

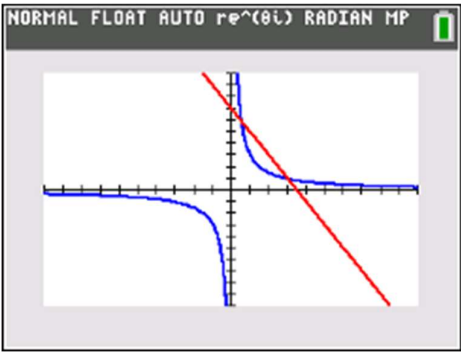


Raffles Institution
H2 Mathematics (9758)
Solution for 2018 A-Level Paper 1

Question 1

No.	Suggested Solution	Remarks for Student
(i)	$y = \frac{\ln x}{x}$ $\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$	
(ii)	$\begin{aligned}\int_1^e \frac{\ln x}{x^2} dx &= \int_1^e \frac{\ln x - 1 + 1}{x^2} dx \\&= \int_1^e \frac{1}{x^2} dx - \int_1^e \frac{1 - \ln x}{x^2} dx \\&= \left[-\frac{1}{x} \right]_1^e - \left[\frac{\ln x}{x} \right]_1^e \\&= 1 - \frac{1}{e} - \frac{1}{e} \\&= 1 - \frac{2}{e}\end{aligned}$	

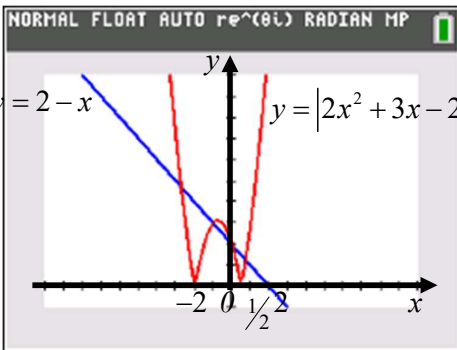
Question 2

No.	Suggested Solution	Remarks for Student
(i)	 Sub $y = \frac{3}{x}$ into $y + 2x = 7$ $\Rightarrow \frac{3}{x} + 2x = 7$ $\Rightarrow 2x^2 - 7x + 3 = 0$ $(2x - 1)(x - 3) =$ $x = \frac{1}{2}, \quad x = 3$ $y = 6, \quad y = 1$	
(ii)	Required volume $= \pi \int_{\frac{1}{2}}^3 (7 - 2x)^2 - \frac{9}{x^2} dx$ $= \pi \left[-\frac{1}{6}(7 - 2x)^3 \right]_{\frac{1}{2}}^3 + \pi \left[\frac{9}{x} \right]_{\frac{1}{2}}^3$ $= \pi \left[-\frac{1}{6} + 36 + 3 - 18 \right]$ $= \frac{125\pi}{6}$	Note that question said exact and did not mention answer in terms of π

Question 3

No.	Suggested Solution	Remarks for Student
(i)	$x \frac{dy}{dx} = 2y - 6 \quad \dots(1)$ $y = ux^2 \quad \dots(2)$ $\frac{dy}{dx} = x^2 \frac{du}{dx} + 2xu \quad \dots(3)$ Sub (2) and (3) into (1): $x \left(x^2 \frac{du}{dx} + 2xu \right) = 2ux^2 - 6$ $x^3 \frac{du}{dx} = -6$ $\frac{du}{dx} = -\frac{6}{x^3}$	
(ii)	$u = \frac{3}{x^2} + C$ $\frac{y}{x^2} = \frac{3}{x^2} + C \Rightarrow y = 3 + Cx^2$ $y = 2$ when $x = 1$: $2 = 3 + C \Rightarrow C = -1$ $\therefore y = 3 - x^2$	

Question 4

No.	Suggested Solution	Remarks for Student
(i)	$ 2x^2 + 3x - 2 = 2 - x$ $(2x^2 + 3x - 2)^2 = (2 - x)^2$ $(2x^2 + 3x - 2)^2 - (2 - x)^2 = 0$ $(2x^2 + 3x - 2 + 2 - x)(2x^2 + 3x - 2 - 2 + x) = 0$ $(2x^2 + 2x)(2x^2 + 4x - 4) = 0$ $x(x+1) = 0 \text{ or } x^2 + 2x - 2 = 0$ $x = 0, x = -1, x = \frac{-2 \pm \sqrt{12}}{2}$ $= -1 \pm \sqrt{3}$	
(ii)	 $y = 2 - x$ $y = 2x^2 + 3x - 2$ $-1 - \sqrt{3} < x < -1 \text{ or } 0 < x < -1 + \sqrt{3}$	

Question 5

No.	Suggested Solution	Remarks for Student
	$f : x \mapsto \frac{x+a}{x+b} \quad \text{for } x \in \mathbb{R}, x \neq -b, a \neq -1$ $g : x \mapsto x \quad \text{for } x \in \mathbb{R}$ Given $ff = g$, $\frac{\frac{x+a}{x+b} + a}{\frac{x+a}{x+b} + b} = x$ $\frac{x+a+ax+ab}{x+a+bx+b^2} = x$ $\frac{x(1+a)+a(1+b)}{x(1+b)+a+b^2} = x$ $x(1+a)+a(1+b) = x^2(1+b)+x(a+b^2)$ Comparing coefficients of x^2: $b = -1$ $ff(x) = x$ $f^{-1}(x) = f(x) = \frac{x+a}{x-1}$	$ff = g$, we assume question is just referring to the rule.

Question 6

No.	Suggested Solution	Remarks for Student
	Given $a \times 3b = 2a \times c$	
(i)	$a \times (3b - 2c) = a \times 3b - a \times 2c$ $= 2a \times c - a \times 2c$ $= 0$ $\therefore a \parallel (3b - 2c)$ Thus, $3b - 2c = \lambda a$, where λ is a constant.	
(ii)	$b \cdot c = b c \cos 60^\circ = 2$ $(3b - 2c) \cdot (3b - 2c) = \lambda a \cdot \lambda a$ $9 b ^2 - 12b \cdot c + 4 c ^2 = \lambda^2 a ^2$ $144 - 24 + 4 = \lambda^2 \Rightarrow \lambda = \pm\sqrt{124} = \pm 2\sqrt{31}$	

Question 7

No.	Suggested Solution	Remarks for Student
(i)	$\frac{x^2 - 4y^2}{x^2 + xy^2} = \frac{1}{2}$ $2x^2 - 8y^2 = x^2 + xy^2$ Differentiate with respect to x : $4x - 16y \frac{dy}{dx} = 2x + 2xy \frac{dy}{dx} + y^2$ $\frac{dy}{dx}(2xy + 16y) = 2x - y^2$ $\frac{dy}{dx} = \frac{2x - y^2}{2xy + 16y}$	
(ii)	When $x = 1$, $\frac{1 - 4y^2}{1 + y^2} = \frac{1}{2}$ $2 - 8y^2 = 1 + y^2$ $y = \pm \frac{1}{3}$ Let P and Q be $\left(1, \frac{1}{3}\right)$ and $\left(1, -\frac{1}{3}\right)$ respectively, At $P\left(1, \frac{1}{3}\right)$: $\frac{dy}{dx} = \frac{2 - \frac{1}{9}}{\frac{2}{3} + \frac{16}{3}} = \frac{17}{54}$ Tangent: $y - \frac{1}{3} = \frac{17}{54}(x - 1)$ $\frac{17}{54}x - y = -\frac{1}{54}$ At $Q\left(1, -\frac{1}{3}\right)$: $\frac{dy}{dx} = \frac{2 - \frac{1}{9}}{-\frac{2}{3} - \frac{16}{3}} = -\frac{17}{54}$ Tangent: $y + \frac{1}{3} = -\frac{17}{54}(x - 1)$ $\frac{17}{54}x + y = -\frac{1}{54}$ Solving coordinates of N is $\left(-\frac{1}{17}, 0\right)$	

Question 8

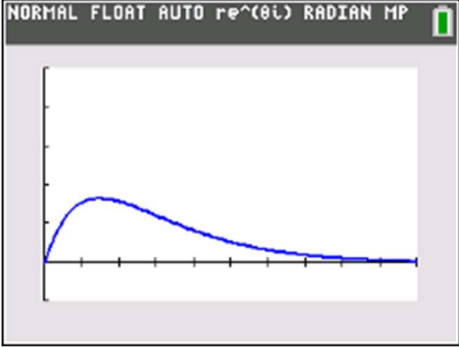
No.	Suggested Solution	Remarks for Student
(i)	$u_1 = 5 \text{ and } u_2 = 15 \Rightarrow 15 = 2(5) + A$ $A = 5$ $u_3 = 2u_2 + 2A = 40$	
(ii)	$u_n = a(2^n) + bn + c$ $u_1 = 5: 2a + b + c = 5 \quad \dots(1)$ $u_2 = 15: 4a + 2b + c = 15 \quad \dots(2)$ $u_3 = 40: 8a + 3b + c = 40 \quad \dots(3)$ Using GC: $a = \frac{15}{2}, b = -5, c = -5$	
(iii)	$\sum_{r=1}^n u_r = \sum_{r=1}^n \left(\frac{15}{2}(2^r) - 5r - 5 \right)$ $= \frac{15}{2} \left(\frac{2(2^n - 1)}{2 - 1} \right) - 5 \left(\frac{1}{2}n(n+1) \right) - 5n$ $= 15(2^n - 1) - \frac{5}{2}n(n+1) - 5n$	

Question 9

No.	Suggested Solution	Remarks for Student
	$x = 2\theta - \sin 2\theta, y = 2\sin^2 \theta$ for $0 \leq \theta \leq \pi$	
(i)	$\frac{dx}{d\theta} = 2 - 2\cos 2\theta, \quad \frac{dy}{d\theta} = 4\sin \theta \cos \theta$ $\frac{dy}{dx} = \frac{4\sin \theta \cos \theta}{2 - 2\cos 2\theta}$ $= \frac{4\sin \theta \cos \theta}{2 - 2(1 - 2\sin^2 \theta)}$ $= \frac{4\sin \theta \cos \theta}{4\sin^2 \theta}$ $= \frac{\cos \theta}{\sin \theta}$ $= \cot \theta$	Ok to have 0 and π included though cot is not defined.
(ii)	Point where $\theta = \alpha$ is $(2\alpha - \sin 2\alpha, 2\sin^2 \alpha)$ Equation of normal: $y - 2\sin^2 \alpha = -\tan \alpha (x - 2\alpha + \sin 2\alpha)$ At point A, $y = 0$, $-2\sin^2 \alpha = -\frac{\sin \alpha}{\cos \alpha} (x - 2\alpha + \sin 2\alpha)$ $2\sin \alpha \cos \alpha = x - 2\alpha + \sin 2\alpha$ $\therefore x = 2\alpha$, that is, $k = 2$	
(iii)	Total length of C = $\int_0^\pi \sqrt{(2 - 2\cos 2\theta)^2 + 4\sin^2 2\theta} \, d\theta$ $= \int_0^\pi \sqrt{(4 - 8\cos 2\theta + 4\cos^2 2\theta) + 4\sin^2 2\theta} \, d\theta$ $= \int_0^\pi \sqrt{(8 - 8\cos 2\theta)} \, d\theta$ $= \int_0^\pi \sqrt{8 - 8(1 - 2\sin^2 \theta)} \, d\theta$ $= \int_0^\pi 4\sin \theta \, d\theta$ $= -4[\cos \theta]_0^\pi$ $= -4(-1 - 1)$ $= 8$	

Question 10

No.	Suggested Solution	Remarks for Student
	$L \frac{dI}{dt} + RI + \frac{q}{C} = V, \text{ where } I = \frac{dq}{dt}$	
(i)	Differentiate with respect to t, $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} \frac{dq}{dt} = \frac{dV}{dt}$ $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = \frac{dV}{dt}, \text{ since } I = \frac{dq}{dt}$ Therefore, $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = 0, \text{ when } \frac{dV}{dt} = 0 \text{ that is, } V \text{ is a constant.}$	
(ii)	$I = Ate^{-\frac{Rt}{2L}}$ $\frac{dI}{dt} = Ae^{-\frac{Rt}{2L}} + Ate^{-\frac{Rt}{2L}} \left(-\frac{R}{2L} \right) = Ae^{-\frac{Rt}{2L}} - \frac{AR}{2L} te^{-\frac{Rt}{2L}}$ $\frac{d^2 I}{dt^2} = -\frac{AR}{2L} e^{-\frac{Rt}{2L}} - \frac{AR}{2L} e^{-\frac{Rt}{2L}} + \frac{AR^2}{4L^2} te^{-\frac{Rt}{2L}} = -\frac{AR}{L} e^{-\frac{Rt}{2L}} + \frac{AR^2}{4L^2} te^{-\frac{Rt}{2L}}$ Sub into $L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = 0$, $-ARe^{-\frac{Rt}{2L}} + \frac{AR^2}{4L} te^{-\frac{Rt}{2L}} + ARe^{-\frac{Rt}{2L}} - \frac{AR^2}{2L} te^{-\frac{Rt}{2L}} + \frac{A}{C} te^{-\frac{Rt}{2L}} = 0$ $-ARe^{-\frac{Rt}{2L}} + ARe^{-\frac{Rt}{2L}} + \frac{AR^2}{4L} te^{-\frac{Rt}{2L}} - \frac{AR^2}{2L} te^{-\frac{Rt}{2L}} + \frac{A}{C} te^{-\frac{Rt}{2L}} = 0$ $-\frac{R^2}{4L} + \frac{1}{C} = 0 \Rightarrow C = \frac{4L}{R^2}$	
(iii)	Note that the given values satisfy (ii), when $R = 4$ and $L = 3$, from (ii), $C = \frac{4L}{R^2} = \frac{3}{4}$	By right, we should sub $R = 4$, $L = 3$ and $C = 0.75$ into I and check that it still a solution of the DE.

	$I = Ate^{-\frac{2}{3}t}$ $\frac{dI}{dt} = 0 \Rightarrow Ae^{-\frac{2}{3}t} - \frac{2A}{3}te^{-\frac{2}{3}t} = 0$ $\Rightarrow t = \frac{3}{2}$ $I = \frac{3A}{2e}$ Sub into $L \frac{d^2I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = 0$, $3 \frac{d^2I}{dt^2} + 4(0) + \frac{1}{0.75} \frac{3A}{2e} = 0$ $\frac{d^2I}{dt^2} = -\frac{A}{1.5e} < 0$	
(iv)	 Remember to label the max point $\left(\frac{3}{2}, \frac{3A}{2e}\right)$	

Question 11

No.	Suggested Solution	Remarks for Student
(i) (a)	$100\left(1 + \frac{0.2}{100}\right)^{12} = 102.4266 \approx 102.43$	Given answer in exam report is 102.42
(b)	$100(1.002) + 100(1.002)^2 + \dots + 100(1.002)^{12}$ $= 100\left(\frac{1.002(1.002^{12} - 1)}{1.002 - 1}\right)$ $= 1215.714974$ ≈ 1215.71	
(c)	$S_n = 100\left(\frac{1.002(1.002^n - 1)}{1.002 - 1}\right) > 3000$ $S_n = 50100(1.002^n - 1) > 3000$ Using GC, $n = 29, S_n = 2988.6$ $n = 30, S_n = 3094.8$ note that $2988.6 + 100 = 3088.6$ So, total will first exceed \$3000 on the first day of June of 2018.	
(ii) (a)	$100 + 12b$	
(b)	$(100 + b) + (100 + 2b) + \dots + (100 + 24b) = 2800$ $2400 + b(1 + 2 + 3 + \dots + 24) = 2800$ $b\left[\frac{24}{2}(1 + 24)\right] = 400$ $b = \frac{4}{3} \approx 1.33$	
(iii)	$100\left(\frac{1.01(1.01^{60} - 1)}{1.01 - 1}\right) = 6000 + b\left[\frac{60}{2}(1 + 60)\right]$ $10100(1.01^{60} - 1) = 6000 + 1830b$ $b = 1.2287632 \approx 1.23$	