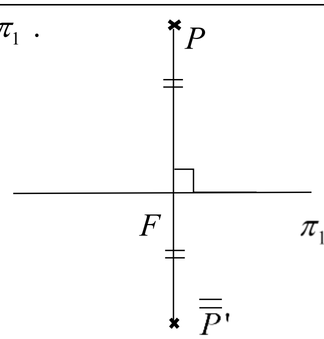
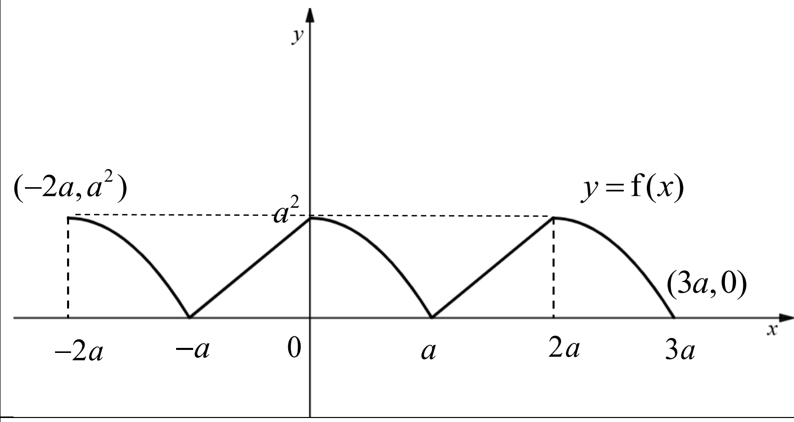
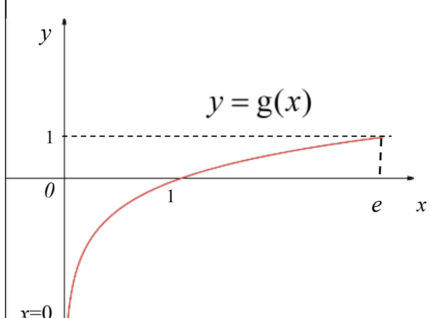
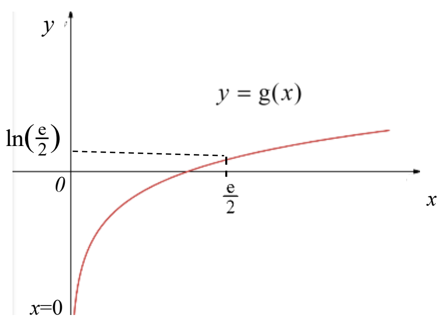
No	Solution	
1	$\mathrm{f}(x)=ax^3+bx^2+cx+d$ $\mathrm{f}(2)=8a+4b+2c+d=10 \quad -(1)$ $\mathrm{f}(-2)=-8a+4b-2c+d=-2 \quad -(2)$ $\mathrm{f}'(x)=3ax^2+2bx+c$ $\mathrm{f}'(-1)=3a-2b+c=0 \quad -(3)$ $\int_{-2}^{2}\mathrm{f}(x)\ \mathrm{d}x=16 \Rightarrow \int_{-2}^{2}(ax^3+bx^2+cx+d)\ \mathrm{d}x=16$ $\left[\frac{1}{4}ax^4+\frac{1}{3}bx^3+\frac{1}{2}cx^2+dx\right]_{-2}^{2}=16$ $\frac{16}{3}b+4d=16 \quad -(4)$ Using GC to solve (1), (2), (3), (4) $a=3, b=0, c=-9, d=4$	
2	Let r and h be the radius and the height of the cylinder respectively. External Surface Area $p=2\pi r^2+2\pi rh$ $\Rightarrow h=\frac{p-2\pi r^2}{2\pi r}=\frac{p}{2\pi r}-r$ Volume $V=\pi r^2 h$ $=\pi r^2\left(\frac{p}{2\pi r}-r\right)$ $=\frac{pr}{2}-\pi r^3$ $\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{p}{2}-3\pi r^2$ For stationary values, $\frac{\mathrm{d}V}{\mathrm{d}r}=\frac{p}{2}-3\pi r^2=0 \quad \Rightarrow \quad r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$ $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=-6\pi r$ When $r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$, $\frac{\mathrm{d}^2V}{\mathrm{d}r^2}=-6\pi\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}<0$ $\therefore$ V is a maximum when $r=\left(\frac{p}{6\pi}\right)^{\frac{1}{2}}$ cm.	

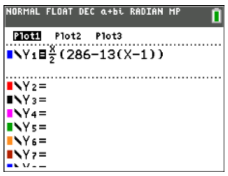
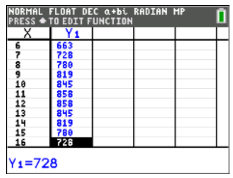
3(i)	$l_1:\ \mathbf{r}=\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix},\ l_2:\mathbf{r}=\begin{pmatrix}0\\5\\-8\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\3\end{pmatrix},\ \lambda\ \&\ \mu\in\mathbb{R}$ $\begin{pmatrix}-2\\1\\2\end{pmatrix}\neq k\begin{pmatrix}2\\-2\\3\end{pmatrix}$, where k is a scalar l_1 is not parallel to l_2. Equate the two lines: $\begin{pmatrix}1\\2\\0\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\2\end{pmatrix}=\begin{pmatrix}0\\5\\-8\end{pmatrix}+\mu\begin{pmatrix}2\\-2\\3\end{pmatrix}$ $2\lambda+2\mu=1$ (1) $-\lambda-2\mu=-3$ (2) $-2\lambda+3\mu=8$ (3) No solution from GC OR Solution for equations (1) and (2) $\lambda=-2$, $\mu=\frac{5}{2}$. check that it satisfies (3). LHS $=-2(2)+3\left(\frac{5}{2}\right)=\frac{7}{2}$ RHS=8 Hence (3) is not satisfied. Since l_1 and l_2 are non-parallel and non-intersecting, they are skew lines.	
(ii)	Let F be foot of perpendicular from P to π_1 . Equation of line through P and F: $\mathbf{r}=\begin{pmatrix}1\\5\\0\end{pmatrix}+\alpha\begin{pmatrix}0\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\5+\alpha\\-\alpha\end{pmatrix},\ \alpha\in\mathbb{R}$ Substitute into equation of plane π_1 : $\begin{pmatrix}1\\5+\lambda\\-\lambda\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}=2$ 	

	$5+2\alpha=2\Rightarrow\ \alpha=-\frac{3}{2}\quad \therefore \overrightarrow{OF}=\begin{pmatrix}1\\ \frac{7}{2}\\ \frac{3}{2}\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}2\\7\\3\end{pmatrix}-\begin{pmatrix}1\\5\\0\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$. **Alternative method to find foot of perpendicular**: $Q(0,2,0)$ is a point on plane π $\overrightarrow{PF}=\left[\overrightarrow{PQ}\bullet\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\frac{1}{\sqrt{1+1}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $=\frac{1}{2}\left[\begin{pmatrix}-1\\-3\\0\end{pmatrix}\bullet\begin{pmatrix}0\\1\\-1\end{pmatrix}\right]\begin{pmatrix}0\\1\\-1\end{pmatrix}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}$ $\overrightarrow{OF}=\frac{-3}{2}\begin{pmatrix}0\\1\\-1\end{pmatrix}+\begin{pmatrix}1\\5\\-0\end{pmatrix}=\begin{pmatrix}1\\ \frac{7}{2}\\ \frac{3}{2}\end{pmatrix}$ $\overrightarrow{OP'}=2\overrightarrow{OF}-\overrightarrow{OP}=\begin{pmatrix}2\\7\\3\end{pmatrix}-\begin{pmatrix}1\\5\\0\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ Coordinates of P' are $(1,2,3)$.	
(iii)	Let $A(1,2,0)$ that lies on l_1. $\overrightarrow{AP}=\overrightarrow{OP}-\overrightarrow{OA}=\begin{pmatrix}1\\5\\0\end{pmatrix}-\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\3\\0\end{pmatrix}$ Normal of plane π_2:	

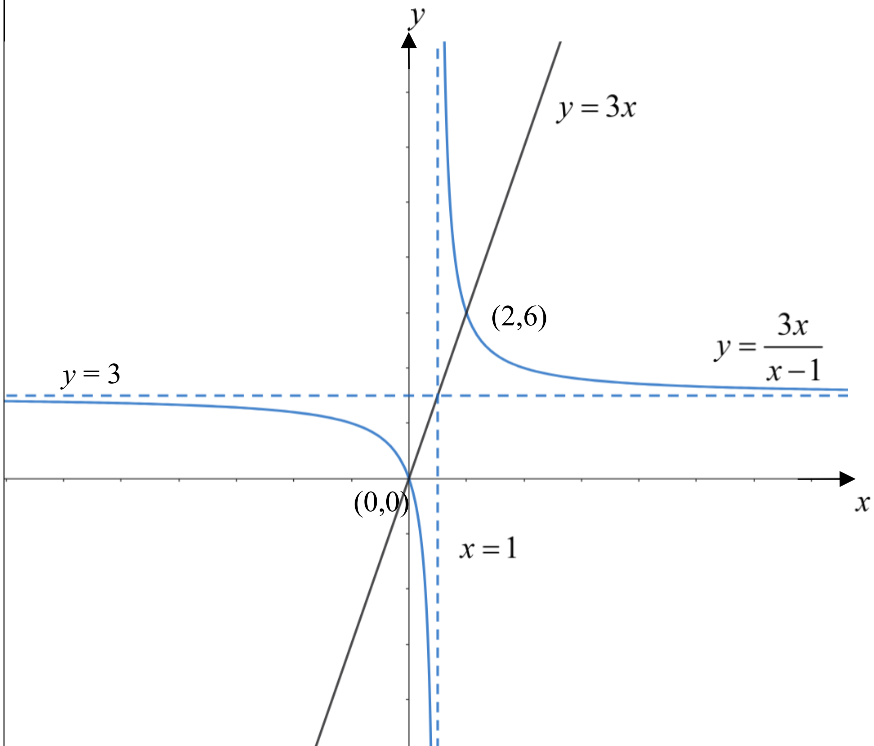
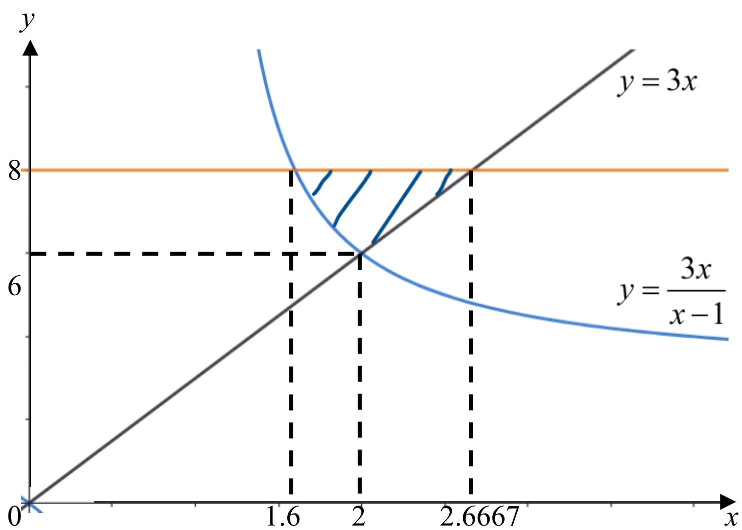
	$\underset{\sim}{n} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ -6 \end{pmatrix} = -6 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ Equation of plane containing P and l_1 : $\underset{\sim}{r} . \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = 1$ Cartesian equation of π_2 is $x + z = 1$	
4(a) (i)		
(ii)	$\mathrm{f}\left(\frac{101a}{2}\right) = \mathrm{f}\left(50a + \frac{a}{2}\right) = \mathrm{f}\left(\frac{a}{2}\right) = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}$	
(b)(i)	 For hg to exist, $\mathrm{R_g} \subseteq \mathrm{D_h}$ $\mathrm{R_g} = (-\infty, 1]$ $\mathrm{D_h} = (0, 4)$ Since $\mathrm{R_g} \not\subset \mathrm{D_h}$, hg does not exist.	
(ii)	Since h is a quadratic function with maximum point at $x = 1$, by symmetry, when $\mathrm{h}(x) = \frac{\mathrm{e}}{2}$, $x = 0$ and $x = 2$. The other value of x is 2. Since h^{-1} exists, h is one to one and $\mathrm{R_h} = \mathrm{D_{h^{-1}}} = \left(0, \frac{\mathrm{e}}{2}\right]$ (given) From the graph of h, restricted $\mathrm{D_h} = [2, 4)$	

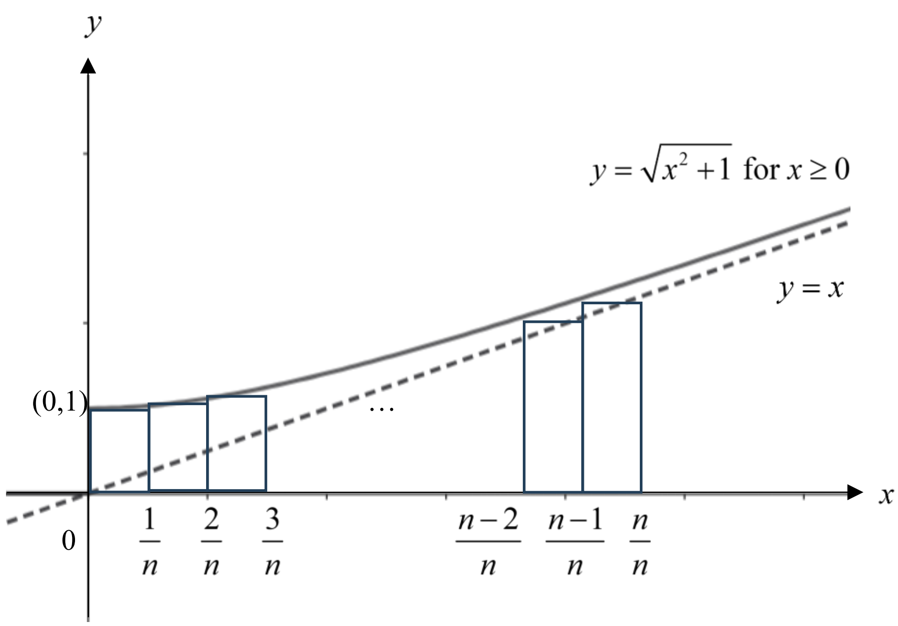
	Alternative Method 1 to find x: Since h is a quadratic function, its roots are at -2 and 4. Let $\mathrm{h}(x)=a(x+2)(x-4)$ $\mathrm{h}(0)=\frac{\mathrm{e}}{2}$ $\frac{\mathrm{e}}{2}=-8a \Rightarrow a=-\frac{\mathrm{e}}{16}$ $\mathrm{h}(x)=-\frac{\mathrm{e}}{16}(x+2)(x-4)$ Solving $\mathrm{h}\left(x\right)=\frac{\mathrm{e}}{2}$, $x=0$ or 2 The other value of x is 2. **Alternative Method 2 to find x:** Since maximum point at $x = 1$, Let $\mathrm{h}(x)=a\left(x-1\right)^2+b$ $\mathrm{h}(4)=0 \Rightarrow 9a+b=0---(1)$ $\mathrm{h}(0)=\frac{\mathrm{e}}{2} \Rightarrow a+b=\frac{\mathrm{e}}{2}---(2)$ $(1)-(2): \quad 8a=-\frac{\mathrm{e}}{2} \Rightarrow a=-\frac{\mathrm{e}}{16}$ $(2) \Rightarrow b=\frac{\mathrm{e}}{2}+\frac{\mathrm{e}}{16}=\frac{9\mathrm{e}}{16}$ $\mathrm{h}(x)=-\frac{\mathrm{e}}{16}\left(x-1\right)^2+\frac{9\mathrm{e}}{16}$ Solving $\mathrm{h}\left(x\right)=\frac{\mathrm{e}}{2}$, $x=0$ or 2 The other value of x is 2. **Alternative Method 3 to find x:** Let $\mathrm{h}(x)=ax^2+bx+c$ $\mathrm{h}(0)=\frac{\mathrm{e}}{2}, \quad c=\frac{\mathrm{e}}{2}$ $\mathrm{h}(4)=0$, $16a+4b+\frac{\mathrm{e}}{2}=0$ ----- (1) $\mathrm{h}'(1)=0$, $\mathrm{h}'(x)=2ax+b$ $2a+b=0$ ----- (2) Solving (1) & (2), (by GC or manually) $a=-\frac{\mathrm{e}}{16}, b=\frac{\mathrm{e}}{8}$ Or $a=-0.1698926143,\ b=0.3397852286$	

	$\therefore \mathrm{h}(x) = -\frac{\mathrm{e}}{16}x^2 + \frac{\mathrm{e}}{8}x + \frac{\mathrm{e}}{2}$ Solving $\mathrm{h}(x) = \frac{\mathrm{e}}{2}$, $x = 0$ or 2 The other value of x is 2.	
(iii)	To find range of gh: $\mathrm{D_h} \xrightarrow{\mathrm{h}} \mathrm{R_h} \xrightarrow{\mathrm{g}} \mathrm{R_{gh}}$ $[2,4) \xrightarrow{\mathrm{h}} \left(0, \frac{\mathrm{e}}{2}\right] \xrightarrow{\mathrm{g}} (-\infty, 1-\ln 2]$  $\therefore\ \mathrm{R_{gh}} = \left(-\infty, \ln\frac{\mathrm{e}}{2}\right]$ or $(-\infty, 1-\ln 2]$ or $(-\infty, 0.307]$	
5(a)	$\frac{a+8d}{a+2d} = \frac{a+10d}{a+8d}$ $(a+8d)^2 = (a+2d)(a+10d)$ $a^2 + 16ad + 64d^2 = a^2 + 12ad + 20d^2$ $4ad + 44d^2 = 0$ $4d(a+11d) = 0$ $d = 0$ or $a + 11d = 0$ (reject since terms of AP, GP are distinct) $d = -\frac{1}{11}a$	

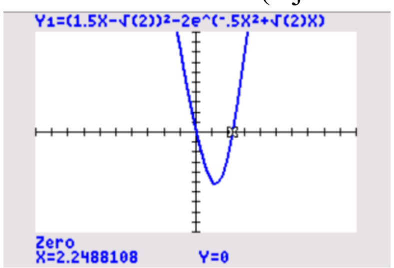
(b)	$S_1 = a = 143$ $d = -\frac{1}{11}(143) = -13$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $= \frac{n}{2}\left[286 - 13(n-1)\right]$ From GC, largest $S_n = 858$  	
(c)	$r = \frac{a+8d}{a+2d} = \frac{-11d+8d}{-11d+2d} = \frac{-3d}{-9d} = \frac{1}{3}$ Since $\lvert r \rvert = \frac{1}{3} < 1$,the geometric series is convergent.	
(d)	Let u_n be the nth term of the geometric progression. $u_{n+1} + u_{n+2} + \ldots$ $= S_\infty - S_n$ $= \frac{b}{1-r} - \frac{b(1-r^n)}{1-r}$ $= \frac{br^n}{1-r}$ $= \frac{b\left(\frac{1}{3}\right)^n}{1-\frac{1}{3}}$ $= \frac{3}{2}\left(\frac{1}{3}\right)^n b$ $= \frac{b}{2}\left(\frac{1}{3}\right)^{n-1} \le \frac{1}{2}b \quad \text{since } \left(\frac{1}{3}\right)^{n-1} \le 1$ Alternative:	

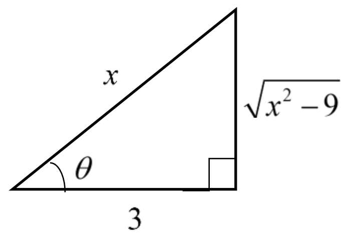
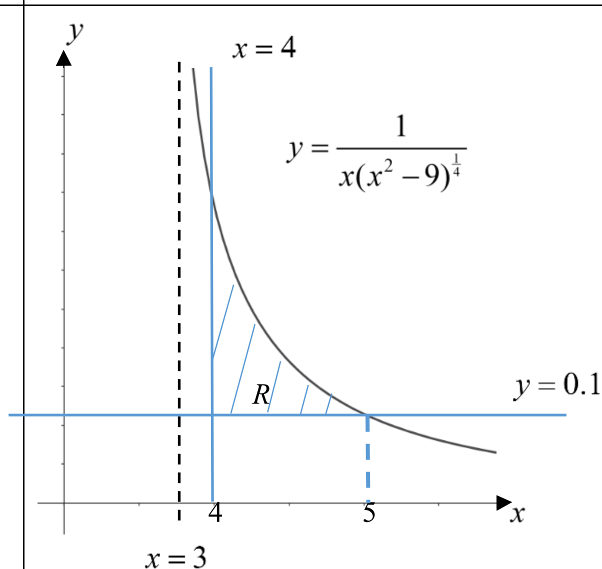
	$u_{n+1}+u_{n+2}+u_{n+3}+\ldots$ $=b\left(\frac{1}{3}\right)^{n}+b\left(\frac{1}{3}\right)^{n+1}+b\left(\frac{1}{3}\right)^{n+2}+\ldots$ $=\dfrac{b\left(\frac{1}{3}\right)^{n}}{1-\frac{1}{3}}$ $=\frac{3}{2}b\left(\frac{1}{3}\right)^{n}$ $=\frac{b}{2}\left(\frac{1}{3}\right)^{n-1}\leq\frac{b}{2}$ since $\left(\frac{1}{3}\right)^{n-1}\leq 1$	
6 (i)	1. Reflection of the graph in the x-axis followed by 2. Scaling of the resulting graph by a factor $\frac{1}{3}$ parallel to the x-axis.	
(ii)	From (i), $\mathrm{g}(x)=-\mathrm{f}(3x)$ $\therefore\ a=-1,\ b=3\ ,\ c=0,\ d=0.$	
(iii)	$\mathrm{g}'(x)=-3\mathrm{f}'(3x)$ $\mathrm{g}'(2)=-3\mathrm{f}'(6)$ Since $\mathrm{f}'(6)=4$ $\mathrm{g}'(2)=-3\mathrm{f}'(6)=-3(4)=-12$	

7(a) (i)		
(a) (ii)	 Area of region bounded by both curves and $y = 8$ $= \int_6^8 \left(\frac{y}{3} - \frac{y}{y-3} \right) \mathrm{d}y$ $= 1.13 \text{ units}^2 \text{ (3 s.f.)}$	

	Alternative Method: Area of region bounded by both curves and $y = 8$ $=(2.6667-1.6)(8)-\int_{1.6}^{2}\frac{3x}{x-1}\,\mathrm{d}x-\int_{2}^{2.6667}3x\,\mathrm{d}x$ $=8.5336-2.7325-4.6669$ $=1.13\text{ units}^2$ (3 s.f.)	
7(b) (i)	 If we split the area of the region enclosed by the curve $y=\mathrm{f}(x)=\sqrt{x^2+1}$, the x-axis and the lines $x = 0$ and $x = 1$ into n rectangles, each of width $\frac{1}{n}$ as shown, the sum of the area of the n rectangles is given by $A=\frac{1}{n}\times\mathrm{f}(0)+\frac{1}{n}\times\mathrm{f}\left(\frac{1}{n}\right)+\ldots+\frac{1}{n}\times\mathrm{f}\left(\frac{n-1}{n}\right)$ $=\frac{1}{n}\left\{\mathrm{f}(0)+\mathrm{f}\left(\frac{1}{n}\right)+\ldots+\mathrm{f}\left(\frac{n-1}{n}\right)\right\}$. $=\frac{1}{n}\left\{1+\sqrt{\frac{1}{n^2}+1}+\sqrt{\frac{4}{n^2}+1}+\ldots+\sqrt{\frac{(n-1)^2}{n^2}+1}\right\}$ As the number of rectangles increases, i.e. $n\to\infty$, the width of each rectangle decreases, then the sum of the area of the n	

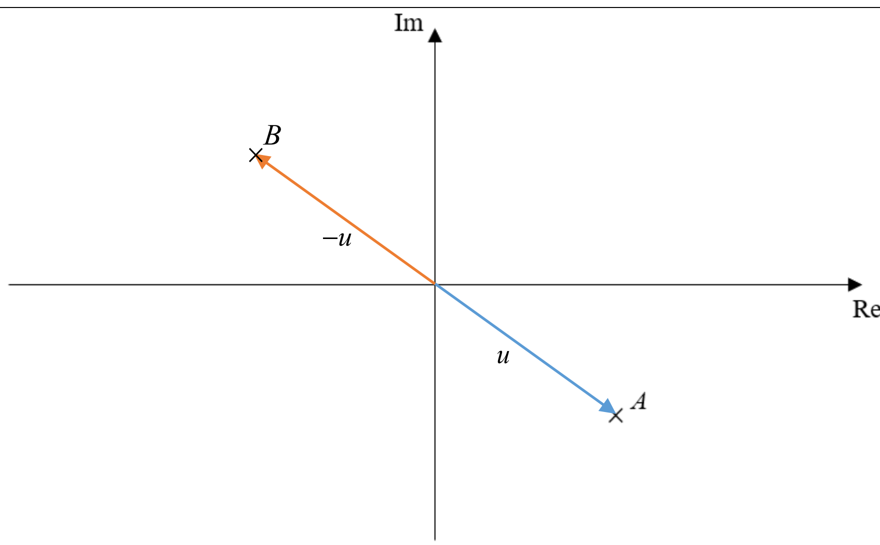
	rectangle approaches the area of the region enclosed by curve $y=\sqrt{x^2+1}$, the x-axis and the lines x = 0 and x = 1 which is given by $\int_0^1 \sqrt{x^2+1}\ \mathrm{d}x$. Hence $S=\lim\limits_{n\to\infty} A$ $=\lim\limits_{n\to\infty}\dfrac{1}{n}\left\{1+\sqrt{\dfrac{1}{n^2}+1}+\sqrt{\dfrac{4}{n^2}+1}+\ldots+\sqrt{\dfrac{(n-1)^2}{n^2}+1}\right\}$ $=\int_0^1 \mathrm{f}(x)\ \mathrm{d}x$ $=\int_0^1 \sqrt{1+x^2}\ \mathrm{d}x$	
(b)(ii)	Actual area under the graph $y=\sqrt{x^2+1}$ from $x=0$ to $x=1$ $\int_0^1 \sqrt{x^2+1}\ \mathrm{d}x=1.1478=1.15$ (3 s.f.)	
8(i)	$(x-y)^2=2\mathrm{e}^{xy}$ Differentiate with respect to x $2(x-y)\left(1-\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=2\mathrm{e}^{xy}\left(x\dfrac{\mathrm{d}y}{\mathrm{d}x}+y\right)$ $(x-y)-(x-y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=x\mathrm{e}^{xy}\dfrac{\mathrm{d}y}{\mathrm{d}x}+y\mathrm{e}^{xy}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{x-y-y\mathrm{e}^{xy}}{x-y+x\mathrm{e}^{xy}}$ (shown)	
(ii)	When $x=0$, $(0-y)^2=2\mathrm{e}^{(0)y}$ $y=\pm\sqrt{2}$ $y=\sqrt{2}\ (\because y>0)$ At (0, $\sqrt{2}$), $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{0-\sqrt{2}-\sqrt{2}\mathrm{e}^{0(\sqrt{2})}}{0-\sqrt{2}+(0)\mathrm{e}^{0(\sqrt{2})}}$ $=\dfrac{-2\sqrt{2}}{-\sqrt{2}}$ $=2$	

	Gradient of normal $=-\frac{1}{2}$ Equation of the normal to the curve at (0, $\sqrt{2}$) is $y-\sqrt{2}=-\frac{1}{2}(x-0)$ $y=-\frac{1}{2}x+\sqrt{2}$	
(iii)	Substitute $y=-\frac{1}{2}x+\sqrt{2}$ into $(x-y)^2=2e^{xy}$, $\left(x+\frac{1}{2}x-\sqrt{2}\right)^2=2e^{x\left(-\frac{1}{2}x+\sqrt{2}\right)}$ $\left(\frac{3}{2}x-\sqrt{2}\right)^2=2e^{-\frac{1}{2}x^2+\sqrt{2}x}$ Using G.C., $x=2.2488$ or $x=0$ (reject $\because$ it's point A)  when $x=2.2488,\ y=-\frac{1}{2}(2.2488)+\sqrt{2}=0.28980$ (5 s.f.) $\therefore B(2.25, 0.290)$	

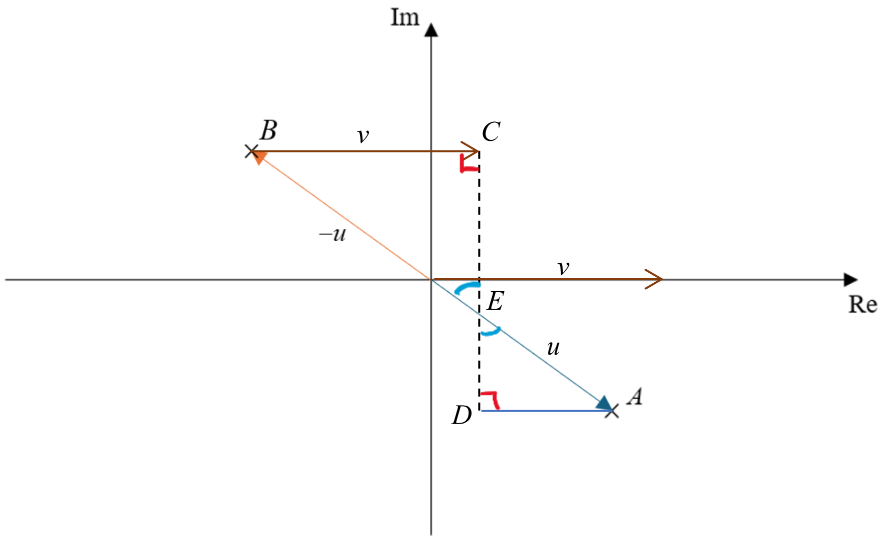
9(a)	$\displaystyle\int \frac{1}{x^2\sqrt{x^2-9}}\,\mathrm{d}x$ let $x=3\sec\theta$ $\displaystyle=\int \frac{1}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}\frac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta$ $\displaystyle\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\sec\theta\tan\theta$ $\displaystyle=\int \frac{1}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}(3\sec\theta\tan\theta)\,\mathrm{d}\theta$ $\displaystyle=\int \frac{1}{9\sec\theta\sqrt{\sec^2\theta-1}}(\tan\theta)\,\mathrm{d}\theta$ $\displaystyle=\int \frac{1}{9\sec\theta\sqrt{\tan^2\theta}}(\tan\theta)\,\mathrm{d}\theta$  $\displaystyle=\int \frac{1}{9\sec\theta}\,\mathrm{d}\theta$ $\displaystyle=\frac{1}{9}\int\cos\theta\,\mathrm{d}\theta$ $\displaystyle=\frac{1}{9}\sin\theta+C$ $x=3\sec\theta$ => $\cos\theta=\dfrac{3}{x}$ $\displaystyle=\frac{\sqrt{x^2-9}}{9x}+C$ where C is an arbitrary constant	
9(b)	 	

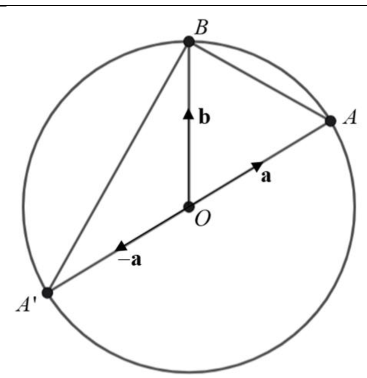
	Volume of solid $= \pi \int_4^5 y^2 \ \mathrm{d}x - \pi (0.1)^2 (1)$ $= \pi \int_4^5 \frac{1}{x^2 \sqrt{x^2-9}} \mathrm{d}x - \pi (0.1)^2 (1)$ $= \pi \left[\frac{\sqrt{x^2-9}}{9x} \right]_4^5 - \frac{\pi}{100}$ $= \pi \left(\frac{\sqrt{5^2-9}}{9\times 5} - \frac{\sqrt{4^2-9}}{9\times 4} \right) - \frac{\pi}{100}$ $= \pi \left(\frac{4}{45} - \frac{\sqrt{7}}{36} \right) - \frac{\pi}{100}$ $= \pi \left(\frac{71}{900} - \frac{\sqrt{7}}{36} \right)$ units^3	
10(a)	$\lvert z \rvert + 5w = 0$ (1) $\mathrm{i}z - 4w = -4 + 7\mathrm{i}$ (2) From (1): $w = -\frac{1}{5} \lvert z \rvert$ (3) Substitute (3) into (2): $\mathrm{i}z + \frac{4}{5} \lvert z \rvert = -4 + 7\mathrm{i}$ Let $z = x + y\mathrm{i}$, $\mathrm{i}(x + y\mathrm{i}) + \frac{4}{5}\sqrt{x^2+y^2} = -4 + 7\mathrm{i}$ $\left(-y + \frac{4}{5}\sqrt{x^2+y^2} \right) + x\mathrm{i} = -4 + 7\mathrm{i}$ Comparing imaginary parts: $x = 7$ Comparing real parts:	

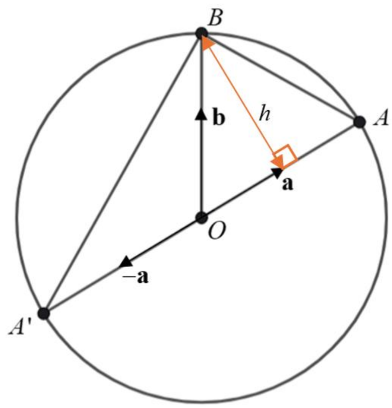
<table>
<tr><td></td><td>$-y+\frac{4}{5}\sqrt{7^2+y^2}=-4$
$5(y-4)=4\sqrt{49+y^2}$ (*)
$25(16-8y+y^2)=16(49+y^2)$
$9y^2-200y-384=0$
$(y-24)(9y+16)=0$
$y=24$ or $y=-\frac{16}{9}$
Note that $y=-\frac{16}{9}$ does not satisfy (*) and hence $y=24$

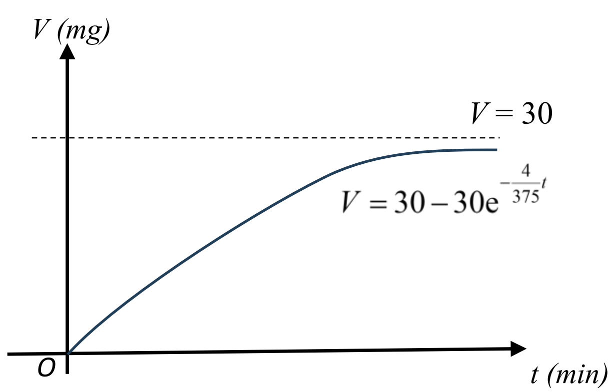
$\therefore\ z=7+24\mathrm{i}$
Substitute into (3):
$w=-\frac{1}{5}\sqrt{7^2+24^2}$
$\quad=-\frac{1}{5}\sqrt{625}$
$\therefore\ w=-5$</td><td></td></tr>
<tr><td>10(b)
(i)</td><td>
</td><td></td></tr>
<tr><td>10(b)
(ii)</td><td>As points C and D represent complex numbers that form a complex conjugate pair, therefore line segment CD must be vertical, and points C and D must be equidistant from the real axis.

Given that $\angle CDA=90^\circ$, and CD is vertical, thus we require line segment DA to be horizontal.</td><td></td></tr>
</table>

	Since A and B are also equidistant from the real axis, DA being horizontal implies that BC must also be horizontal. Since C represents the complex number $v - u$, the vector BC must represent the complex number v. Since BC is horizontal, v must be a real number. Hence, Im(v) = 0.  Alternatively, $u = a + ib$ $v = c + id$ $v - u = (c-a) + (d-b)i$ $(v-u)^* = (c-a) + (b-d)i$ $A(a,b)$ $C(c-a, d-b)$ $D(c-a, b-d)$ C and D share the same x coordinate. Therefore CD is vertical. $\angle CDA = 90^\circ$ implies that DA must be horizontal, hence A and D must share the same y-coordinate. $b - d = b \Rightarrow d = 0 \Rightarrow \text{Im}(v) = 0$	
	Let $\overrightarrow{OA} = \begin{pmatrix} a \\ b \end{pmatrix}$ where $u = a + b\text{i}$, where $a, b \in \mathbb{R}$. Let $\overrightarrow{OV} = \begin{pmatrix} c \\ d \end{pmatrix}$ where $v = c + d\text{i}$, where $c, d \in \mathbb{R}$.	

	Then $\overrightarrow{OC}=\overrightarrow{OV}-\overrightarrow{OA}=\begin{pmatrix}c\\d\end{pmatrix}-\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}c-a\\d-b\end{pmatrix}$. $\overrightarrow{OD}=\overrightarrow{OV^*}-\overrightarrow{OA^*}=\begin{pmatrix}c-a\\-(d-b)\end{pmatrix}=\begin{pmatrix}c-a\\b-d\end{pmatrix}$ Since $\angle CDA=90^\circ$, then $\overrightarrow{CD}\bullet\overrightarrow{AD}=0$ $\overrightarrow{CD}=\overrightarrow{OD}-\overrightarrow{OC}=\begin{pmatrix}c-a\\b-d\end{pmatrix}-\begin{pmatrix}c-a\\d-b\end{pmatrix}=\begin{pmatrix}0\\2b-2d\end{pmatrix}$ $\overrightarrow{DA}=\overrightarrow{OA}-\overrightarrow{OD}=\begin{pmatrix}a\\b\end{pmatrix}-\begin{pmatrix}c-a\\b-d\end{pmatrix}=\begin{pmatrix}2a-c\\d\end{pmatrix}$ $\overrightarrow{CD}\bullet\overrightarrow{DA}=0$ $\begin{pmatrix}0\\2b-2d\end{pmatrix}\bullet\begin{pmatrix}2a-c\\d\end{pmatrix}=0$ $2d(b-d)=0$ $d=0$ or $d=b$ If $d=b$, then $\overrightarrow{OC}=\begin{pmatrix}c-a\\0\end{pmatrix}$ and $\overrightarrow{OD}=\begin{pmatrix}c-a\\0\end{pmatrix}$ then *C* and *D* are the same point. Then $\angle CDA\neq 90^\circ$. Hence $d=0 \Rightarrow \text{Im}(v)=0$ (shown)	
11 (i)	 Since AA' is a straight line passing through the origin *O*, $\therefore \overrightarrow{OA'}=-\mathbf{a}$ $\overrightarrow{BA}=\overrightarrow{OA}-\overrightarrow{OB}$ $=\mathbf{a}-\mathbf{b}$ $\overrightarrow{BA'}=\overrightarrow{OA'}-\overrightarrow{OB}$ $=-\mathbf{a}-\mathbf{b}$	

	$\overrightarrow{BA}\cdot\overrightarrow{BA'} = (\mathbf{a}-\mathbf{b})\cdot(-\mathbf{a}-\mathbf{b})$ $= -\mathbf{a}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{b} + \mathbf{b}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{b}$ $= -\lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 \quad (\lvert\mathbf{a}\rvert = \lvert\mathbf{b}\rvert \because \text{ radius of circle})$ $= 0$ $\therefore \overrightarrow{BA} \perp \overrightarrow{BA'}$, hence $\measuredangle ABA' = 90^\circ$ (shown).	
(ii)	By Vector Product, $\Delta OAB = \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert$  From the above diagram, $\frac{1}{2}\times\lvert\overrightarrow{OA}\rvert\times h = \text{area of } \Delta OAB = \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert \qquad (1)$ Area of $\Delta ABA' = \frac{1}{2}\times AA'\times h$ $= \frac{1}{2}\times 2\lvert\overrightarrow{OA}\rvert\times h$ $= \left[\frac{1}{2}\times\lvert\overrightarrow{OA}\rvert\times h\right]\times 2$, substituting (1) $= \frac{1}{2}\lvert\mathbf{a}\times\mathbf{b}\rvert\times 2$ $= \lvert\mathbf{a}\times\mathbf{b}\rvert$ where $k = 1$	
12 (i)	$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V_{in}}{\mathrm{d}t} - \frac{\mathrm{d}V_{out}}{\mathrm{d}t}$ $= 0.32 - kV$	

<table>
<tr><td></td><td>
When $\dfrac{dV}{dt} = 0.12$ and $V = 18.75$ mg,
$0.12 = 0.32 - k(18.75)$
$k = \dfrac{0.2}{18.75} = \dfrac{4}{375}$
$\dfrac{dV}{dt} = 0.32 - \dfrac{4}{375}V = \dfrac{1}{375}(120-4V) = \dfrac{4}{375}(30-V)$ (shown)
</td><td></td></tr>
<tr><td>(ii)</td><td>
$\dfrac{dV}{dt} = \dfrac{4}{375}(30-V)$
$\displaystyle\int \frac{1}{30-V}\, dV = \int \frac{4}{375}\, dt$
$-\ln|30-V| = \dfrac{4}{375}t + C$
$\ln|30-V| = -\dfrac{4}{375}t - C$
$|30-V| = e^{-\frac{4}{375}t - C}$
$30 - V = \pm e^{-C} e^{-\frac{4}{375}t}$
$30 - V = Ae^{-\frac{4}{375}t}$ where $A = \pm e^{-C}$
$V = 30 - Ae^{-\frac{4}{375}t}$
When $t = 0, V = 0$
$A = 30$
$V = 30 - 30e^{-\frac{4}{375}t}$
</td><td></td></tr>
<tr><td>(iii)</td><td>


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(iv)	As the maximum mass of particulates is 30 mg, the extraction device is effective enough to meet the recommendation.	
(v)	$\frac{\mathrm{d}V}{\mathrm{d}t} = 0.32 - \frac{2}{375}V = \frac{1}{375}(120 - 2V) = \frac{2}{375}(60 - V)$	
(vi)	At steady state, $\frac{\mathrm{d}V}{\mathrm{d}t} = 0$ $0 = 60 - V$ $V = 60$	