1 Given $10+2\sqrt{21}=(\sqrt{a}+\sqrt{b})^2$, where a and b are integers. Find the values of a and b. [4]

Solution	
$10+2\sqrt{21}=(\sqrt{a}+\sqrt{b})^2$ $10+2\sqrt{21}=a+b+2\sqrt{ab}$ M1 $a+b=10$ M1 (for forming either one eqn) $ab=21$ $a=\frac{21}{b}$ $\frac{21}{b}+b=10$ $21+b^2-10b=0$ $b^2-10b+21=0$ $(b-3)(b-7)=0$ $b=3$ or $b=7$ $a=7$ $\sqrt{7}+\sqrt{3}$ A1, A1	

2 Over a period of 5 years from 1 January 2000 to 1 January 2005, the number of bacteria decreases exponentially. The number of bacteria, P, at time t years from 2000, is given by the formula $P = Ae^{-kt}$, where A and k are constants. The table below shows the population, in thousands, for $1 < t < 5$. One value of P has been recorded incorrectly.

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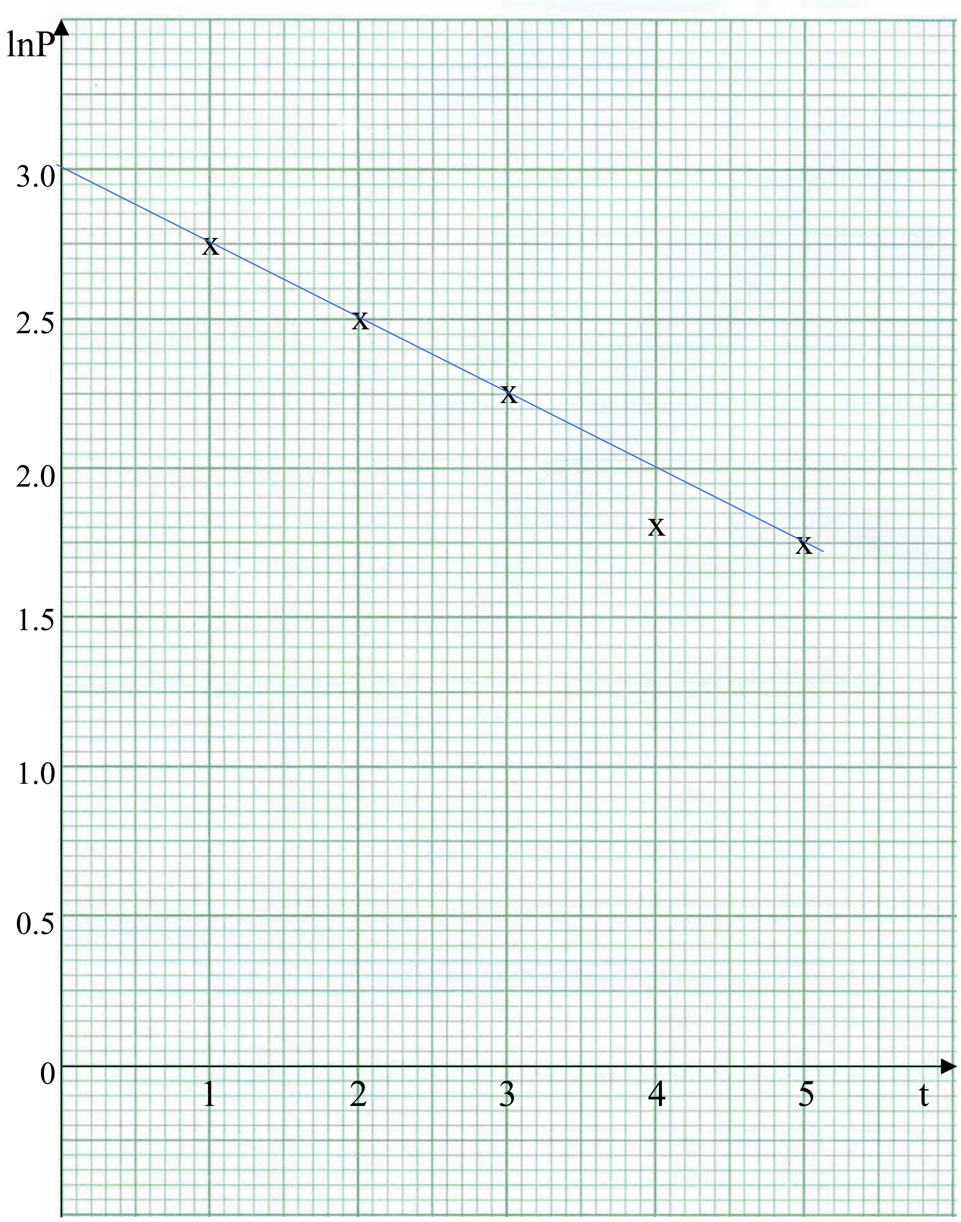
(a) Draw a straight line graph to illustrate the information. [2]


lnP
3.0
2.5
2.0
1.5
1.0
0.5
0
1
2
3
4
5
t
x
x
x
x
x

Solution	
$P = Ae^{-kt}$ $\ln P = \ln A - kt$ $y - axis : \ln P$ $x - axis : t$ $\ln P = 2$ $P = 7.4$	

t	1	2	3	4	5
P	15.6	12.1	9.5	6	5.7
lnP	2.75	2.49	2.25	1.79	1.74

(b) Use your graph to estimate

(i) the correct value of P that is recorded incorrectly, [2]

Solution	
$\ln P = 2$ M1 $P = 7.4$ A1	

(ii) the value of A and k, [3]

Solution	
$\ln A = 3$ $A = 20.0855$ $A = 20.1$ B1 $k = \frac{3-2}{-4}$ M1 $k = \frac{1}{4}$ A1	

(iii) the number of years for 50% of the bacteria to be present. [2]

Solution	
$t = 0,\ \ln P = 3$ $P = e^3$ 50% present means P = $\frac{1}{2}e^3$ $\ln P = \ln\left(\frac{1}{2}e^3\right)$ M1 $\ln P = 2.31$ $t = 2.9 \approx 3$ A1	

3 **(a)** Divide x^3-1 by x^3+x^2. [1]

Solution	
$\frac{x^3-1}{x^3+x^2}=1+\frac{-x^2-1}{x^3+x^2}$ B1	

(b) Express $\frac{x^3-1}{x^3+x^2}$ in partial fractions. [4]

Solution	
$\frac{-x^2-1}{x^3+x^2}=\frac{-x^2-1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$ M1 $-x^2-1=Ax(x+1)+B(x+1)+Cx^2$ $x=0,\quad B=-1$ M1 $x=-1,\ -2=C$ M1 $x=1,\ -2=2A-2-2$ M1 $A=1$ $\frac{x^3-1}{x^3+x^2}=1+\frac{1}{x}-\frac{1}{x^2}-\frac{2}{x+1}$ or A1	

4 Find the range of values of x for which the gradient of $y = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 12x + 5$ is greater than the gradient of $y = 2x^2 + 16x + 5$. [4]

Solution	
$y = \frac{2}{3}x^3 + \frac{3}{2}x^2 - 12x + 5$ $\frac{dy}{dx} = 2x^2 + 3x - 12$ M1 $y = 2x^2 + 16x + 5$ $\frac{dy}{dx} = 4x + 16$ M1 $2x^2 + 3x - 12 > 4x + 16$ M1 $(2x+7)(x-4) > 0$ $x < -\frac{7}{2}$ or $x > 4$ A1	

5 **(a)** Find the first three terms in the expansion of $(1 + \frac{x}{4})^n$ in ascending powers of x, simplifying each term. [3]

Solution	Remarks
$(1 + \frac{x}{4})^n$ $= 1 + \binom{n}{1}\frac{x}{4} + \binom{n}{2}\left(\frac{x}{4}\right)^2 + ...$ M1 $= 1 + \frac{n}{4}x + \frac{n(n-1)}{2} \cdot \frac{x^2}{16} + ...$ $= 1 + \frac{n}{4}x + \frac{n(n-1)}{32}x^2 + ...$ A1x2	

(b) Given that the first two terms in the expansion of $(2-5x)(1+\frac{x}{4})^n$, in ascending powers of x are $a+bx^2$, where a and b are constants.

(i) Find the value of n. [2]

Solution	Remarks
$(2-5x)(1+\frac{x}{4})^n$ $=(2-5x)\left(1+\frac{n}{4}x+\frac{n(n-1)}{32}x^2+.....\right)$ $\frac{n}{4}\times 2-5=0$ M1 $n=10$ A1	

(ii) Hence, find the value of a and of b. [3]

Solution	Remarks
$(2-5x)(1+\frac{x}{4})^n$ $=(2-5x)\left(1+\frac{n}{4}x+\frac{n(n-1)}{32}x^2+.....\right)$ $a=2$ B1 $\frac{2n(n-1)}{32}-\frac{5n}{4}=b$ M1 $\frac{10(9)}{16}-\frac{50}{4}=b$ $b=-\frac{55}{8}$ $b=-6\frac{7}{8}$ A1	

6 A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 9e^{-3x}$. The curve passes through the point $P(0,\ -9)$ and has a stationary point at the point $x = -\frac{1}{3}$. Find the equation of the curve.

[6]

Solution	Remarks
$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 9e^{-3x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \int 9e^{-3x}\mathrm{d}x$ M1 $\frac{\mathrm{d}y}{\mathrm{d}x} = -3e^{-3x} + C$ or M1 $x = -\frac{1}{3},\ \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $-3e^{-3(-\frac{1}{3})} + C = 0$ M1 $C = 3e$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -3e^{-3x} + 3e$ $y = \int 3e - 3e^{-3x}\mathrm{d}x$ M1 $y = 3ex + e^{-3x} + D$ M1 $P(0,\ -9)$ $-9 = 1 + D$ M1 $D = -10$ $y = 3ex + e^{-3x} - 10$ A1	

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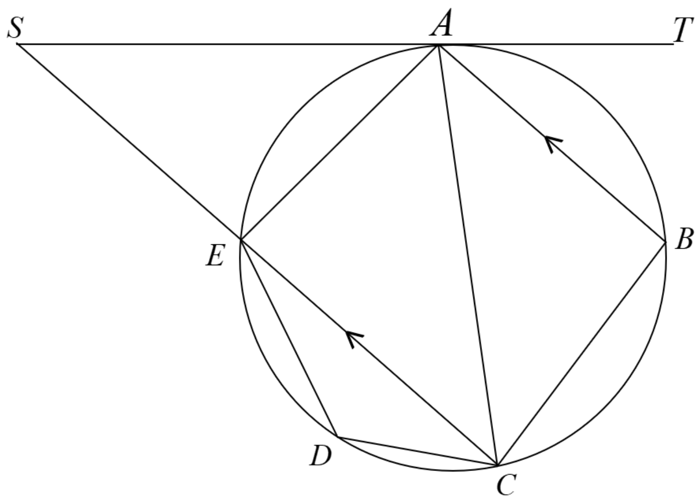


In the diagram, A, B, C, D and E lie on a circle such that CA bisects angle BCE and CE is parallel to BA. The line ST is a tangent to the circle at A and meets CE produced at S. Let angle $EAS = \theta$.

(a) Show that triangle ABC is an isosceles triangle. [3]

Solution	Remarks
Let angle EAS = θ angle $ACE = \theta$ (angle in alternate segment) B1 angle $CAB = \theta$ (alternate angles) B1 angle $BCA = \theta$ (CA bisects angle BCE) B1 $= \text{angle } CAB$ Triangle ABCis an isocceles triangle.	

(b) Show that angle CDE = 3θ. [3]

Solution	Remarks
angle $BAT = \theta$ (angle in alternate segment) B1 angle $CAE = 180 - 3\theta$ (angles on a straight line) B1 angle $CDA = 180 - (180 - 3\theta)$ (angles in opp segment) B1 $= 3\theta$	

8 **(a)** Find the range of values of c for which the line $y = x + c$ intersects the curve $y = -\frac{2}{x} - x$. [4]

Solution	Remarks
$x + c = -\frac{2}{x} - x$ M1 $x^2 + x(x + c) + 2 = 0$ $2x^2 + cx + 2 = 0$ $(c)^2 - (4)(2)(2) \geq 0$ M1 $c^2 - 16 \geq 0$ M1 $c^2 - 16 \geq 0$ $(c-4)(c+4) \geq 0$ $c \leq -4$ or $c \geq 4$ A1	

(b) Given that the line $y = x + c$ is a tangent to the curve $y = -\frac{2}{x} - x$ at point P, find the possible x-coordinates of point P. [4]

Solution	Remarks
$\frac{dy}{dx}(-\frac{2}{x} - x) = 1$ M1 $\frac{2}{x^2} - 1 = 1$ M1 for differentiation $\frac{2}{x^2} = 2$ $x^2 = 1$ $x = 1$ or $x = -1$ A1, A1	

9 The equation of a curve is $y = 2x^3 + ax^2 + bx + 3$, where k is a constant.

(a) Show that if y is always increasing, then $a^2 < 6b$. [4]

Solution	Remarks
$y = 2x^3 + ax^2 + bx + 3$ $\frac{dy}{dx} = 6x^2 + 2ax + b < 0$ $4a^2 - 4(6)b < 0$ $a^2 < 6b$	

(b) In the case where $a = 3$ and $b = 4$, explain why the curve has exactly 1 root. [4]

Solution	Remarks
$f(x) = 2x^3 + 3x^2 + 4x + 3 = 0$ $f(-1) = -2 + 3 - 4 + 3$ M1 for trial and error $= 0$ $(x+1)$ is a factor of $2x^3 + 3x^2 + 4x + 3$ M1 for $(x+1)$ $2x^3 + 3x^2 + 4x + 3 = (x+1)(2x^2 + kx + 3)$ Compare x^2 : $3 = 2 + k$ $k = 1$ $2x^3 + 3x^2 + 4x + 3 = 0$ $(x+1)(2x^2 + x + 3) = 0$ M1 for $(2x^2 + x + 3)$ $x = -1$ or $2x^2 + x + 3 = 0$ $1 - 4(2)(3) < 0$, no real roots M1 f(x) only has 1 real root.	

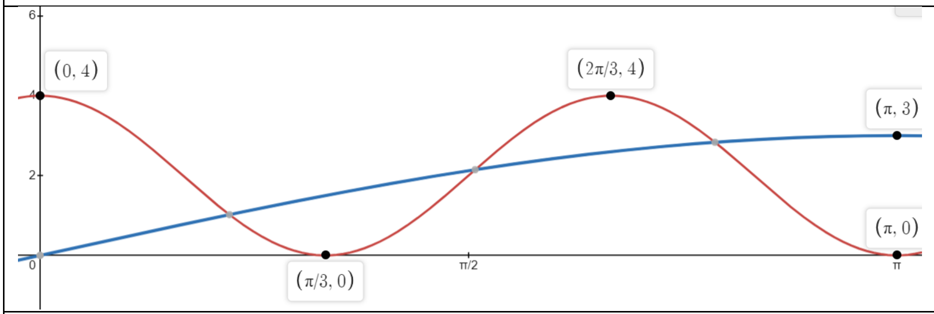
10 **(a)** Find the amplitude and period of [3]

(i) $2\cos 3x + 2$,

(ii) $3\sin\dfrac{x}{2}$

Solution	Remarks
$2\cos 3x+2$ $Period = 120°$ or $\dfrac{\pi}{3}$ B1 $Amplitude = 2$ $3\sin\dfrac{x}{2}$ $Period = 720°$ or 4π B1 $Amplitude = 3$ *Both* amplitude correct = B1	

(b) Sketch, on the same diagram, the curves $y = 2\cos 3x + 2$ and $y = 3\sin\dfrac{x}{2}$ for $0 \le x \le \pi$. [4]

Solution	Remarks
	
Correct shape for each A1 x 2 Correct turning point for BOTH A1 x 2	

11. A curve has an equation $y=(k-8)x^2+6x+k$.

(a) In the case where $k=5$, explain why $y \le 8$. [3]

Solution	Remarks
$y=-3x^2+6x+5$ $y=-3(x^2-2x)+5$ $y=-3\left((x-1)^2-1\right)+5$ M1 $y=-3(x-1)^2+8$ M1 $-3(x-1)^2 \le 0$ $-3(x-1)^2+8 \le 8$ M1 $y \le 8$	

(b) In the case where the curve $y=(k-8)x^2+6x+k$ has two distinct real roots and a minimum point, find the range of values of *k*. [5]

Solution	Remarks
$y=(k-8)x^2+6x+k$ $6^2-4(k-8)k>0$ M1 $36-4k^2+32k>0$ $4k^2-32k-36<0$ $k^2-8k-9<0$ M1 $-1<k<9$ A1 For maximum point, $k-8>0$ M1 $k>8$ $8<k<9$ A1	

12 A particle P starting from rest from a fixed point A and moves in a straight line so that, t s after leaving A, its velocity, $v\,\mathrm{m\,s^{-1}}$, is given by $v=\frac{5\pi}{3}\sin\frac{\pi}{3}t$. Find

(a) the time when the particle is at its first instantaneous rest for $0\le t\le 4$, [2]

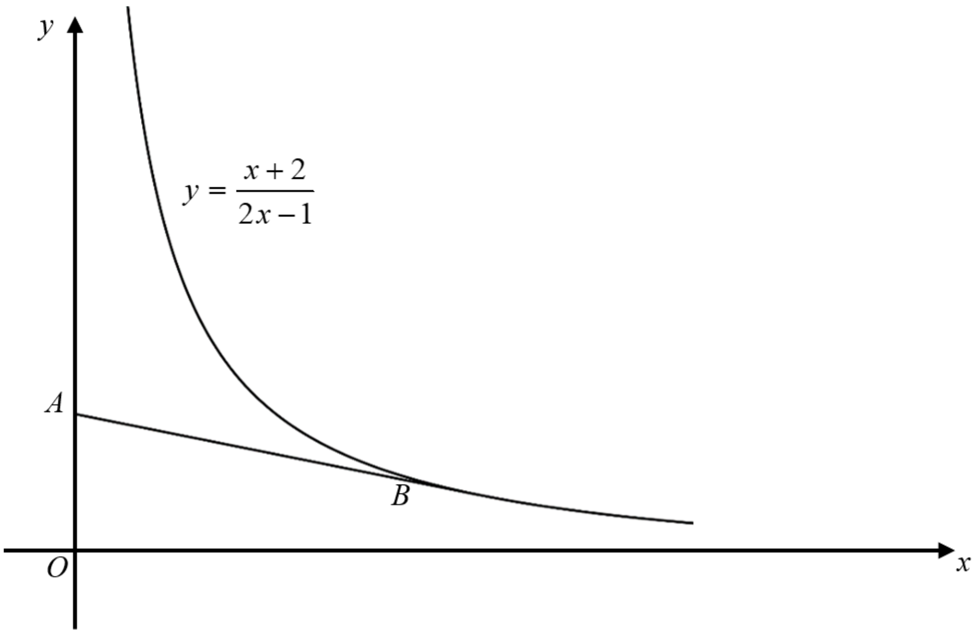
Solution	Remarks
$v=\frac{5\pi}{3}\sin\frac{\pi}{3}t=0$ M1 $\frac{\pi}{3}t=0,\ \pi$ $t=0,\ 3$ $t=3$ A1	

(b) the total distance travelled by P in the first 4 seconds. [5]

Solution	Remarks
Method 1 $s=\int_0^3\frac{5\pi}{3}\sin\frac{\pi}{3}t\ dt+(-\int_3^4\frac{5\pi}{3}\sin\frac{\pi}{3}t\ dt)$ M1, M1 $s=10-0+(-(\frac{15}{2}-10))$ M1, M1 $s=12\frac{1}{2}$ A1	

Method 2 $s = -5\cos\frac{\pi}{3}t + C$ M1 $t = 0, s = 0,\ c = 5$ $s = -5\cos\frac{\pi}{3}t + 5$ B1 $s(0) = 0$ M1 Either one $s(3) = 10$ $s(4) = -5\cos\frac{4\pi}{3} + 5$ $= \frac{15}{2}$ M1 Total distance $= 10 + (10 - \frac{15}{2})$ M1 $= 12\frac{1}{2}$ A1	

13 The diagram shows part of the curve $y = \dfrac{x+2}{2x-1}$ for $x > \dfrac{1}{2}$.



(a) Explain why the curve does not have a stationary point. [3]

Solution	
$\dfrac{dy}{dx} = \dfrac{(1)(2x-1)-(2)(x+2)}{(2x-1)^2}$ $= \dfrac{2x-1-2x-4}{(2x-1)^2}$ $= -\dfrac{5}{(2x-1)^2}$ B1 $(2x-1)^2 \neq 0$ M(FT) 1 $-\dfrac{5}{(2x-1)^2} \neq 0$ $\dfrac{dy}{dx} \neq 0$ A1	

(b) The point B lies on the curve and the gradient of the curve at B is $-\frac{1}{5}$. The tangent to the curve at B meets the y-axis at A. Find the area of triangle AOB. [5]

Solution	
When $\frac{dy}{dx}=-\frac{1}{5}$ $-\frac{5}{(2x-1)^2}=-\frac{1}{5}$ M1 $25=(2x-1)^2$ $(2x-1)=5$ or -5 $x=3$ or $x=-2$ (reject $x=-2$ since B is on positive side of x-axis) When $x=3$, $y=\frac{3+2}{2(3)-1}=\frac{5}{5}=1$ M1 for finding B $\therefore B(3,1)$ Eqn of line AB: $y-1=-\frac{1}{5}(x-3)$ M1 for finding equation $y=-\frac{1}{5}x+\frac{8}{5}$ $x=0$, $y=\frac{8}{5}$ M1 for finding A A(0, $\frac{8}{5}$) Area = $\frac{1}{2}\times\frac{8}{5}\times 3$ $=2\frac{2}{5}$ units A1	

(c) The line $y = k$ intersects the curve. By expressing $\frac{x+2}{2x-1}$ in the form $a + \frac{b}{2x-1}$, where a, b and k are constants, explain why $k > \frac{1}{2}$. [2]

Solution	
$\frac{x+2}{2x-1} = \frac{1}{2} + \frac{5}{2(2x-1)}$ M1 $\frac{1}{2} + \frac{5}{2(2x-1)} > \frac{1}{2}$ $y > \frac{1}{2}$ M1 (or equivalent) Therefore for intersection, $k > \frac{1}{2}$	

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