2025 PRELIM EXAM
CHUNG CHENG HIGH/YISHUN)
ADDITIONAL MATH (G3)
PAPER 2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 In 2020, there were an estimated 7 million insects of a particular species in a certain country. In 2021, the numbers were estimated to have fallen to 5 million. Scientists believe that the number of these insects, N million, can be modelled by the formula $N = 3 + ae^{-kt}$, where t is the time in years after 2020.

(a) Find the exact values of a and of k. [4]

(b) What is the approximate size of the population for large values of t? [1]

2 The equation of a curve is $y = px^2 - (p+2)x + 1$, where p is a constant.

(a) Given that $p > 0$, by completing the square, find the minimum value of y.

Express your answer in the form $\dfrac{a-p^2}{bp}$, where a and b are integers. [3]

(b) Given instead that $p < 0$, find the coordinates of the maximum point in terms of p. [2]

3 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [2]

(b) A curve has equation $y = \mathrm{e}^{-ax}\cot x$, where a is a positive constant. There is exactly one point in the interval $-\frac{1}{2}\pi < x < 0$ at which the tangent is parallel to the x-axis.

Find the value of a and state the exact x-coordinate of this point. [5]

4 It is given that $f(x)=3x^3+ax^2-7x+3$, where a is a constant, has a factor of $x+1$.

(a) Find the value of a and factorise $f(x)$ completely. [4]

(b) Solve the equation $f(\cot^2\theta)=0$ for $-90°<\theta<90°$. [3]

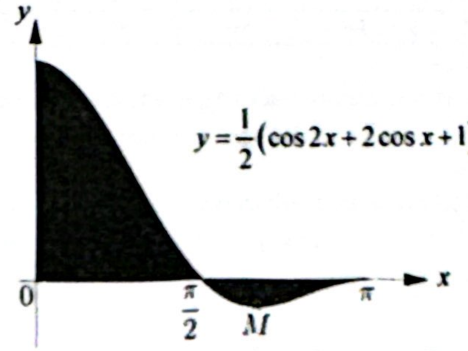
5 **(a)** Prove the identity $\sin 2x(a\cot x + b\tan x) = a + b + (a-b)\cos 2x$, where a and b are constants. [4]

(b) Hence solve the equation $\sin\frac{1}{2}\theta\left(3\cot\frac{1}{4}\theta + 5\tan\frac{1}{4}\theta\right) = 7$ for $0 \le \theta \le 4\pi$. [3]

6 (a) Solve the equation $\log_5(3y+1)-\dfrac{1}{\log_y 5}+\log_2 8=4$. [4]

(b) By expressing the equation $\ln(6e^x + 1) + x = 0$ as a quadratic equation in e^x, solve the equation $\ln(6e^x + 1) + x = 0$, giving values of x in logarithmic form. [4]

7 The diagram shows the curve $y=\frac{1}{2}(\cos 2x+2\cos x+1)$ for $0 \le x \le \pi$ radians.



The point M is the minimum point of the curve, where the x-coordinate of M lies in the interval $\frac{\pi}{2} < x < \pi$.

(a) Find the exact coordinates of M. [5]

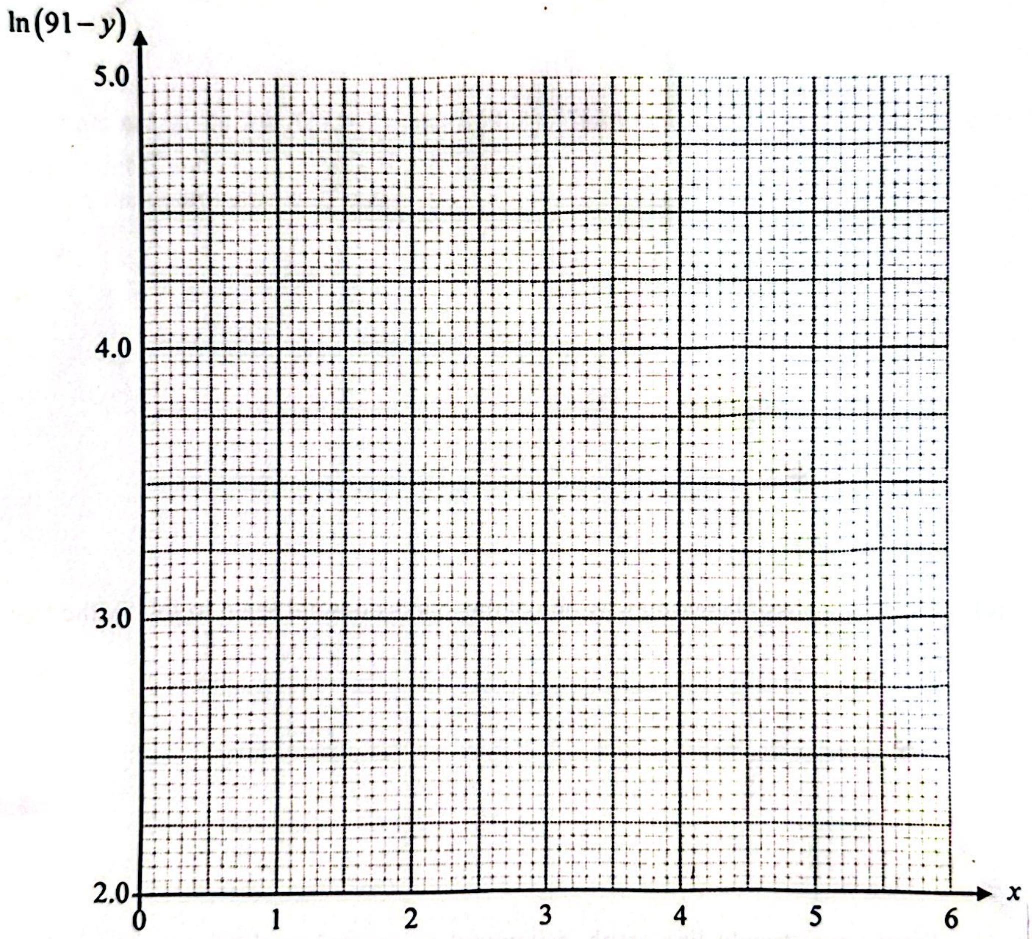
(b) Find the exact total area of the shaded regions bounded by the curve and the axes. **[4]**

8 The table shows Andy's marks for his Mathematics practice papers each week.

Week x	1	2	3	4	5	6
Marks y	45	59	68	74	71	82

He believed that these figures can be modelled by the formula $y = 91 - Ae^{kx}$, where A and k are constants, by excluding one datapoint that does not follow the trend.

(a) On the grid below, by ignoring the datapoint that does not follow the trend, plot $\ln(91-y)$ against x and draw a straight line graph. [2]



(b) Use the graph to estimate the value of each of the constants A and k. [5]

(c) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(d) From your straight line graph, estimate the expected marks for the datapoint that was excluded. [2]

9 The equation of a circle is $x^2+y^2-8x-4y+15=0$.

(a) Find the radius and coordinates of the centre of the circle. [3]

Two points are given by $A(5,-2)$ and $B(9,2)$. The perpendicular bisector of AB cuts the circle at point C and D.

(b) Find the coordinates of C and of D. [6]

(c) Find the shortest distance from the origin O to the line segment CD, giving your answer in the form $k\sqrt{2}$, where k is a constant to be determined. [3]

10 A particle travels in a straight line so that its displacement, s m, from O at time t seconds, where $t \geq 0$, is modelled by $s = t^3 - \frac{9}{2}t^2 + 6t + 1$.

(a) Find the values of t for which the particle is instantaneously at rest. [3]

(b) The particle's acceleration is 15 m/s^2 at T seconds. Find the total distance travelled by the particle in the interval $t = 0$ to $t = T$. [4]

(c) Explain clearly why the answer found in part (b) **should not** be obtained by finding the value of s when $t = T$. [2]

11 **(a)** A curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = ax^2 - 6x + b$, where a and b are constants.

(i) Given that the curve is always increasing, what conditions must apply to a and b? **[3]**

It is now given that $a = 5$.

(ii) The curve intersects the y-axis at $(0, 4)$ and passes through the points $(-3, -80)$ and $(3, k)$. Find the value of k. **[5]**

This question is not related to part **(a)**.

(b) It is given that $y = g(x)$ is a solution of the equation $e^{-2x}\left(\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y\right) = -e^k$, where k is a constant. The point $(1, 0)$ is a stationary point on the curve $y = g(x)$.
Find the nature of this stationary point. [3]

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CHUNG CHENG HIGH SCHOOL (YISHUN)

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2025 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME		INDEX NUMBER	
FORM CLASS / SUBJECT GROUP	/	DATE	

ADDITIONAL MATHEMATICS 4049/02

Paper 2

28 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	5	
2	5	
3	7	
4	7	
5	7	
6	8	
7	9	
8	10	
9	12	
10	9	
11	11	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **18** printed pages.

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For the equation $ax^2 + bx + c = 0$,

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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(a) Find the exact values of a and of k. [4]

(a)	When $t = 0, N = 7$. $7 = 3 + ae^{-k(0)}$ $a = 7 - 3$ $= 4$ When $t = 1, N = 5$. $5 = 3 + 4e^{-k(1)}$ $2 = 4e^{-k}$ $e^{-k} = \frac{2}{4}$ $-k = \ln \frac{1}{2}$ $k = \ln 2$	

(b) What is the approximate size of the population for large values of t? [1]

(b)	$N = 3 + 4e^{-(\ln 2)t}$ When t is large, $e^{-(\ln 2)t}$ becomes close to zero. Thus, the approximate size is 3 million.	

2 The equation of a curve is $y = px^2 - (p+2)x + 1$, where p is a constant.

(a) Given that $p > 0$, by completing the square, find the minimum value of y.

Express your answer in the form $\dfrac{a-p^2}{bp}$, where a and b are integers. [3]

(a)	$y = px^2 - (p+2)x + 1$ $= p\left(x^2 - \dfrac{p+2}{p}x\right) + 1$ $= p\left[x^2 - \dfrac{p+2}{p}x + \left(\dfrac{p+2}{2p}\right)^2 - \left(\dfrac{p+2}{2p}\right)^2\right] + 1$ $= p\left[\left(x - \dfrac{p+2}{2p}\right)^2 - \left(\dfrac{p+2}{2p}\right)^2\right] + 1$ $= p\left(x - \dfrac{p+2}{2p}\right)^2 + 1 - \dfrac{p^2+4p+4}{4p^2} \times p$ $= p\left(x - \dfrac{p+2}{2p}\right)^2 + \dfrac{4p - p^2 - 4p - 4}{4p}$ $= p\left(x - \dfrac{p+2}{2p}\right)^2 + \dfrac{-p^2-4}{4p}$ Hence, the minimum value of y is $\dfrac{-p^2-4}{4p}$.	

(b) Given instead that $p < 0$, find the coordinates of the maximum point in terms of p. [2]

(b)	$\left(\dfrac{p+2}{2p}, \dfrac{-p^2-4}{4p}\right)$	

3 **(a)** Show that $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$. [2]

(a)	$\frac{d}{dx}(\cot x)$ $= \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$ $= \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$ $= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ $= \frac{-1}{\sin^2 x}$ or $-1-\cot^2 x$ $= -\operatorname{cosec}^2 x$ $\frac{d}{dx}(\cot x)$ $= \frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{d}{dx}(\tan x)^{-1}$ $= -1(\tan x)^{-2}(\sec^2 x)$ $= -\frac{1}{\tan^2 x \cos^2 x}$ $= \frac{-1}{\sin^2 x}$ $= -\operatorname{cosec}^2 x$	

(b) A curve has equation $y = e^{-ax}\cot x$, where a is a positive constant. There is exactly one point in the interval $-\frac{1}{2}\pi < x < 0$ at which the tangent is parallel to the x-axis. Find the value of a and state the exact x-coordinate of this point. [5]

(b)	$\frac{dy}{dx} = e^{-ax}(-\operatorname{cosec}^2 x) - ae^{-ax}\cot x$ $= -e^{-ax}(\operatorname{cosec}^2 x + a\cot x)$ The tangent is parallel to x-axis means $\frac{dy}{dx} = 0$. $-e^{-ax}(\operatorname{cosec}^2 x + a\cot x) = 0$ Since $e^{-ax} > 0$ for all values of x, $-(\operatorname{cosec}^2 x + a\cot x) = 0$ $\operatorname{cosec}^2 x + a\cot x = 0$ $\cot^2 x + 1 + a\cot x = 0$ Since there is only 1 point, the above equation only has 1 solution, i.e. discriminant = 0. $a^2 - 4(1)(1) = 0$ $a = \pm 2$ $a = 2$ $(a > 0)$ $\cot^2 x + 2\cot x + 1 = 0$ $(\cot x + 1)^2 = 0$ $\cot x = -1$ $\tan x = -1$ $x = -\frac{\pi}{4}$	

4 It is given that $f(x) = 3x^3 + ax^2 - 7x + 3$, where a is a constant, has a factor of $x + 1$.

(a) Find the value of a and factorise $f(x)$ completely. [4]

(a)	$f(-1) = 3(-1)^3 + a(-1)^2 - 7(-1) + 3$ $0 = -3 + a + 7 + 3$ $a = -7$ By long division, $f(x) = 3x^3 - 7x^2 - 7x + 3$ $= (x+1)(3x^2 - 10x + 3)$ $= (x+1)(3x-1)(x-3)$	

(b) Solve the equation $f(\cot^2\theta) = 0$ for $-90° < \theta < 90°$. [3]

(b)	$\cot^2\theta = -1$ (rej) or $\cot^2\theta = \frac{1}{3}$ or $\cot^2\theta = 3$ $\cot\theta = \pm\frac{1}{\sqrt{3}}$ $\cot\theta = \pm\sqrt{3}$ $\theta = \pm 60°$ $\theta = \pm 30°$	

5 **(a)** Prove the identity $\sin 2x(a\cot x + b\tan x) = a + b + (a-b)\cos 2x$, where a and b are constants. [4]

(a)	$\sin 2x(a\cot x + b\tan x) = 2\sin x\cos x\left(\frac{a\cos x}{\sin x} + \frac{b\sin x}{\cos x}\right)$ $= 2a\cos^2 x + 2b\sin^2 x$ $= a(\cos 2x + 1) + b(1 - \cos 2x)$ $= a\cos 2x + a + b - b\cos 2x$ $= a + b + (a-b)\cos 2x$	

(b) Hence solve the equation $\sin\frac{1}{2}\theta\left(3\cot\frac{1}{4}\theta + 5\tan\frac{1}{4}\theta\right) = 7$ for $0 \le \theta \le 4\pi$. [3]

(b)	$3 + 5 + (3-5)\cos 2\left(\frac{1}{4}\theta\right) = 7$ $-2\cos\left(\frac{1}{2}\theta\right) = -1$ $\cos\left(\frac{1}{2}\theta\right) = \frac{1}{2}$ basic angle $= \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ $\frac{1}{2}\theta$ is in 1st or 4th quadrant. $\frac{1}{2}\theta = \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3}$ $\theta = \frac{2\pi}{3}$ or $\frac{10\pi}{3}$	

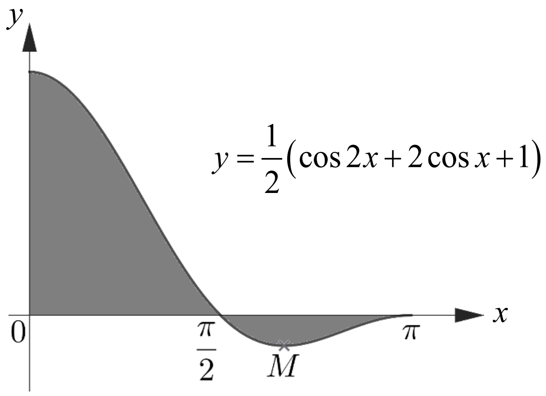
6 **(a)** Solve the equation $\log_5(3y+1)-\dfrac{1}{\log_y 5}+\log_2 8=4$. [4]

(a)	$\log_5(3y+1)-\dfrac{1}{\log_y 5}+\log_2 8=4$ $\log_5(3y+1)-\log_5 y=4-\log_2 8$ $\log_5\left(\dfrac{3y+1}{y}\right)=4-\log_2 2^3$ $\log_5\left(3+\dfrac{1}{y}\right)=4-3$ $3+\dfrac{1}{y}=5^1$ $\dfrac{1}{y}=2$ $y=\dfrac{1}{2}$	

(b) By expressing the equation $\ln(6e^x+1)+x=0$ as a quadratic equation in e^x, solve the equation $\ln(6e^x+1)+x=0$, giving values of x in logarithmic form. [4]

(b)	$\ln(6e^x+1)+x=0$ $\ln(6e^x+1)=-x$ $6e^x+1=e^{-x}$ $6(e^x)^2+(e^x)-1=0$ $(3e^x-1)(2e^x+1)=0$ $e^x=\dfrac{1}{3}$ or $e^x=-\dfrac{1}{2}$ (rej) $x=\ln\dfrac{1}{3}$ or $-\ln 3$	

7 The diagram shows the curve $y=\frac{1}{2}(\cos 2x+2\cos x+1)$ for $0\le x\le \pi$ radians.



The point M is the minimum point of the curve, where the x-coordinate of M lies in the interval $\frac{\pi}{2}<x<\pi$.

(a) Find the exact coordinates of M. [5]

(a)	$y=\frac{1}{2}(\cos 2x+2\cos x+1)$ $\frac{dy}{dx}=\frac{1}{2}(-2\sin 2x-2\sin x)$ $=-\sin 2x-\sin x$ At min pt, $\frac{dy}{dx}=0$. $-\sin 2x-\sin x=0$ $-2\sin x\cos x-\sin x=0$ $-\sin x(2\cos x+1)=0$ $-\sin x=0$ or $\cos x=-\frac{1}{2}$ $x=0$ or $\frac{\pi}{2}$ (rej) $\quad x=\pi-\frac{\pi}{3}$ or $\pi+\frac{\pi}{3}$ (rej) $y=-\frac{1}{4}$ $\therefore M\left(\frac{2\pi}{3},-\frac{1}{4}\right)$	

(b) Find the exact total area of the shaded regions bounded by the curve and the axes. [4]

(b)	$\text{area} = \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\cos 2x + 2\cos x + 1\right)\ \mathrm{d}x - \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\left(\cos 2x + 2\cos x + 1\right)\ \mathrm{d}x$ $= \frac{1}{2}\left[\frac{\sin 2x}{2} + 2\sin x + x\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin 2x}{2} + 2\sin x + x\right]_{\frac{\pi}{2}}^{\pi}$ $= \left[\left(0 + 1 + \frac{\pi}{4}\right) - \left(0 + 0 + 0\right)\right] - \left[\left(0 + 0 + \frac{\pi}{2}\right) - \left(0 + 1 + \frac{\pi}{4}\right)\right]$ $= 2 \text{ units}^2$	

8 The table shows Andy's marks for his Mathematics practice papers each week.

Week x	1	2	3	4	5	6
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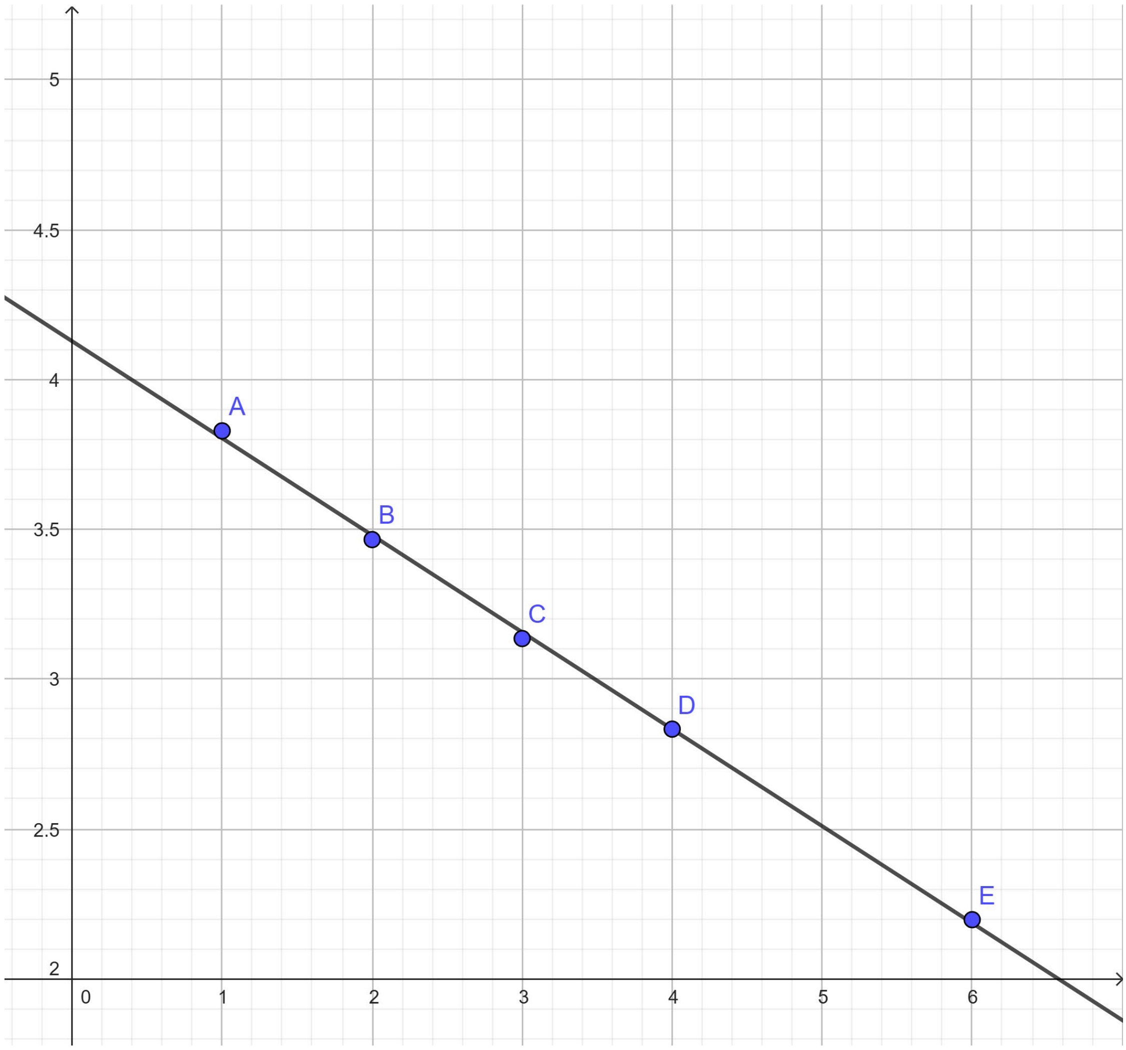
He believed that these figures can be modelled by the formula $y = 91 - Ae^{kx}$, where A and k are constants, by excluding one datapoint that does not follow the trend.

(a) On the grid below, by ignoring the datapoint that does not follow the trend, plot $\ln(91 - y)$ against x and draw a straight line graph. [2]



(b) Use the graph to estimate the value of each of the constants A and k. [5]

(b)	Transforming the equation: $y = 91 - Ae^{kx}$ $91 - y = Ae^{kx}$ $\ln(91-y) = \ln A + \ln e^{kx}$ $\ln(91-y) = \ln A + kx$ Using 2 points on the line (1, 3.8) and (3.5, 3), $\text{gradient} = \frac{3.8-3}{1-3.5} = -0.32$ Reading off the Y-intercept, we have 4.15 The equation of line is $\ln(91-y) = -0.32x + 4.15$ By comparing, $\ln A = 4.15$ $\quad$ $k = -0.32$ $A = 63.4$ (3 s.f.)	

(c) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(c)	Any reason within the context - The lower mark on week 5 was likely to be due to a harder paper. - Andy might have been ill.	

(d) From your straight line graph, estimate the expected marks for the datapoint that was excluded. [2]

(d)	The datapoint is (5, 71) Based on the straight line graph, the reading should be 2.5. The expected marks is $\ln(91-y) = 2.5$ $y = 79$	

9 The equation of a circle is $x^2 + y^2 - 8x - 4y + 15 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [3]

(a)	$x^2 + y^2 - 8x - 4y + 15 = 0$ $x^2 - 8x + 4^2 + y^2 - 4y + 2^2 = 4^2 + 2^2 - 15$ $(x-4)^2 + (y-2)^2 = 5$ Radius is $\sqrt{5}$ units Centre (4, 2)	

Two points are given by $A(5,-2)$ and $B(9,2)$. The perpendicular bisector of AB cuts the circle at point C and D.

(b) Find the coordinates of C and of D. [6]

(b)	$m_{AB} = \dfrac{-2-2}{5-9}$ $= 1$ Midpoint of $AB = \left(\dfrac{5+9}{2}, \dfrac{-2+2}{2}\right)$ $= (7, 0)$ Equation of perpendicular bisector is $y - 0 = -1(x-7)$ $y = -x + 7$ Substituting equation of line into equation of circle: $(x-4)^2 + (-x+7-2)^2 = 5$ $x^2 - 8x + 16 + x^2 - 10x + 25 = 5$ $2x^2 - 18x + 36 = 0$ $x^2 - 9x + 18 = 0$ $(x-3)(x-6) = 0$ $x = 3$ or $x = 6$ $y = 4$ $y = 1$ The coordinates are (3, 4) and (6, 1).	

(c) Find the shortest distance from the origin O to the line segment CD, giving your answer in the form $k\sqrt{2}$, where k is a constant to be determined. [3]

(c)	Area of $OCD = \frac{1}{2}\begin{vmatrix} 0 & 6 & 3 & 0 \\ 0 & 1 & 4 & 0 \end{vmatrix}$ $= \frac{1}{2}\left[(0+24+0)-(0+3+0)\right]$ $= 10.5$ Length of $CD = \sqrt{(3-6)^2+(4-1)^2}$ $= \sqrt{18}$ $= 3\sqrt{2}$ shortest dist $= \frac{10.5}{\frac{1}{2}\times 3\sqrt{2}}$ $= \frac{7}{\sqrt{2}}$ $= \frac{7}{2}\sqrt{2}$ *Alternatively,* The perpendicular line from O to CD has gradient 1, since the gradient of CD is –1. Thus the perpendicular line has equation $y = x$, since the y-intercept is 0. Intersecting the 2 lines $y = x$ and $y = -x+7$, we have $x = -x+7$ $2x = 7$ $x = \frac{7}{2}$ $y = \frac{7}{2}$ The point on CD shortest distance from O is $\left(\frac{7}{2}, \frac{7}{2}\right)$. Thus, the distance is $\sqrt{\left(\frac{7}{2}-0\right)^2+\left(\frac{7}{2}-0\right)^2} = \sqrt{2\left(\frac{7}{2}\right)^2}$ $= \frac{7}{2}\sqrt{2}$	

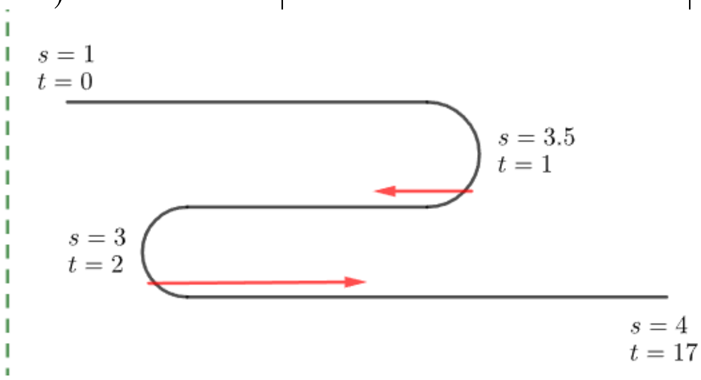
10 A particle travels in a straight line so that its displacement, s m, from O at time t seconds, where $t \geq 0$, is modelled by $s = t^3 - \frac{9}{2}t^2 + 6t + 1$.

(a) Find the values of t for which the particle is instantaneously at rest. [3]

(a)	$v = \frac{\mathrm{d}s}{\mathrm{d}t} = 3t^2 - 9t + 6$ When particle is at instantaneously rest, $v = 0$. $3t^2 - 9t + 6 = 0$ $t^2 - 3t + 2 = 0$ $(t-1)(t-2) = 0$ $t = 1$ or $t = 2$	

(b) The particle's acceleration is 15 m/s^2 at T seconds. Find the total distance travelled by the particle in the interval $t = 0$ to $t = T$. [4]

(b)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 9$ When a = 15, $6T - 9 = 15$ $T = 4$ When $t = 0, s = 1$ When $t = 1, s = 3.5$ When $t = 2, s = 3$ When $t = 4, s = 17$ Total distance $= (3.5-1) + (3.5-3) + (17-3)$ $= 17$	



(c) Explain clearly why the answer found in part **(b) should not** be obtained by finding the value of s when $t = T$. [2]

(c)	Even though the numerical value is the same, the particle turned twice (at least once), thus will need to account for the additional distance. Also, evaluating s at $t = T$ only gave the distance of the particle from O, which is not what is required.	

11 **(a)** A curve $y = f(x)$ is such that $f'(x) = ax^2 - 6x + b$, where a and b are constants.

(i) Given that the curve is always increasing, what conditions must apply to a and b? [3]

(a)	For the curve to be always increasing, i.e. $f'(x) > 0$, that is $ax^2 - 6x + b > 0$ This means that there are no roots, i.e. discriminant < 0. $(-6)^2 - 4(a)(b) < 0$ $36 - 4ab < 0$ $36 < 4ab$ $ab > 9$ and $a > 0$	

It is now given that $a = 5$.

(ii) The curve intersects the y-axis at $(0, 4)$ and passes through the points $(-3, -80)$ and $(3, k)$. Find the value of k. [5]

(b)	$\frac{dy}{dx} = 5x^2 - 6x + b$ $y = \frac{5}{3}x^3 - 3x^2 + bx + 4$ When $x = -3$, $y = 80$. $80 = \frac{5}{3}(-3)^3 - 3(-3)^2 + b(-3) + 4$ $-80 = -45 - 27 - 3b + 4$ $3b = 12$ $b = 4$ When $x = 3$, $y = k$. $k = \frac{5}{3}(3)^3 - 3(3)^2 + 4(3) + 4$ $= 45 - 27 + 12 + 4$ $= 34$	

This question is not related to part **(a)**.

(b) It is given that $y = \mathrm{g}(x)$ is a solution of the equation $\mathrm{e}^{-2x}\left(\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + y\right) = -\mathrm{e}^k$, where k is a constant. The point $(1, 0)$ is a stationary point on the curve $y = \mathrm{g}(x)$.
Find the nature of this stationary point. [3]

(b)	Since it is a stationary point, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$. Together with (1, 0), we can substitute $x = 1,\ y = 0,\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$. The equation is now reduced to $\mathrm{e}^{-2(1)}\left(\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2(0) + 0\right) = -\mathrm{e}^k$ $\mathrm{e}^{-2}\left(\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) = -\mathrm{e}^k$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\mathrm{e}^{2+k}$ Since exponential functions are always positive, we have $-\mathrm{e}^{2+k}$ to be negative. Since $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} < 0$, by 2nd derivative test, (1, 0) is a maximum point.	