Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

[4]

[Turn over

$$y - x = 4,$$
$$4x^2 - 6xy - 6y = 4.$$

2 Solve the equation $9 \sin x = -2 \sec x$ for $-\pi \leq x \leq 0$. [4]

For Examiner's Use

3 **(a)** Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

[Turn over

For Examiner's Use

(b) Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$. [4]

For Examiner's Use

4 Express $\dfrac{2x^3+4}{x\left(x+2\right)^2}$ in partial fractions. [5]

[Turn over

For Examiner's Use

5 The function f is given by $\mathrm{f}(x) = \dfrac{x+a}{x^2+3}$.

(a) Find $\mathrm{f}'(x)$. [2]

It is given that f increases for $b < x < 1$.

(b) Find the values of a and b. [4]

For Examiner's Use

6 Find the set of values of the constant k for which the curve $y = (k-6)x^2 - 8x + k$ does not intersect the x-axis and has a minimum point. [6]

7 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]$. [2]

(b) Hence find $\int x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

For Examiner's Use

8 The radius of a circle, r cm, decreases at a rate of $\dfrac{2}{(t+1)^2}$ cm per minute.

(a) Given that the initial radius is 4 cm, find an expression for r in terms of t. [3]

(b) Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]

9 **(a)** Find the value of the constant k such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [3]

(b) Solve $\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers. [4]

For Examiner's Use

10 An isosceles triangle PQR has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
$PQ = QR$.

(a) Find the area of the triangle PQR. [2]

(b) If S is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of S. [5]

11 **(a)** Solve the equation $2^{x} + 4^{x-1} = 24$. [4]

[Turn over

For Examiner's Use

(b) Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$. [4]

For Examiner's Use

12 The gradient at any point on a curve y is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

(a) Show that $k = 2$. [2]

(b) Hence find the x-coordinates of the stationary points. [2]

[Turn over

For Examiner's Use

(c) Find $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and explain whether the gradient $6x - \frac{1}{kx^3}$ has a turning point. [4]

For Examiner's Use

13 The cost in thousands of dollars, C, of producing n hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

(a) Find the initial cost of production. [1]

(b) Express C in the form $a(x + h)^2 + k$ where a, h and k are constants to be determined. [3]

(c) Explain the meaning of the values of h and k in the formula as expressed in **part (b)**. [1]

(d) Find the maximum number of pairs of running shoes for which the cost will be [3]

[Turn over

For Examiner's Use

50 thousand dollars.

For Examiner's Use

14 The line $y = -x + 4$ is a tangent to a circle at the point A and the centre of the circle is at $B(-4, -2)$.

(a) Find coordinates of A. [3]

(b) Find the radius and the equation of the circle. [3]

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(c) Find the equation of another tangent to the circle which is parallel to $y = -x + 4$. [4]

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