

CANDIDATE
NAME

CLASS

ADMIN NUMBER

2025 Preliminary Examination

Pre-University 3

MATHEMATICS

9758/02

Paper 2

August/September 2025

3 hours

Additional Materials: Printed Answer Booklet
 List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

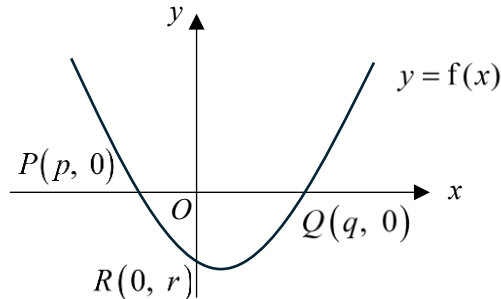
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **8** printed pages.

Section A: Pure Mathematics [40 marks]

- 1** The diagram shows the curve with equation $y = f(x)$. The curve passes through the points $P(p, 0)$, $Q(q, 0)$ and $R(0, r)$.



- (a) The curve $y = f(x)$ is transformed onto the curve with equation $y = f(2x-1)$. Find the coordinates of the points on the graph of $y = f(2x-1)$ which correspond to the points P , Q and R on curve $y = f(x)$. [3]
- (b) It is given that $I = \int_p^q f(x) dx$. Find, in terms of I , the area of the finite region bounded by
- (i) the curve with equation $y = -f(x)$ and the x -axis, [1]
 - (ii) the curve with equation $y = \frac{1}{2}f(x+3)$ and the x -axis. [1]
- (c) Find the value of $\int_0^q f'(x) dx$, justifying your answer. [1]

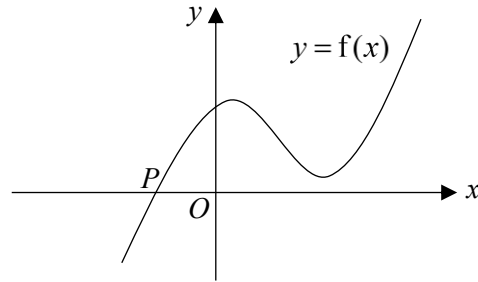
- 2** A curve C has parametric equations $x = \sqrt{t}$, $y = t^2 + 4$, where $0 \leq t \leq 16$.
- (i) Find the equation of the tangent to C at the point P with parameter p . [3]
 - (ii) Given that the tangent in part (i) passes through the point $(0,1)$, find the exact coordinates of the point P . [3]
 - (iii) Sketch the curve C . [2]

- 3 (a) The complex numbers z and w satisfy the simultaneous equations

$$z + 2z^* + w = 6 + i \quad \text{and} \quad iz - w = 3.$$

Find z and w , leaving your answers in the form $a + bi$, where a and b are real numbers. [4]

- (b) The diagram shows the curve with equation $y = f(x)$, where $f(x)$ is given by $x^3 + mx^2 + nx + 5$, and m and n are real constants. The curve crosses the x -axis at the point P .



One of the roots of the equation $f(x) = 0$ is $2 + i$. Determine the coordinates of the point P . [4]

- 4 A sequence is defined by $T_{n+2} = T_{n+1} + T_n$ with $T_1 = 3$, $T_2 = 1$ for $n \geq 1$. Another sequence is defined by $r_n = \frac{T_{n+1}}{T_n}$ for $n \geq 1$.

- (a) Find T_4 . [1]
- (b) Show that $r_{n+1} = 1 + \frac{1}{r_n}$ for all for $n \geq 1$. [2]
- (c) It is given that r_n converges to a limit L . Find the exact value of L . [3]

A third sequence u_n for $n \geq 1$, is an arithmetic sequence with first term 5 and common difference 2.

- (d) Find the least value of n such that $\sum_{k=1}^n u_k > 930$. [2]

5 The function g is given by $g : x \mapsto \frac{x-3}{x-1}$, for $x \in \mathbb{R}$, $x \neq 1$.

- (i) Sketch the graph of $y = g(x)$, stating clearly the coordinates of any points of intersection with the axes and the equation of any asymptotes. [3]
- (ii) Explain if g^{-1} exists. [1]
- (iii) Find $g^{-1}(x)$ and state the domain of g^{-1} . [3]

A function f is said to be self-inverse if $f(x) = f^{-1}(x)$ for all $x \in \mathbb{R}$ in the domain of f .

- (iv) Show that g is self-inverse. [1]
- (v) Without the use of a calculator, solve the equation $g^2(x) = -g^{-1}(x)$. [2]

Section B: Probability and Statistics [60 marks]

- 6 The waiting times for customers to get their pastries from a popular bakery is modelled by a normal distribution with mean μ minutes and standard deviation σ minutes. On average, 10% of customers wait for more than 25 minutes and the same percentage of customers wait for less than 5 minutes.

- (i) Find the values of μ and σ , leaving your answers to the nearest minute. [3]
- (ii) Explain if the normal distribution is a suitable model for the waiting times for customers to get pastries from this bakery. [1]

- 7 A deck of 12 cards consists of 2 gold cards and 10 silver cards. The cards are identical other than their colours. In a game, three cards are drawn from the deck of cards at random without replacement. The number of gold cards drawn is denoted by G .

- (i) Explain the significance of stating that the cards are identical other than their colours. [1]
- (ii) Show that $P(G=2) = \frac{1}{22}$ and find the probabilities of all other possible combinations of the cards drawn. [4]

A gold card is worth 4 points while a silver card is worth 1 point. The score X is the sum of the points of each of the three drawn cards.

- (iii) Using your answers in part (ii), find the probability distribution of X . [2]
- (iv) If the player of the game wins \$10 x for a score of x points, what is the expected winnings of the player? [1]

- 8 For events A and B , it is given that $P(A) = \frac{11}{20}$, $P(A \cup B) = \frac{4}{5}$ and $P(B|A) = \frac{4}{11}$.

- (i) Find
 - (a) $P(B)$, [3]
 - (b) $P(A \cap B')$. [1]
- (ii) Determine if the events A and B' are mutually exclusive. [1]

A third event C is such that $P(C) = 0.6$ and B and C are independent.

- (iii) Find $P(B \cap C)$. [1]
- (iv) Find the range of values of $P(A \cap B' \cap C)$. [2]

- 9 In a particular stage of a computer game, a player is given a first set of 20 tasks,
- If he completes fewer than 13 tasks, he has to retry the stage.
 - If he completes more than 16 tasks, he progresses directly to the next stage.
 - Otherwise, he is given a second set of 10 tasks.
 - If he completes more than 7 of these 10 tasks, he progresses to the next stage.
 - Otherwise, he has to retry the stage.

It is known that the player's performances in the two sets of tasks are independent.

Let X and Y be the number of tasks completed in the first and second sets of tasks respectively.

- (i) State two assumptions needed for X to be well-modelled by a binomial distribution. [2]

Assume now that X has the distribution $B(20, 0.7)$ and Y has the distribution $B(10, 0.8)$,

- (ii) Find the probability that the player is given a second set of tasks. [2]
- (iii) Using a tree diagram or otherwise, find the probability that the player
- (a) progresses to the next stage, [3]
 - (b) completes exactly 15 tasks in the first set of tasks given he does not progress to the next stage. [2]

- 10 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters.

In a fruit shop, the masses, in kg, of a certain type of apple and pear are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Apple	0.2	0.05
Pear	0.3	0.08

- (i) An apple and a pear are chosen at random.
- (a) Find the probability that the mass of the apple is more than 0.25 kg and the mass of the pear is less than 0.25 kg. [1]
 - (b) Find the probability that the mass of the apple is within 0.075 kg of the mass of the pear. [4]

Apples are sold at \$5 per kg and pears at \$7 per kg.

- (ii) Find the probability that the total price of 10 randomly chosen apples and 5 randomly chosen pears exceeds \$22. [4]

- 11** Cardiovascular fitness is commonly measured using VO_2 max, the maximum amount of oxygen the body can use during intense exercise. The higher the value of VO_2 max, the fitter the person is. In a study on cardiovascular fitness, the resting heart rate x , measured in beats/min and the VO_2 max y , measured in ml/kg/min, of 8 randomly selected people are given in the table below.

x	78	75	65	64	62	56	53	48
y	36	41	47	48	48	56	58	63

- (i) Draw a scatter diagram for these values, labelling the axes. [1]
- (ii) Find the product moment correlation coefficient between y and x and comment on its value in the context of the data. [2]
- (iii) Use an appropriate regression line to estimate the VO_2 max of a person with a resting heart rate of 60 beats/min, leaving your answer to the nearest integer. Comment on the reliability of your estimate. [3]
- (iv) Use a specific example to explain why extrapolating using your regression line in part (iii) to predict the VO_2 max of people with very high or very low resting heart rate is not advisable. [1]

Due to an instrumentation issue, all the resting heart rate values are to be increased by v beats/min.

- (v) Find the value of the product moment correlation coefficient, and the gradient and y -intercept of the regression line of y on x after these adjustments, leaving your answer in terms of v where applicable. [3]

- 12** Supernova School is a school which specialises in sports. The mean reaction time of the students in this school is 250 milliseconds. To reduce the reaction times of students, the teachers implemented a reflex training programme. After its completion, 50 students were randomly selected from the school, and their reaction times x (in milliseconds) were recorded as follows:

$$\sum(x - 200) = 2159.62 \quad \sum(x - 200)^2 = 112081.09$$

- (i) Calculate the unbiased estimates of the population mean and variance of the reaction time of students. [2]
- (ii) Test, at 1% level of significance, whether the programme was effective. You should state your hypotheses and define any symbols you use. [5]
- (iii) Explain why the test in part (ii) may not be appropriate if only a sample of 10 students was taken. [1]

In another school, Galaxy School, the reaction times of its students follow a normal distribution with mean μ_0 milliseconds and variance 153 milliseconds². The school also implemented the reflex training programme with its students.

- (iv) A random sample of 40 students is taken and the mean reaction time of this sample is computed to be 241 milliseconds. A test carried out at 5% level of significance indicated that the reflex training programme was not successful. Find the range of values of μ_0 . [4]