ANDERSON SERANGOON JUNIOR COLLEGE

H2 MATHEMATICS **9758**

JC1 Promo Paper (100 marks)

7 Oct 2025

3 hours

Additional Material(s): List of Formulae (MF 27)

CANDIDATE NAME

CLASS

READ THESE INSTRUCTIONS FIRST

Write your name and class in the boxes above.
Please write clearly and use capital letters.
Write in dark blue or black pen. HB pencil may be used for graphs and diagrams only.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions and write your answers in this booklet.
Do not tear out any part of this booklet.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

All work must be handed in at the end of the examination. If you have used any additional paper, please insert them inside this booklet.
The number of marks is given in brackets [ ] at the end of each question or part question.

Question number	Marks
1	
2	
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10	
11	
Total	

This document consists of 6 printed pages and 2 blank pages.

1 (a) Find $\int \left(\operatorname{cosec} 2x \cot 2x - \frac{1}{\sqrt{9-4x^2}} \right) \mathrm{d}x$. [2]

(b) Find the exact value of $\int_0^3 \left| x^2 - x \right| \mathrm{d}x$. Show your workings clearly. [3]

2 A curve has equation $2x^2 + 3xy + y^2 = 12$.

(a) Find the gradient of the normal to the curve at the point $(1, 2)$. Hence, determine the equation of the normal at this point. [4]

(b) Find the exact area of the triangle enclosed by the normal obtained in part **(a)** and the axes. [2]

3 Using an algebraic method, find the range of values of x for which

$$3 + \frac{10}{x-1} \leq \frac{1}{x+2}.$$ [4]

Hence find the range of values of x for which

$$3 + \frac{10}{\mathrm{e}^{-x}-1} \leq \frac{1}{\mathrm{e}^{-x}+2}.$$

Show your workings clearly. [2]

4 A curve C has equation $y = \frac{ax^2 + bx + c}{2x + d}$, where a, b, c and d are real constants.

(a) (i) It is given that curve C has two asymptotes. One of the asymptotes is the line $y = 2 - 2x$ and the other asymptote intersects this line at the point $(1, 0)$. Find the value of d. [1]

(ii) If the curve passes through the point $(0, -2.5)$, find the values of a, b and c. [3]

(b) Taking $a = 4$, $b = -4$, $c = 2$ and $d = -1$, state a sequence of transformations that will transform the curve C on to the curve with equation $y = x + \frac{1}{x}$. [3]

5 The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2x+a}{x+b}, \quad x \in \mathbb{R}, x \neq -b \text{ and } a \neq 2b,$$

where b is a real constant.

(a) (i) Find $\mathrm{f}^{-1}(x)$ in terms of a and b. State the domain of f^{-1}. [2]

The function f is such that $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ for all x in the domain of f.

(ii) Find the possible values of a and b. [2]

(b) The function g is defined by

$$\mathrm{g}: x \mapsto 7 + \sqrt{x-3}, \qquad x \in \mathbb{R},\ x \geq 3.$$

Using the values $a = -1$ and $b = 1$,

(i) Show that fg exists. [2]

(ii) Find the value of fg(7). [1]

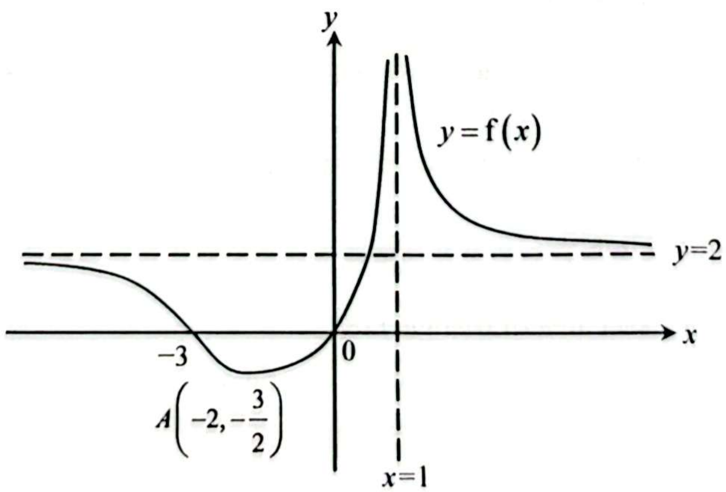
(iii) Find the exact range of fg. [2]

6 **(a)** The diagram shows the graph of $y = \mathrm{f}(x)$ with asymptotes $x = 1$ and $y = 2$, and a minimum point at $A\left(-2, -\frac{3}{2}\right)$. The curve cuts the x-axis at $(0,0)$ and $(-3,0)$. On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

(ii) $y = \mathrm{f}'(x)$, [3]

stating, in each case, the equations of asymptotes, the exact coordinates of any axial intercepts, turning points and points corresponding to A where possible.



6 **(b)** The function g is defined by

$$\mathrm{g}(x)=\begin{cases}2\cos\left(\dfrac{\pi x}{4}\right)+2 & \text{for } 0\le x<4,\\ x-2 & \text{for } 4\le x<6,\end{cases}$$

and $\mathrm{g}(x)=\mathrm{g}(x+6)$ for $x\in\mathbb{R}$.

Sketch the graph of $y=\mathrm{g}(x)$ for $-6\le x<4$, showing clearly the end points values. Find the exact value of g(2025). [4]

7 **(a)** The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by

$$S_n=2n^2-n+\ln\left(3^n\right),\ n\ge 1.$$

Show that the sequence is an arithmetic progression. [4]

(b) On 1 January 2025 Mr *A* puts \$1000 into a savings account. On the first day of each subsequent month, he puts another \$200 into the account. The saving account pays an interest of 0.1% per month, so that on the last day of each month the amount in the account on that day is increased by 0.1%.

(i) Show that the amount in Mr *A*'s account on the last day of the nth month is given by $201201\left(1.001^{n-1}\right)-200200$. [3]

(ii) Hence find the month and year in which the value of Mr *A*'s account first became greater than \$6500. [3]

8 Referred to the origin *O*, the points *A*, *B* and C are such that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OB}=\mathbf{b}$ and $\overrightarrow{OC}=\mathbf{c}$. The point *P* lies on *AB* produced such that $AP:BP=5:2$.

(a) Find $\overrightarrow{OP}$ in terms of **a** and **b**. [1]

(b) Given that $\mathbf{c}=\dfrac{7}{2}\mathbf{b}-\dfrac{5}{2}\mathbf{a}$, prove that the points *A*, *C* and *P* are collinear. [2]

It is known that **b** is a unit vector, the magnitude of **a** is $2\sqrt{3}$ and the angle in radian between **a** and **b** is $\dfrac{5}{6}\pi$.

(c) Find the exact area of triangle *OBC*. [3]

(d) Find the exact length of projection of **c** on **a**. [4]

9 **(a)** Write down the constants A and B such that, for all values of x,

$1-x=A(2x-4)+B$. Hence find $\int \frac{1-x}{x^2-4x+1}\,dx$. [4]

(b)(i) Find $\frac{d}{dx}\left(e^{\frac{1}{x}}\right)$. [1]

(ii) Hence, evaluate $\int_{\frac{1}{2}}^{1} \frac{1}{x^3}e^{\frac{1}{x}}\,dx$, giving your answer in terms of e. [3]

(c) By using the substitution $x=3\sec\theta$ for $0\le\theta\le\pi$, find

$$\int_3^6 \frac{\sqrt{x^2-9}}{x}\,dx,$$

leaving your answer in the form $a\sqrt{b}+c\pi$, where a, b, c are constants to be found. [5]

10 The plane p_1 has equation $x+y+z=3$ while the line l_1 passes through the origin and is parallel to $-2\mathbf{i}+3\mathbf{j}-2\mathbf{k}$. It is given that the point A has position vector $\mathbf{i}$.

(a) Find the acute angle, in degrees, between l_1 and p_1. [2]

(b) Find the exact coordinates of the foot of perpendicular from the point A to p_1. [4]

(c) Point B is a point on the line l_1. It is given that the shortest distance from point B to the plane p_1 is $\sqrt{27}$ units, find the possible coordinates of B. [3]

The plane p_2 has equation $3x+4y+\alpha z=7+\alpha,\ \alpha\in\mathbb{R}$.

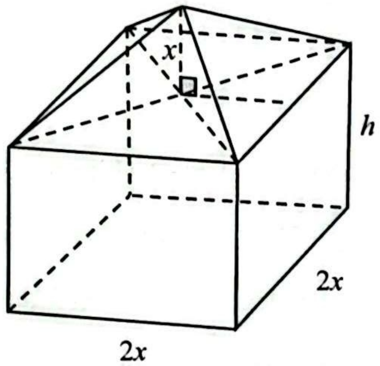
(d) It is given that the point C with coordinates $(1,1,1)$ lies on p_1. Show that point C also lies on p_2. Hence find a vector equation of the line l_2, the line of intersection between p_1 and p_2, in terms of α. [3]

11 The town council in a HDB housing estate is planning to construct a shed to store gardening equipment for a community garden.

The shed is made up of three parts.

- The roof is modelled by the four identical triangular surfaces of a right pyramid. This right pyramid has a square base with sides $2x$ metres and a height of x metres.
- The four vertical walls are modelled by rectangles, each of sides $2x$ metres by h metres.
- The floor is modelled by a square of sides $2x$ metres.

The three parts are joined together as shown in the diagram below. The shed is made of material of negligible thickness.



Due to budgeting, the material used to build this shed must be exactly 72 m^2 of treated plywood (including the flooring). To support the needs of the garden, the total volume of the shed must be at least 36 m^3.

(a) By finding the height h in terms of x, show that the volume V of the shed is given by $V = 36x - \left(\frac{2}{3} + 2\sqrt{2}\right)x^3$. [5]

(b) Find the exact value of x that maximises the volume of the shed and determine whether the maximum volume meets the needs of the garden. [5]

(c) It was decided that $x = 1.8$ m for the actual construct of the shed. Due to wear and tear, a hole was formed at the top of the roof. During a heavy downpour, rainwater flows into the shed at a rate of $1\ \text{m}^3\text{min}^{-1}$. Assuming that there is no leakage, determine the time taken to flood the shed up to the base of the pyramid-shaped roof. [2]

[Volume of right pyramid = $= \frac{1}{3} \times (\text{area of base}) \times (\text{height of pyramid})$]

End of Paper

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