Year	2025	**Level & Stream**	4E5N
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P1

Qns	Working
1	$4x^2 - 6xy - 6y = 4$ ----(1) $y - x = 4$ ------(2) Sub (2) into (1): $4x^2 - 6x(4+x) - 6(4+x) = 4$ $4x^2 - 24x - 6x^2 - 24 - 6x = 4$ $2x^2 + 30x + 28 = 0$ $x^2 + 15x + 14 = 0$ $(x+1)(x+14) = 0$ $x = -14$ or $x = -1$ $y = -10$ or $y = 3$
2	$9\sin x = -2\sec x$ $9\sin x = -\dfrac{2}{\cos x}$ $\sin x\cos x = -\dfrac{2}{9}$ $2\sin x\cos x = -\dfrac{4}{9}$ $\sin 2x = -\dfrac{4}{9}$ Basic angle = $\sin^{-1}\left(\dfrac{4}{9}\right)$ $= 0.46055$ $2x = -0.46055,\ -(\pi - 0.46055)$ $= -0.46055$ or -2.68104 $x = -0.230$ or -1.34

Qns	Working
3a	Principal range of $\cos^{-1}x$ is $0 \le \cos^{-1}x \le \pi$, hence principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$.
3b	L.H.S: $\cot A + \tan A = \frac{1}{\tan A} + \tan A$ $= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$ $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \sec A \operatorname{cosec} A = \text{RHS (shown)}$ **OR** RHS: $\sec A \operatorname{cosec} A = \frac{1}{\cos A} \times \frac{1}{\sin A}$ $\frac{1}{\cos A \sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$ $= = \frac{\sin^2 A}{\cos A \sin A} + \frac{\cos^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \tan A + \cot A = \text{LHS (shown)}$

Qns	Working
4	By long division, $\frac{2x^3+4}{x(x+2)^2}=\frac{2x^3+4}{x^3+4x^2+4x}$ $=2+\frac{-8x^2-8x+4}{x(x+2)^2}$ $\frac{-8x^2-8x+4}{x(x+2)^2}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $-8x^2-8x+4=A(x+2)^2+Bx(x+2)+Cx$ Let $x=0$: $4=4A$ $A=1$ Let $x=-2$: $-12=-2C$ $C=6$ Let $x=1$: $-8-8+4=9+3B+6$ $B=-9$ $\therefore \frac{2x^3+4}{x(x+2)^2}=2+\frac{1}{x}-\frac{9}{x+2}+\frac{6}{(x+2)^2}$
5a	$f'(x)=\frac{(x^2+3)-2x(x+a)}{(x^2+3)^2}$ $=\frac{-x^2-2ax+3}{(x^2+3)^2}$
5b	$\frac{-x^2-2ax+3}{(x^2+3)^2}>0,\text{ for } b<x<1$ Since $(x^2+3)^2>0$, $-x^2-2ax+3>0$ $x^2+2ax-3<0$ Given that $b<x<1$ $(x-b)(x-1)<0$ $x^2-bx-x+b<0$ By comparing: $b=-3$ $2a=-b-1$ $2a=3-1$ $a=1$

Qns	Working
6	$y=(k-6)x^2-8x+k$ Discriminant $=(-8)^2-4(k-6)(k)$ $=64-4k^2+24k$ For curve that does not intersect the x-axis, Discriminant < 0 $k^2-6k-16>0$ $(k+2)(k-8)>0$ $k<-2$ or $k>8$ The curve has a minimum point: $k-6>0$ $k>6$ $\therefore k>8$
7a 7b	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]=\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}$ $=2\mathrm{e}^{-x}-x\mathrm{e}^{-x}$ $\int 2\mathrm{e}^{-x}-x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $\int 2\mathrm{e}^{-x}\mathrm{d}x-\int x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $-\int x\mathrm{e}^{-x}\mathrm{d}x=-\int 2\mathrm{e}^{-x}\mathrm{d}x+(x-1)\mathrm{e}^{-x}+c$ $\int x\mathrm{e}^{-x}\mathrm{d}x=-2\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}+c$ $=-2\mathrm{e}^{-x}-x\mathrm{e}^{-x}+\mathrm{e}^{-x}+c$ $=-\mathrm{e}^{-x}-x\mathrm{e}^{-x}+c$

Qns	Working
8a	$\frac{dr}{dt} = -\frac{2}{(t+1)^2}$ $r = -\int \frac{2}{(t+1)^2} dt$ $= -\int 2(t+1)^{-2} dt$ $= 2(t+1)^{-1} + c$ $r = \frac{2}{t+1} + c$ At $r = 4$ and $t = 0$, $4 = 2 + c$ $c = 2$ $r = \frac{2}{t+1} + 2$
8b	$A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ At $r = 2.6$, $\frac{dA}{dr} = 5.2\pi$, $2.6 = \frac{2}{t+1} + 2$ $0.6 = \frac{2}{t+1}$ $t = \frac{7}{3}$ $\frac{dr}{dt} = -\frac{2}{\left(\frac{7}{3}+1\right)^2} = -0.18$ $\frac{dA}{dt} = 5.2\pi \times -0.18$ $\frac{dA}{dt} = -0.936\pi \text{ cm}^2/\text{min}$

Qns	Working
9a	$y = kx + 6$ ---- (1) $2x^2 - xy = 3$ -----(2) Sub (1) into (2): $2x^2 - x(kx+6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2-k)x^2 - 6x - 3 = 0$ Discriminant $= (-6)^2 - 4(2-k)(-3)$ $= 60 - 12k$ For tangent, $60 - 12k = 0$ $k = 5$
9b	$\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$ $4x - 2x\sqrt{2} + \sqrt{2} = 3x\sqrt{2} - 1$ $\sqrt{2} + 1 = 3x\sqrt{2} + 2x\sqrt{2} - 4x$ $\sqrt{2} + 1 = x\left(5\sqrt{2} - 4\right)$ $x = \dfrac{\sqrt{2}+1}{5\sqrt{2}-4} \times \dfrac{5\sqrt{2}+4}{5\sqrt{2}+4}$ $= \dfrac{5(2) + 4\sqrt{2} + 5\sqrt{2} + 4}{\left(5\sqrt{2}\right)^2 - 4^2}$ $= \dfrac{9\sqrt{2} + 14}{50 - 16}$ $= \dfrac{9}{34}\sqrt{2} + \dfrac{7}{17}$

Qns	Working
10a	Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2}[3+4-2]$ $= 2.5 \text{ units}^2$
10b	$m_{PR} = \frac{3-0}{2-1}$ $= 3$ $m_{QS} = -\frac{1}{3}$ Equation of QS: $y-2=-\frac{1}{3}(x-0)$ $y=-\frac{1}{3}x+2 --- (1)$ $x-y=5 --- (2)$ Sub (1) into (2): $x+\frac{1}{3}x-2=5$ $x=\frac{21}{4}$ $y=\frac{1}{4}$ coordinates of $S = \left(\frac{21}{4}, \frac{1}{4}\right)$

Qns	Working
11a	$2^x + 4^{x-1} = 24$ $2^x + \left(4^x\right)\left(4^{-1}\right) = 24$ $2^x + \frac{2^{2x}}{4} = 24$ Let $y = 2^x$, $y + \frac{y^2}{4} = 24$ $4y + y^2 = 96$ $y^2 + 4y - 96 = 0$ $(y-8)(y+12) = 0$ $y = 8$ or $y = -12$ $2^x = 8$ or $2^x = -12$ $\left(\text{Rejected since } 2^x > 0\right)$ $2^x = 2^3$ $x = 3$
11b	$\log_{\sqrt{3}}(x-1) = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3\sqrt{3}} = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = \log_3(x+5)$ $2\log_3(x-1) = \log_3(x+5)$ $\log_3(x-1)^2 = \log_3(x+5)$ $(x-1)^2 = x+5$ $x^2 - 2x + 1 - x - 5 = 0$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $x + 1 = 0$ $x = -1$ [N.A. as $\log_{\sqrt{3}}(x-1)$ will be undefined] or $x - 4 = 0$ $x = 4$

Qns	Working
12a	$4x-4y=3$ $y=x-\frac{3}{4}$ m of normal $=1$ m of tangent $=-1$ At $x=\frac{1}{2}$ and $\frac{dy}{dx}=-1$, $6\left(\frac{1}{2}\right)-\frac{1}{k\left(\frac{1}{2}\right)^3}=-1$ $3-\frac{8}{k}=-1$ $k=2$ (shown)
12b	At stat points, $6x-\frac{1}{2x^3}=0$ $6x=\frac{1}{2x^3}$ $x^4=\frac{1}{12}$ $x=\pm\sqrt[4]{\frac{1}{12}}$ $x=-0.537$ or 0.537
12c	$\frac{d^2y}{dx^2}=6+\frac{3}{2x^4}$ $6>0$ $\frac{3}{2x^4}>0$ $\therefore \frac{d^2y}{dx^2}=6+\frac{3}{2x^4}>0$ Since $\frac{d^2y}{dx^2}=6+\frac{3}{2x^4}>0$ the gradient has no turning point.

Qns	Working
13a	$C = 1.2n^2 - 14.4n + 53.7$ At $n = 0$, $C = \$53.7$ thousands
13b	$C = 1.2n^2 - 14.4n + 53.7$ $= 1.2\left[n^2 - 12n + 44.75\right]$ $= 1.2\left[(n-6)^2 - 6^2 + 44.75\right]$ $= 1.2(n-6)^2 + 10.5$
13c	For 600 pairs of running shoes produced, the minimum cost of 10.5 thousand dollars will be incurred.
13d	$1.2(n-6)^2 + 10.5 = 50$ $1.2(n-6)^2 = 39.5$ $(n-6)^2 = \frac{39.5}{1.2}$ $n - 6 = \pm 5.7373$ $n = 11.7373$ or 0.2627 Maximum number of pairs of running shoes for which the cost is at most 50 thousand dollars = 11.7373 = 11.7 hundreds

Qns	Working
14a	$m_{AB} = 1$ Equation of AB: $y+2 = x+4$ $y = x+2 ----(1)$ $y = -x+4 ----(2)$ $(1) = (2): x+2 = -x+4$ $2x = 2$ $x = 1$ $y = 3$ Coordinates of A = (1, 3)
14b	Radius $= \sqrt{(1+4)^2 + (3+2)^2}$ $= \sqrt{50}$ units Equation of circle: $(x+4)^2 + (y+2)^2 = 50$
14c	Let the coordinates of the other end of the diameter passes through A = $C(x, y)$. Since AC is the diameter, B is the midpoint of AC. $\left(\frac{x+1}{2}, \frac{y+3}{2}\right) = (-4, -2)$ $\frac{x+1}{2} = -4, \quad \frac{y+3}{2} = -2$ $x = -9, \ y = -7$ Equation of another tangent which is parallel to $y = -x+4$: $y+7 = -1(x+9)$ $y = -x-16$