2024 PRELIM
SINGAPORE SPORTS SCH
ADDITIONAL MATH (G3)
PAPER 2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The mass of a certain radioactive substance is known to decay with time.
At 12 noon, it was estimated that 100 grams of the radioactive substance were present.
After 40 hours, the mass estimated mass was estimated to have decreased to 90 grams.
The mass, M grams, of radioactive substance present can be modelled by the formula $M = Ae^{-kt}$, where A and k are constants and t is the number of hours after 12 noon.

(a) Estimate the mass of the substance after 2 days. [4]

(b) Find the time, to the nearest hour, for the mass to decay to less than 50% of its original mass. [2]

2 It is given that $f(x) = x^3 + ax^2 + bx + 6$, where a and b are constants, has a factor of $x - 2$ and leaves a remainder of -60 when divided by $x + 3$.

(a) Find the values of a and b. [4]

(b) Using these values of a and b, factorise $f(x)$ completely. [3]

3 **(a)** Prove the identity $\dfrac{\sin x + \cos x}{1 + \sin 2x + \cos 2x} = \dfrac{1}{2}\sec x$. [4]

(b) Hence solve the equation $\dfrac{\sin x + \cos x}{1 + \sin 2x + \cos 2x} = \dfrac{9}{4}\sin x$ for $0° \le x \le 360°$. [4]

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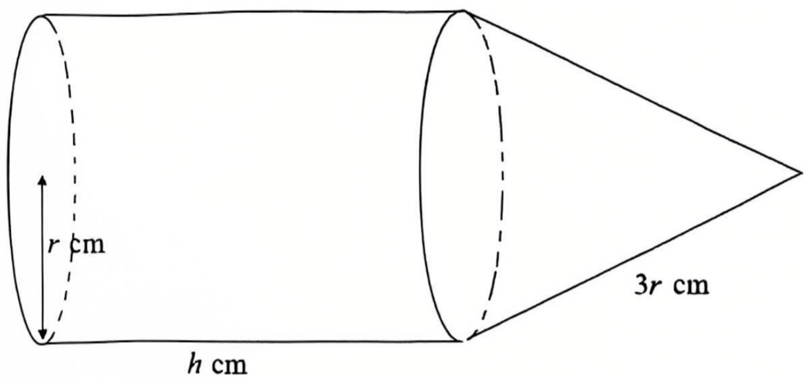

The diagram shows a closed container which is made up of a cone fixed to a right circular cylinder of radius r cm and height h cm. The slant height of the cone is $3r$ cm.

[Curved surface area of a cone with radius r and slant height $l = \pi r l$;
Curved surface area of cylinder with radius r and height $h = 2\pi r h$]

(a) Given that volume of the cylinder is 96π cm^3. Express h in terms of r. [1]

(b) Show that the total surface area of the entire container is $4\pi\left(\frac{48}{r}+r^2\right)$. [3]

(c) Given that r can vary, find the exact value of r for which the total surface area is stationary. [4]

(d) Determine whether this value of r gives a maximum or minimum value for the total surface area of the container. [2]

5 **A calculator must not be used in this question.**

(a) Show that $\sin 105° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [4]

(b) Use the result in **(a)** to find an expression for $\cot^2 105°$ in the form $a + b\sqrt{3}$, where a and b are integers. [4]

6 (a) Solve the equation $7-3^{1-x}=2(3^{x})$. [5]

(b) The curve $y = e^{\sqrt{x}-2} + 5$ intersects the line $y = 10$ at the point P.

(i) Find the x-coordinates of P, in logarithmic form where appropriate. [2]

(ii) Find the exact gradient of the curve at point P, in the form $\dfrac{A}{B\ln 5 + C}$, where A, B and C are constants. [2]

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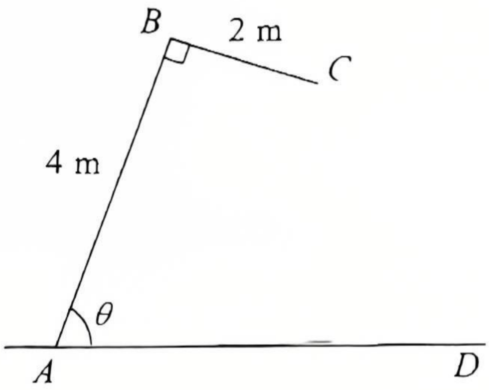


In the figure, the lengths of AB and BC are 4 m and 2 m respectively.
Given that angle $BAD = = \theta\,°$, angle $ABC = 90\,°$ and AD is a straight line.

(i) Show that the shortest distance from C to AD is $4\sin\theta - 2\cos\theta$. [2]

(ii) Express $4\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. **[4]**

(iii) Find the value of θ if the shortest distance of C from AD is 4 m. **[3]**

8 A curve has the equation $y=(2x-1)\sqrt{4x+1}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{12x}{\sqrt{(4x+1)}}$. [4]

(b) Find the rate of change of x when $x=6$, given that y is changing at a constant rate of 2 units per second. [2]

(c) Use the result from (a) to evaluate $\int_0^2 \frac{3x-1}{\sqrt{(4x+1)}}\,dx$. [4]

9 The equation of a circle C_1 is $x^2+y^2+2kx-14y-35=0$, where k is a positive constant, has a radius of 10 units.

(a) Find the value of k. [3]

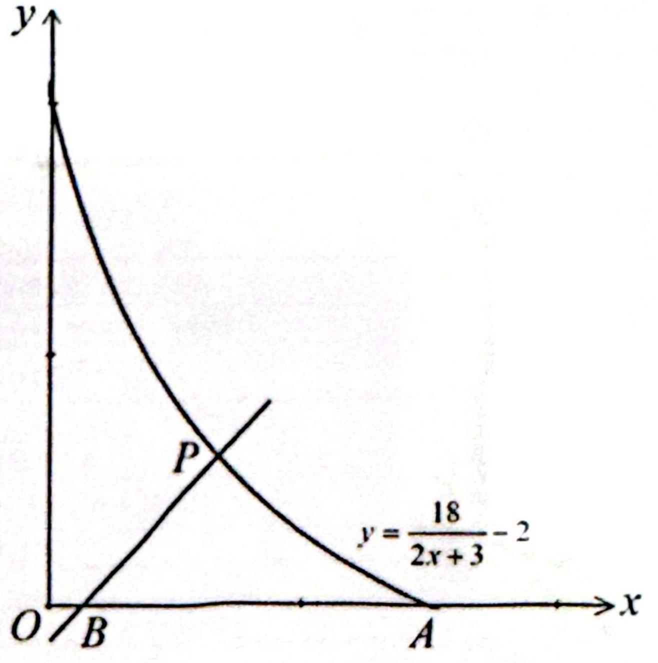
The line $y = 3x + 9$ intersects the circle C_1 at the points P and Q.

(b) Find the shortest distance between the centre of the circle to the the line $y = 3x + 9$. [5]

(c) A second circle C_2 passes through the points $A(2, 4)$ and $B(-3, 5)$ and its centre lies on the line $y = 3x + 9$. Find the coordinates of the centre of circle C_2.

[3]

10 The diagram below shows part of the curve $y=\frac{18}{2x+3}-2$, which crosses the x-axis at A. Point P lies on the curve and the gradient of the curve at P is -1. The normal to the curve at point P meets the x-axis at point B.



(a) Find the coordinates of point P. [3]

(b) Hence, find the area of the region ABP. [9]

1 The mass of a certain radioactive substance is known to decay with time.
At 12 noon, it was estimated that 100 grams of the radioactive substance were present.
After 40 hours, the mass estimated mass was estimated to have decreased to 90 grams.
The mass, M grams, of radioactive substance present can be modelled by the formula $M = Ae^{-kt}$, where A and k are constants and t is the number of hours after 12 noon.

(a) Estimate the mass of the substance after 2 days. [4]

Solution	
$M = Ae^{-kt}$ $t = 0, M = 100$ $100 = A$ B1 $t = 40, M = 90$ $90 = 100e^{-40k}$ M1 $\frac{9}{10} = e^{-40k}$ $-40k = \ln\frac{9}{10}$ $k = -\frac{1}{40}\ln\frac{9}{10}$ $M = 100e^{\frac{1}{40}\ln\frac{9}{10}t}$ $t = 48$ $M = 100e^{\frac{1}{40}\ln\frac{9}{10}\times(48)}$ M(FT) 1 $M = 88.1$ grams A1	

(b) Find the time, to the nearest hour, for the mass to decay to less than 50% of its original mass. [2]

Solution	
$100e^{\frac{1}{40}\ln\frac{9}{10}t} < 50$ $e^{\left(\frac{1}{40}\ln\frac{9}{10}\right)t} < \frac{1}{2}$ M(FT) 1 $\left(\frac{1}{40}\ln\frac{9}{10}\right)t < \ln\frac{1}{2}$ t> 263.2 hours t=264 (to nearest hours) A1	

2 It is given that $f(x) = x^3 + ax^2 + bx + 6$, where a and b are constants, has a factor of $x-2$ and leaves a remainder of -60 when divided by $x+3$.

(a) Find the values of a and b. [4]

Solution	
$f(x) = x^3 + ax^2 + bx + 6$ $f(2) = 0$ $8 + 4a + 2b + 6 = 0$ M1 $4a + 2b = -14$ $2a + b = -7 ------(1)$ $f(-3) = -60$ $-27 + 9a - 3b + 6 = -60$ M1 $9a - 3b = -39$ $3a - b = -13 -----(2)$ $5a = -20$ $a = -4$ A1 $b = 1$ A1	

(b) Using these values of a and b, factorise $f(x)$ completely. [3]

Solution	
$x^3 - 4x^2 + x + 6 = (x-2)(x^2 + kx - 3)$ M1 compare x terms: $1 = -3 - 2k$ $k = -2$ $x^3 - 4x^2 + x + 6$ $= (x-2)(x^2 - 2x - 3)$ $(x-2)(x+1)(x-3)$ A1, A1	

3 **(a)** Prove the identity $\dfrac{\sin x + \cos x}{1 + \sin 2x + \cos 2x} = \dfrac{1}{2}\sec x$. [4]

Solution	
$\dfrac{\sin x + \cos x}{1 + \sin 2x + \cos 2x}$ $= \dfrac{\sin x + \cos x}{1 + 2\sin x \cos x + 2\cos^2 x - 1}$ M1 for sin2x, M1 for cos2x $= \dfrac{\sin x + \cos x}{2\cos x(\sin x + \cos x)}$ M1 for factorising $= \dfrac{1}{2\cos x}$ M1 $= \dfrac{1}{2}\sec x$	

(b) Hence solve the equation $\dfrac{\sin x+\cos x}{1+\sin 2x+\cos 2x}=\dfrac{9}{4}\sin x$ for $0°\le x\le 360°$. [4]

Solution	
$\dfrac{\sin x+\cos x}{1+\sin 2x+\cos 2x}=\dfrac{9}{4}\sin x$ $\dfrac{1}{2}\sec x=\dfrac{9}{4}\sin x$ $2\sec x=9\sin x$ $9\sin x\cos x=2$ $2\sin x\cos x=\dfrac{4}{9}$ M1 $\sin 2x=\dfrac{4}{9}$ M1 $\alpha=26.387°$ $2x=26.387°,\ 180\text{-}26.387°, 360+26.387°, 360+180\text{-}26.387°$ $2x=26.387,\ 153.612,\ 386.387,\ 513.612$ $x=13.2°,\ 76.8°,\ 193.2°,\ 256.8°$ A2 (A1 for any 2 correct)	

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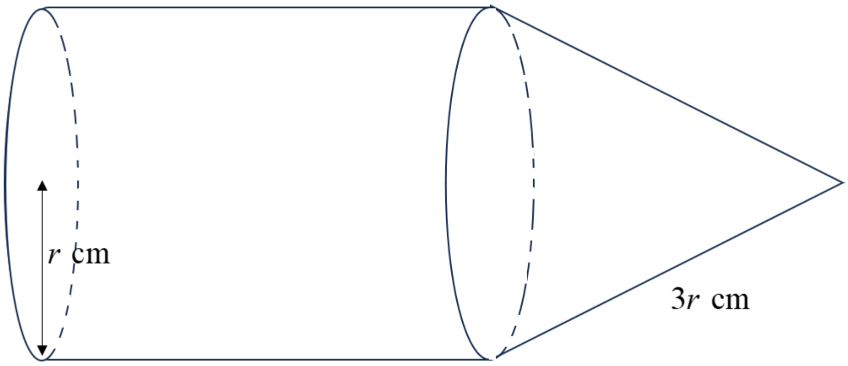

The diagram shows a closed container which is made up of a cone fixed to a right circular cylinder of radius r cm and height h cm. The slant height of the cone is $3r$ cm.

[Curved surface area of a cone with radius r and slant height $l = \pi rl$;
Curved surface area of cylinder with radius r and height $h = 2\pi rh$]

(a) Given that volume of the cylinder is 96π cm^3. Express h in terms of r. [1]

Solution	
$\pi r^2 h = 96\pi$ B1 $h = \dfrac{96}{r^2}$	

(b) Show that the total surface area of the entire container is $4\pi\left(\dfrac{48}{r} + r^2\right)$. [3]

Solution	
$A = \pi r(3r) + 2\pi r\left(\dfrac{96}{r^2}\right) + \pi r^2$ B3 for each correct area, ECF for h $A = 4\pi r^2 + 2\pi\left(\dfrac{96}{r}\right)$ $A = 4\pi(r^2 + \dfrac{48}{r})$	

(c) Given that r can vary, find the exact value of r for which the total surface area is stationary. [4]

Solution	
$A = 4\pi(r^2 + \frac{48}{r})$ $\frac{dA}{dr} = 4\pi(2r - \frac{48}{r^2})$ M1x2 for each correct differentiation $\frac{dA}{dr} = 4\pi(2r - \frac{48}{r^2})=0$ M1 for $\frac{dA}{dr}=0$ $2r - \frac{48}{r^2} = 0$ $2r^3 - 48 = 0$ $r^3 = 24$ $r = \sqrt[3]{24}$ A1	

(d) Determine whether this value of r gives a maximum or minimum value for the total surface area of the container. [2]

Solution	
$\frac{dA}{dr} = 4\pi(2r - \frac{48}{r^2})$ $\frac{d^2A}{dr^2} = 4\pi(2 + \frac{96}{r^3})$ M1 $r = \sqrt[3]{24}$ $\frac{d^2A}{dr^2} = 4\pi(2 + \frac{96}{24})$>0, min area A1	

5 **A calculator must not be used in this question.**

(a) Show that $\sin 105° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [4]

Solution	
$\sin 105°$ $= \sin(60° + 45°)$ M1 $= \sin 60° \cos 45° + \cos 60° \sin 45°$ $= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ M2 (1mark for any 2 correct) $= \frac{\sqrt{2}+\sqrt{6}}{4}$ A1	

(b) Use the result in **(a)** to find an expression for $\cot^2 105°$ in the form $a + b\sqrt{3}$, where a and b are integers. [4]

Solution	
$\cos ec105° = \frac{4}{\sqrt{2}+\sqrt{6}}$ B1 $\cot^2 105°$ $= \cos ec^2 105° - 1$ $= \left(\frac{4}{\sqrt{2}+\sqrt{6}}\right)^2 - 1$ M1 $= \left(\frac{4}{\sqrt{2}+\sqrt{6}} \times \frac{\sqrt{2}-\sqrt{6}}{\sqrt{2}-\sqrt{6}}\right)^2 - 1$ M1 for rationalisation $= \left(\frac{4(\sqrt{2}-\sqrt{6})}{2-6}\right)^2 - 1$ $= (\sqrt{2}-\sqrt{6})^2 - 1$ $= 2 - 2\sqrt{12} + 6 - 1$ $= 7 - 4\sqrt{3}$ A1	

6 **(a)** Solve the equation $7-3^{1-x}=2(3^x)$. [5]

Solution	
$7-3^{1-x}=2(3^x)$ $7-\frac{3}{3^x}=2(3^x)$ M1 $3^x=u$ $7-\frac{3}{u}=2(u)$ $2u^2-7u+3=0$ M1 $(u-3)(2u-1)=0$ $u=3$ or $u=\frac{1}{2}$ $3^x=3$ or $3^x=\frac{1}{2}$ $x=1$ or $x\ln 3=\ln\frac{1}{2}$ M1 for taking ln on both sides $x=1$ *or* $x=-0.631$ A1, A1	

(b) The curve $y=e^{\sqrt{x}-2}+5$ intersects the line $y=10$ at the point *P*.

(i) Find the *x*-coordinates of *P*, in logarithmic form where appropriate. [2]

Solution	
$e^{\sqrt{x}-2}+5=10$ M1 $e^{\sqrt{x}-2}=5$ $\sqrt{x}-2=\ln 5$ $x=(\ln 5+2)^2$ A1	

(ii) Find the exact gradient of the curve at point P, in the form $\dfrac{A}{B\ln 5 + C}$, where A, B and C are constants. [2]

Solution	
$y = e^{\sqrt{x}-2} + 5$ $\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}-2}}{2\sqrt{x}}$ M1 $\dfrac{dy}{dx} = \dfrac{e^{\ln 5 + 2 - 2}}{2(\ln 5 + 2)}$ $\dfrac{dy}{dx} = \dfrac{5}{2\ln 5 + 4}$ A1	

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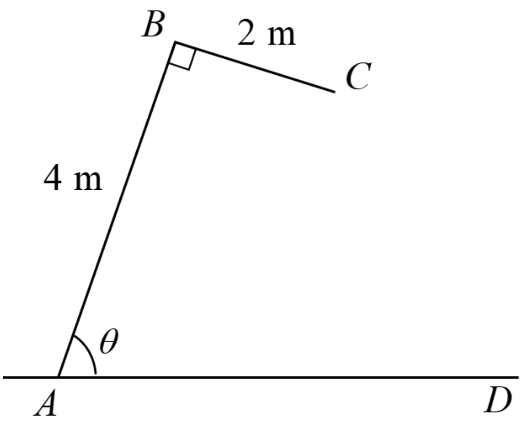
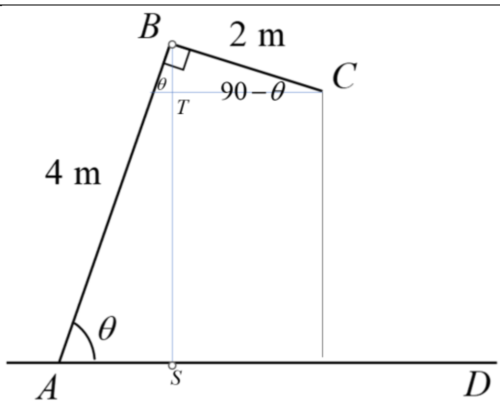
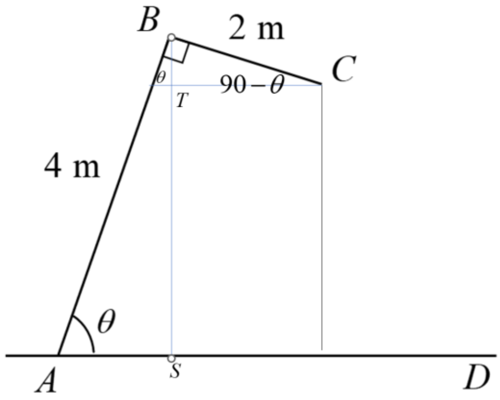


In the figure, the lengths of AB and BC are 4 m and 2 m respectively.
Given that angle $BAD = \; = \theta°$, angle $ABC = 90°$ and AD is a straight line.

(i) Show that the shortest distance from C to AD is $4\sin\theta - 2\cos\theta$. [2]

Solution	
	
$BT = 2\sin(90-\theta)$ $BT = 2\cos\theta$ B1 $BS = 4\sin\theta$ B1 $TS = 4\sin\theta - 2\cos\theta$	

(ii) Express $4\sin\theta - 2\cos\theta$ in the form $R\sin(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [4]

Solution	
	
$4\sin\theta - 2\cos\theta = R\sin(\theta - \alpha)$ $4\sin\theta - 2\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R\cos\alpha = 4$ $R\sin\alpha = 2$ $R = \sqrt{20} = 2\sqrt{5}$ B1 $\tan\alpha = \frac{1}{2}$ M1 $\alpha = 26.567$ $\alpha = 26.6$ A1 $4\sin\theta - 2\cos\theta = 2\sqrt{5}\sin(\theta - 26.6°)$ $B1$	

(iii) Find the value of θ if the shortest distance of C from AD is 4 m. [3]

Solution	
$2\sqrt{5}\sin(\theta-26.6°)=3$ $\sin(\theta-26.6°)=\dfrac{3}{2\sqrt{5}}$	M1
$\theta-26.567°=42.130°$ $\theta=26.56°+42.130°$	M1
$\theta=68.7°$	A1

8 A curve has the equation $y=(2x-1)\sqrt{4x+1}$.

(a) Show that $\dfrac{dy}{dx}=\dfrac{12x}{\sqrt{(4x+1)}}$. [4]

Solution	
$y=(2x-1)\sqrt{4x+1}$ $=(2x-1)(4x+1)^{\frac{1}{2}}$	
$\dfrac{dy}{dx}=(2x-1)\dfrac{1}{2}(4x+1)^{-\frac{1}{2}}(4)+2(4x+1)^{\frac{1}{2}}$ M1 for product rule M1 for $\dfrac{1}{2}(4x+1)^{-\frac{1}{2}}$ M1 for (1)	
$=(4x+1)^{-\frac{1}{2}}[2(2x-1)+2(4x+1)]$ M1 for correct factorisation $=\dfrac{12x}{\sqrt{(4x+1)}}$	

(b) Find the rate of change of x when $x = 6$, given that y is changing at a constant rate of 2 units per second. [2]

Solution	
$\frac{dy}{dt} = 2$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $2 = \frac{12(6)}{\sqrt{(4x+1)}} \times \frac{dx}{dt}$ M1 $\frac{dx}{dt} = \frac{2(5)}{12(6)}$ $\frac{dx}{dt} = \frac{5}{36}$ unit/s A1	

(c) Use the result from **(a)** to evaluate $\int_0^2 \frac{3x-1}{\sqrt{(4x+1)}}\,\mathrm{d}x$. [4]

Solution	
$\int_0^2 \frac{3x-1}{\sqrt{(4x+1)}}\,\mathrm{d}x$ $=\int_0^2 \frac{3x}{\sqrt{(4x+1)}}-\frac{1}{\sqrt{(4x+1)}}\,\mathrm{d}x$ M1 for splitting $=\left[\frac{1}{4}(2x-1)\sqrt{4x+1}-\frac{\sqrt{(4x+1)}}{\frac{1}{2}\times 4}\right]_0^2$ M1x2 for each correct or equivalent $=\left[\frac{1}{4}(2x-1)\sqrt{4x+1}-\frac{\sqrt{(4x+1)}}{2}\right]_0^2$ $=\left[\frac{1}{4}(3)\sqrt{9}-\frac{\sqrt{9}}{2}\right]-\left[\frac{1}{4}(-1)\sqrt{1}-\frac{\sqrt{1}}{2}\right]$ $=\frac{9}{4}-\frac{3}{2}+\frac{1}{4}+\frac{1}{2}$ $=1\frac{1}{2}$ A1	

9 The equation of a circle C_1 is $x^2+y^2+2kx-14y-35=0$, where k is a positive constant, has a radius of 10 units.

(a) Find the value of k. [3]

Solution	
$x^2+y^2+2kx-14y-35=0$ $x^2++2kx+y^2-14y-35=0$ $(x+k)^2-k^2+(y-7)^2-49-35=0$ M1 for completing the square $(x+k)^2+(y-7)^2=k^2+84$ $k^2+84=10^2$ M1 for equating to r^2 $k=4$ A1	

The line $y=3x+9$ intersects the circle C_1 at the points P and Q.

(b) Find the shortest distance between the centre of the circle to the the line $y=3x+9$. [5]

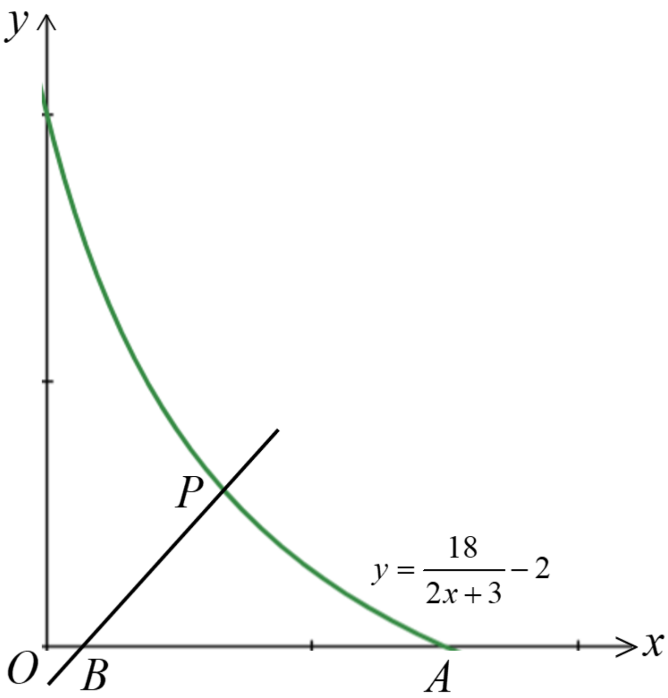
Solution	
$(x+4)^2+(3x+9-7)^2=100$ M1 $x^2+8x+16+9x^2+12x+4=100$ $10x^2+20x-80=0$ $x^2+2x-8=0$ $x=-4$ or $x=2$ M1 for finding x or y $y=-3$ or $y=15$ $P(-4,-3)$ or $Q(2,15)$ Mid point $=\left(\frac{-4+2}{2},\frac{-3+15}{2}\right)$ M1 $=(-1,\ 6)$ $Center=(-4,\ 7)$ *Shortest Distance between the centre of circle to the linePQ* $=\sqrt{(-1-(-4))^2+(6-7)^2}$ M1 $=\sqrt{10}$ units or 3.16 units A1	

(c) A second circle C_2 passes through the points $A(2, 4)$ and $B(-3, 5)$ and its centre lies on the line $y = 3x + 9$. Find the coordinates of the centre of circle C_2.

[3]

Solution	
$A(2,\ 4)$ and $B(-3,\ 5)$ $(x,\ 3x+9)$ $\sqrt{(x+3)^2+(3x+9-5)^2}=\sqrt{(x-2)^2+(3x+9-4)^2}$ M1, M1 $x^2+6x+9+9x^2+24x+16=x^2-4x+4+9x^2+30x+25$ $30x+25=26x+29$ $4x=4$ $x=1$ $y=12$ $(1,\ 12)$ A1	

10 The diagram below shows part of the curve $y = \dfrac{18}{2x+3} - 2$, which crosses the x-axis at A. Point P lies on the curve and the gradient of the curve at P is -1. The normal to the curve at point P meets the x-axis at point B.



(a) Find the coordinates of point P. [3]

Solution	
Graident at P= -1 $-18\left(\dfrac{2}{(2x+3)^2}\right) = -1$ M1 $(2x+3)^2 = 36$ $2x+3=6 \quad (x>0)$ $x = \dfrac{3}{2}$ M1 $y=1$ $P\left(\dfrac{3}{2},1\right)$ A1	

(b) Hence, find the area of the region *ABP*. [9]

Solution	
$y=\dfrac{18}{2x+3}-2$ *at* A, y =0 $\dfrac{18}{2x+3}-2=0$ M1 $\dfrac{18}{2x+3}=2$ $2x+3=9$ $x=3$ $A(3,0)$ A1	
Eqn of PB $y-1=1(x-\dfrac{3}{2})$ M1 $y=x-\dfrac{1}{2}$ $B\left(\dfrac{1}{2},0\right)$ A1	
$\int_{\frac{3}{2}}^{3}\dfrac{18}{2x+3}-2\ \mathrm{d}x+\dfrac{1}{2}\left(\dfrac{3}{2}-\dfrac{1}{2}\right)(1)$ M1(FT) for correct limit M1(FT) for area of triangle $=\left[9\ln(2x+3)-2x\right]_{\frac{3}{2}}^{3}+\dfrac{1}{2}$ M1 for correct integration $=9\ln 9-6-(9\ln 6-3)+\dfrac{1}{2}$ $=9\ln 9-6-9\ln 6+3+\dfrac{1}{2}$ M1 $=9(\ln\dfrac{3}{2})-\dfrac{5}{2}$ A1	

Working for Question 10 continues here:

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