Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

SUGGESTED ANSWERS

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

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Vetter: Ms Sabrina Tan

This question paper consists of **23** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1	**(a)**	Differentiate $\ln\left(\dfrac{3x}{x^2+1}\right)$ with respect to x.	[3]
	(b)	Hence find $\displaystyle\int \frac{2x}{x^2+1}\,\mathrm{d}x$.	[2]

1	(a)	*Let* $y=\ln\left(\dfrac{3x}{x^2+1}\right)=\ln(3x)-\ln\left(x^2+1\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x}-\dfrac{2x}{x^2+1}$ OR $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\dfrac{3\left(x^2+1\right)-3x(2x)}{\left(x^2+1\right)^2}}{\dfrac{3x}{x^2+1}}$ $=\dfrac{3x^2+3-6x^2}{\left(x^2+1\right)^2}\times\dfrac{x^2+1}{3x}$ $=\dfrac{1-x^2}{x\left(x^2+1\right)}$	
	(b)	$\displaystyle\int\left(\frac{1}{x}-\frac{2x}{x^2+1}\right)\mathrm{d}x=\ln\left(\frac{3x}{x^2+1}\right)+c$ $\displaystyle\int\frac{2x}{x^2+1}\,dx=\int\frac{1}{x}\,\mathrm{d}x-\ln\left(\frac{3x}{x^2+1}\right)+c$ $\displaystyle=\ln x-\ln\left(\frac{3x}{x^2+1}\right)+c$ OR $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1-x^2}{x\left(x^2+1\right)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$	

Solve for A, B & C.

$A=1,\ B=-2,\ C=0$

$$\frac{1-x^2}{x\left(x^2+1\right)}=\frac{1}{x}-\frac{2x}{x^2+1}$$

$$\int\left(\frac{1}{x}-\frac{2x}{x^2+1}\right)\mathrm{d}x\ =\ln\left(\frac{3x}{x^2+1}\right)\ +\ c$$

$$\int\frac{2x}{x^2+1}\,dx=\int\frac{1}{x}\,\mathrm{d}x-\ln\left(\frac{3x}{x^2+1}\right)\ +\ c$$

$$=\ln x-\ln\left(\frac{3x}{x^2+1}\right)+\ c$$

2	It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$. Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers.	[4]

2	$\cos A = \frac{1}{2},\ \sin A = \frac{\sqrt{3}}{2}$ $\sin B = -\frac{1}{\sqrt{2}},\ \cos B = -\frac{1}{\sqrt{2}}$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$ $= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ $= -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$	

3	Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. $\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$	[5]

	Volume of cone, $V = \frac{1}{3}\pi r^2 (18r) = 6\pi r^3$ $\frac{dV}{dr} = 18\pi r^2$ After 3s, $V = 2\pi \times 3 = 6\pi$ $6\pi r^3 = 6\pi$ $r = 1$ When $t = 3s$, $r = 1cm$, $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$ $2\pi = 18\pi(1)^2 \times \frac{dr}{dt}$ $\frac{dr}{dt} = \frac{1}{9}$ cm/s Rate of change of radius of cone after 3 s is $\frac{1}{9}$ cm/s	

4	A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.		
	(a)	Find the coordinates of A and of B.	[4]
	(b)	Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB. Is he correct? Justify your answer with workings.	[3]

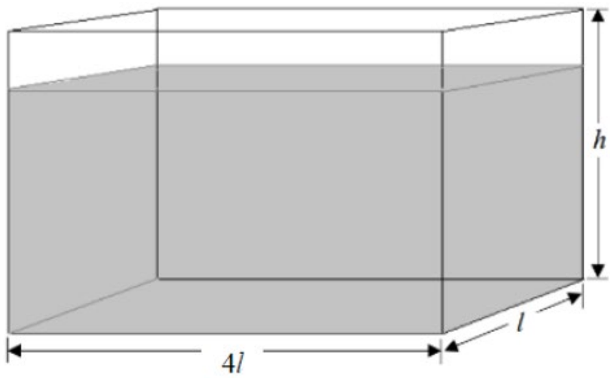
4 **(a)**	$4y = 2x + 1$ ....(1) $y = \dfrac{2x+1}{4}$ ....(2) $3y - x = 4xy$ ....(3) Substitute (2) into (3): $3\left(\dfrac{2x+1}{4}\right) - x = 4x\left(\dfrac{2x+1}{4}\right)$ $\dfrac{3}{2}x + \dfrac{3}{4} - x = 2x^2 + x$ $6x + 3 - 4x = 8x^2 + 4x$ $8x^2 + 2x - 3 = 0$ $(2x-1)(4x+3) = 0$ $x = \dfrac{1}{2}$ or $x = -\dfrac{3}{4}$ $y = \dfrac{1}{2}$ or $y = -\dfrac{1}{8}$ $A = \left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ $B = \left(-\dfrac{3}{4}, -\dfrac{1}{8}\right)$	
(b)	Gradient of $2y - 4x = 5$ is 2 Gradient of AB $= \dfrac{-\frac{1}{8} - \frac{1}{2}}{-\frac{3}{4} - \frac{1}{2}} = \dfrac{1}{2}$ Since $2 \times \dfrac{1}{2} \neq -1$, the 2 lines are not perpendicular to each other. Henry is not correct.	

5	**(a)**	Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\frac{3}{x}\right)^8$.	[2]
	(b)	Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\frac{3}{x}\right)^8$, find the constant term in the expansion.	[4]

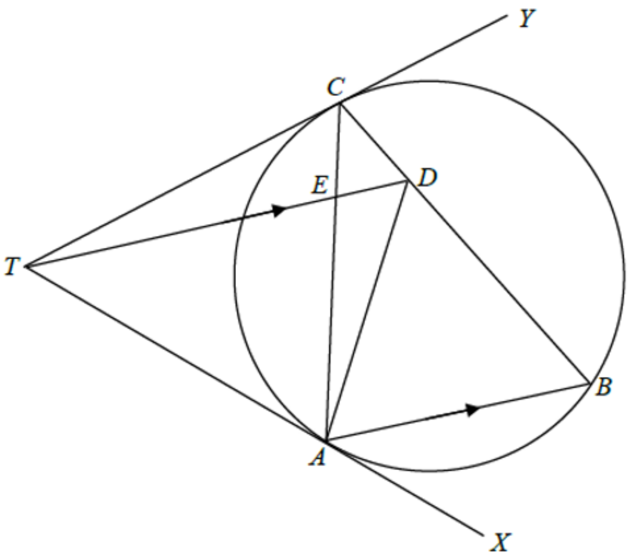
5	**(a)**	$\left(2-\frac{3}{x}\right)^8 = 2^8 + \binom{8}{1}\left(2^7\right)\left(-\frac{3}{x}\right) + \binom{8}{2}\left(2^6\right)\left(-\frac{3}{x}\right)^2 + \text{........}$ $= 256 - \frac{3072}{x} + \frac{16128}{x^2} + \text{.........}$	
	(b)	$\left(1-2x-kx^2\right)\left(1-\frac{2}{x}\right)^8$ $=\left(1-2x-kx^2\right)\left(256-\frac{3072}{x}+\frac{16128}{x^2}+\text{.........}\right)$ $=256-2\left(256\right)x+2\left(3072\right)+3072kx-16128k+\text{......}$ $=256-512x+6144+3072kx-16128k+\text{......}$ Since there is no x term in the expansion, $3072k-512=0$ $k=\frac{1}{6}$ The constant term $=256+6144-16128k$ $=6400-2688$ $=3712$	

6	**(a)**	Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants.	[2]
	(b)	Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions.	[2]

6	**(a)**	$-4x^2 + 4x - 3$ $= -4(x^2 - x) - 3$ $= -4\left[x^2 - x + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2\right] - 3$ $= -4\left(x - \frac{1}{2}\right)^2 + 1 - 3$ $= -4\left(x - \frac{1}{2}\right)^2 - 2$	
	(b)	$4x - 4x^2 - 3 = 0$ $y = 0\ (x - axis)$ Since the maximum value of y is -2, the graph of $y = 4x - 4x^2 - 3$ will not intersect the x-axis, $4x - 4x^2 - 3 = 0$ has no real solutions. OR $-4\left(x - \frac{1}{2}\right)^2 - 2 = 0$ $\left(x - \frac{1}{2}\right)^2 = -\frac{1}{2}$ $\left(x - \frac{1}{2}\right) = \pm\sqrt{-\frac{1}{2}}$ $(no\ real\ solutions)$	

7	Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be $5\ \text{m}^2$. 		
	(a)	Show that the volume of the tank, $V\ \text{m}^3$, is given by $V = \dfrac{2}{5}\left(5l - 4l^3\right)$.	[3]
	(b)	Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim.	[4]

7	(a)	Total outer surface area $=5\,m^2$ $2(4lh)+2(lh)+4l^{\;2}=5$ $10lh+4l^2=5$ $h=\dfrac{5-4l^2}{10l}$ $V=4l\times l\times h$ $=4l^2\times\dfrac{5-4l^2}{10l}$ $=2l\times\dfrac{5-4l^2}{5}$ $=\dfrac{2}{5}\left(5l-4l^3\right)$ $\quad$ $(Shown)$	
	(b)	$V=\dfrac{2}{5}\left(5l-4l^3\right)$ $\dfrac{dV}{dl}=\dfrac{2}{5}\left(5-12l^2\right)$ $\quad or\;\; 2-\dfrac{24}{5}l^2$ $\dfrac{d^2V}{dl^2}=-\dfrac{48}{5}l<0$ (maximum V) $\dfrac{dV}{dl}=0$ $\dfrac{2}{5}\left(5-12l^2\right)=0$ $l^2=\dfrac{5}{12}$ $l=\sqrt{\dfrac{5}{12}}$ Area of rectangular base $=4l^2=4\left(\dfrac{5}{12}\right)=\dfrac{5}{3}\quad or\quad 1.67\;m^2$	

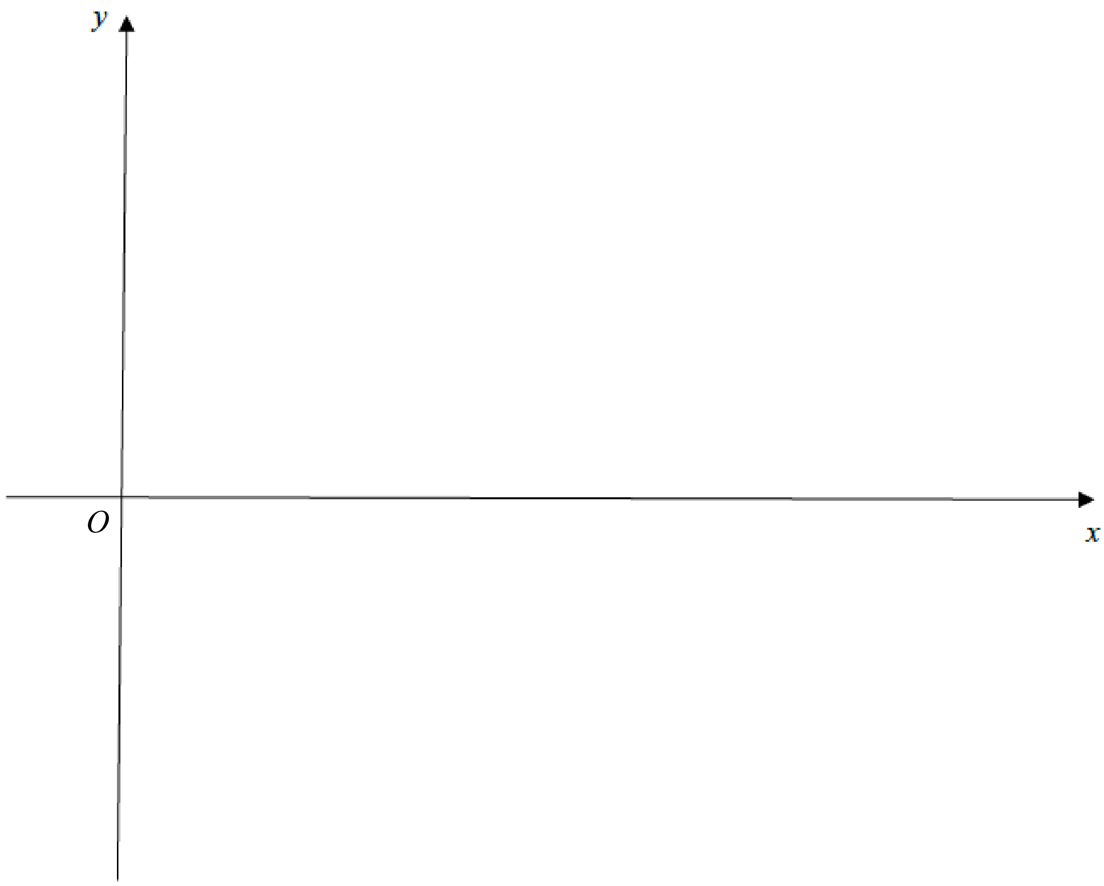
8	In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E and D is on BC such that TD is parallel to AB. 		
	(a)	Prove that angle ACB is equal to angle ATD.	[2]
	(b)	Using the result from part **(a)**, explain whether a circle can be drawn passing through the points T, A, D and C.	[2]
	(c)	Prove that $CE \times EA = DE \times TE$.	[3]

8	(a)	$\angle ATD = \angle XAB$ (corresponding angles, $AB//TD$) $\angle XAB = \angle ACB$ (Alternate Segment Theorem) $\angle ACB = \angle ATD$ (Proved)	
	(b)	From Part (a), $\angle ACD = \angle ATD$, the properties of circles tell us that angles in the same segment of a circle are equal. Therefore, we can draw a circle passing through points T, A, D and C.	
		$\angle ATE = \angle DCE$ (from Part (a)) $\angle TEA = \angle CED$ (vertically opposite angles) Therefore ΔATE is similar to ΔDCE (AA Similarity test) $\frac{TE}{CE} = \frac{EA}{DE}$ $CE \times EA = DE \times TE$	

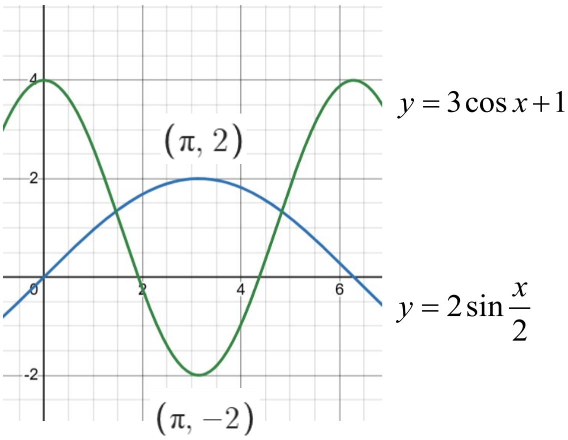
9	**(a)**	A square has an area $\left(17-8\sqrt{2}\right)\text{ cm}^2$. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)\text{ cm}$, where a and b are integers. Show that $2b^4-17b^2+16=0$.	[4]
	(b)	[The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.] The sector of a circle with radius, r, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)$ cm and an area of $\left(3\sqrt{3}-\sqrt{15}\right)\text{ cm}^2$. Show that $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express r in the form $\left(p+q\sqrt{5}\right)$ cm, where p and q are integers.	[5]

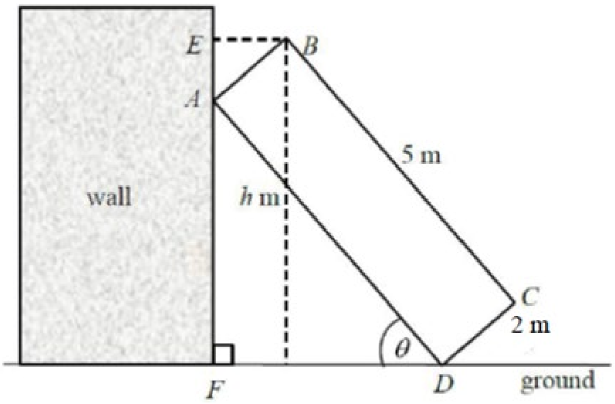
9	(a)	$\left(a+b\sqrt{2}\right)^2=17-8\sqrt{2}$ $a^2+2ab\sqrt{2}+2b^2=17-8\sqrt{2}$ $\Rightarrow a^2+2b^2=17$ ....(1) $2ab=-8$ $a=-\dfrac{4}{b}$ ....(2) Substitute (2) into (1): $\left(-\dfrac{4}{b}\right)^2+2b^2=17$ $\dfrac{16}{b^2}+2b^2=17$ $16+2b^4=17b^2$ $2b^4-17b^2+16=0$ (Shown)	
	(b)	$A=\dfrac{1}{2}r^2\theta$, $S=r\theta$ $\dfrac{A}{S}=\dfrac{r}{2}$ $r=\dfrac{2A}{S}$ $r=\dfrac{2\left(3\sqrt{3}-\sqrt{15}\right)}{\sqrt{15}-\sqrt{3}}=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ $(showed)$ $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}\times\dfrac{\sqrt{15}+\sqrt{3}}{\sqrt{15}+\sqrt{3}}$ $=\dfrac{6\sqrt{45}+18-30-2\sqrt{45}}{15-3}$ $=\dfrac{4\sqrt{45}-12}{12}$ $=\dfrac{\sqrt{9}\sqrt{5}}{3}-1$ $=\sqrt{5}-1$	

10	It is given that $\mathrm{f}(x)=2\sin\frac{x}{2}$ and $\mathrm{g}(x)=3\cos x+1$, where $0\le x\le 2\pi$.		
	(a)	State the period of $\mathrm{f}(x)$.	[1]
	(b)	State the smallest value of $\mathrm{f}(x)$.	[1]
	(c)	State the largest value of $\mathrm{g}(x)$.	[1]
	(d)	Sketch on the same axes, the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ for $0\le x\le 2\pi$. Label your graphs clearly.	[4]



	(e)	The solutions to the equation $\mathrm{f}(x)=\mathrm{g}(x)$ for $0\le x\le 2\pi$ are a and b, where $a<b$. State, in terms of a and b, the range of values of x for which $\mathrm{f}(x)>\mathrm{g}(x)$.	[1]

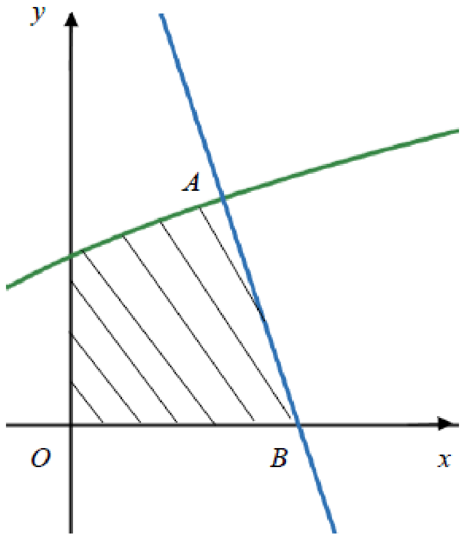
10	**(a)**	Period of $\mathrm{f}(x) = 720°$ *or* 4π	
	(b)	Smallest value of $\mathrm{f}(x) = 0$	
	(c)	Smallest value of $\mathrm{g}(x) = 4$	
	(d)		
	(e)	$a < x < b$	

11	The diagram shows a rectangular wooden plank *ABCD*, 5 m by 2 m, leaning against a vertical wall. *AD* makes an acute angle θ with the horizontal ground. *E* is a point on the vertical wall such that *EB* is parallel to the ground and *F* is a point on the ground such that *AF* is perpendicular to *FD*. The point *B* is *h* m vertically above the ground. 		
	(a)	Show that $h = 5\sin\theta + 2\cos\theta$.	[2]
	(b)	Express *h* in the form $R\sin(\theta+\alpha)$, where $R > 0$, and $0° < \alpha < 90°$.	[3]
	(c)	Find the greatest possible value of *h* and the value of θ at which it occurs.	[2]
	(d)	Find the value of θ for which $h = \sqrt{15}$ m.	[2]

11	(a)	In ΔABE, $\angle EAB = \theta$, $AB = 2\,\text{cm}$, $EA = 2\cos\theta$ In ΔAFD, $\angle ADF = \theta$, $AD = 5\,\text{cm}$, $AF = 5\sin\theta$ $h = AF + EA = 5\sin\theta + 2\cos\theta$ (showed)	
	(b)	$h = 5\sin\theta + 2\cos\theta$ $R = \sqrt{5^2 + 2^2} = \sqrt{29}$ $\alpha = \tan^{-1}\dfrac{2}{5} = 21.801°$ $h = \sqrt{29}\sin(\theta + 21.8°)$	
	(c)	$h = \sqrt{29}\sin(\theta + 21.8°)$ Greatest possible value of $h = \sqrt{29}$ $\sin(\theta + 21.8°) = 1$ $\theta + 21.8° = 90°$ $\theta = 68.2°$	
	(d)	$\sqrt{29}\sin(\theta + 21.8014°) = \sqrt{15}$ $\sin(\theta + 21.8014°) = \sqrt{\dfrac{15}{29}}$ $\theta + 21.8014° = 45.98805°$ $\theta = 24.2°$	

12	**(a)**	By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$.	[1]
	(b)	Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions.	[5]
	(c)	Hence, find $\displaystyle\int \frac{4x^3+5x^2+x-1}{x^2(x+1)}\mathrm{d}x$.	[3]

12	(a)	$\begin{array}{r} 4 \\ x^3+x^2 \overline{\smash{)}4x^3+5x^2+x-1} \\ \underline{4x^3+4x^2} \\ x^2+x-1 \end{array}$	
	(b)	$\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}=4+\dfrac{x^2+x-1}{x^2(x+1)}$ $\dfrac{x^2+x-1}{x^2(x+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+1}$ $x^2+x-1=Ax(x+1)+B(x+1)+Cx^2$ When $x=0$, $B=-1$ When $x=-1$, $C=-1$ When $x=1$, $2A-2-1=1$ $A=2$ $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}=4+\dfrac{2}{x}-\dfrac{1}{x^2}-\dfrac{1}{x+1}$	
	(c)	$\displaystyle\int \frac{4x^3+5x^2+x-1}{x^2(x+1)}\mathrm{d}x$ $=\displaystyle\int 4+\frac{2}{x}-\frac{1}{x^2}-\frac{1}{x+1}\,dx$ $=4x+2\ln x+\dfrac{1}{x}-\ln(x+1)+c$	

13	The diagram shows part of the curve $y = \sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B.  		
	(a)	Find the equation of the normal to the curve at A.	[5]
	(b)	Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis.	[5]

13	(a)	$y=\sqrt{2x+5}$ Coordinates of $A=(2,\,3)$ $\frac{dy}{dx}=\frac{1}{2}(2x+5)^{-\frac{1}{2}}(2)=\frac{1}{\sqrt{2x+5}}$ At A, $\frac{dy}{dx}=\frac{1}{3}$ Equation of normal AB is $y-3=-3(x-2)$ $y=-3x+9$	
	(b)	At $B, y=0$ $-3x+9=0$ $x=3$ $B=(3,\,0)$ Area of shaded region $=\frac{1}{2}(3-2)(3)+\int_0^2\left(\sqrt{2x+5}\right)dx$ $=1.5+\left[\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}\times 2}\right]_0^2$ $=1.5+\left[\left(\frac{9^{\frac{3}{2}}}{3}\right)-\left(\frac{5^{\frac{3}{2}}}{3}\right)\right]$ $=1.5+(5.27322)$ $=6.77\ units^2$	