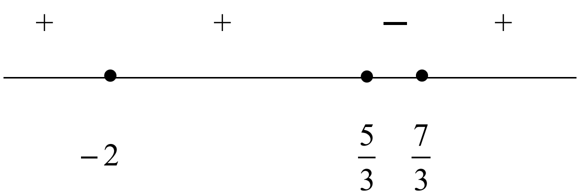
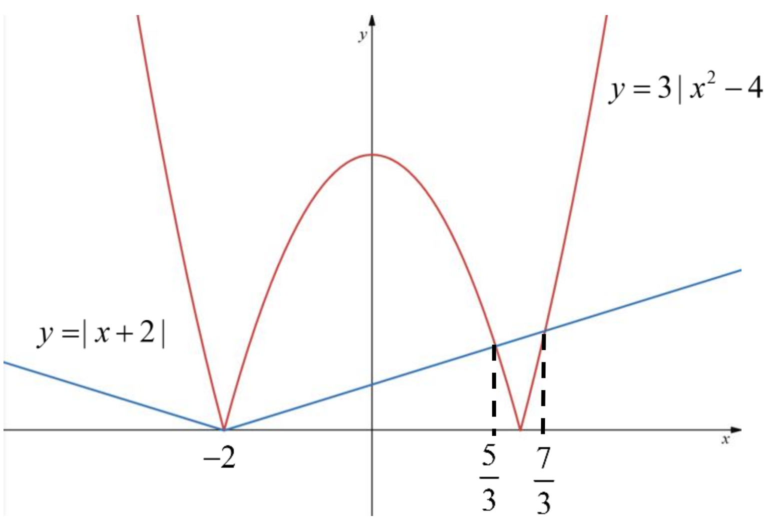
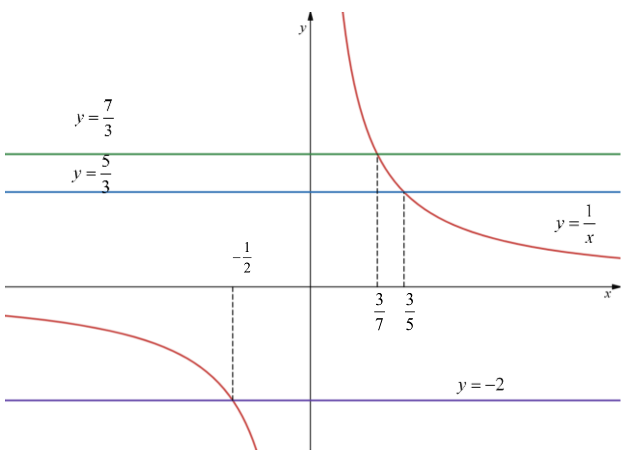
No	**Solution**	
1(i)	**Method 1: Algebraic Method** $3\left\|x^2-4\right\| \le \left\|x+2\right\|$ $3\left\|x^2-4\right\| \le \left\|x+2\right\|$ $9(x^2-4)^2 \le (x+2)^2$ $9(x-2)^2(x+2)^2 \le (x+2)^2$ $(x+2)^2\left[9(x-2)^2-1\right] \le 0$ $(x+2)^2(3x-5)(3x-7) \le 0$  $\frac{5}{3} \le x \le \frac{7}{3}$ or $x=-2$	
	Method 2: Graphical Method  Using GC to find x-coordinates of the points of intersection between $y=\|x+2\|$ and $y=3\|x^2-4\|$: $x=-2$ or $\frac{5}{3}$ or $x=\frac{7}{3}$	

	From the graph, $\frac{5}{3} \le x \le \frac{7}{3}$ or $x = -2$	
(ii)	$\frac{3\left\|1-4x^2\right\|}{x^2} \le \left\|\frac{1+2x}{x}\right\|$ $3\left\|\frac{1-4x^2}{x^2}\right\| \le \left\|\frac{1}{x}+2\right\|$ $3\left\|\frac{1}{x^2}-4\right\| \le \left\|\frac{1}{x}+2\right\|$ Replace x with $\frac{1}{x}$, $\frac{5}{3} \le \frac{1}{x} \le \frac{7}{3}$ or $\frac{1}{x} = -2$  From the graph, $\frac{3}{7} \le x \le \frac{3}{5}$ or $x = -\frac{1}{2}$	
2(a) (i)	Since $x = \alpha$ is a root, $\mathrm{f}(\alpha) = 0$ $\Rightarrow p\alpha^6 + q\alpha^4 + r = 0$ $\mathrm{f}(-\alpha) = p(-\alpha)^6 + q(-\alpha)^4 + r$ $= p\alpha^6 + q\alpha^4 + r$ $= 0$ $\Rightarrow x = -\alpha$ is also a root.	
2 (a) (ii)	$x = 5$ and $x = \beta$ are roots	

	$\Rightarrow x=-5$ and $x=-\beta$ are also roots. Since coefficients of f are all real, $x=\beta^*$ and $x=-\beta^*$ are also roots. Therefore, the remaining roots are $-5,\ \beta^*, -\beta,$ and $-\beta^*$.	
(b)	**Method 1:** Since $1+i$ is a root, $z^3+(3-a)z^2-(2+6\mathrm{i})z-6=(z-1-\mathrm{i})(z^2+Bz+C)$ Comparing coefficients, Constant: $-6=C(-1-\mathrm{i})$ $C=3-3\mathrm{i}$ ---(1) z^2: $3-a=B-1-\mathrm{i}\Rightarrow a=4-B+\mathrm{i}$ ------(2) z: $-2-6\mathrm{i}=3-3\mathrm{i}+B(-1-\mathrm{i})$ $(1+\mathrm{i})B=5+3\mathrm{i}$ $B=\dfrac{5+3\mathrm{i}}{1+\mathrm{i}}$ $B=4-\mathrm{i}$ $a=4-B+\mathrm{i}=4-(4-\mathrm{i})+\mathrm{i}=2\mathrm{i}$ $z^3+(3-a)z^2-(2+6\mathrm{i})z-6=0$ $\Rightarrow(z-1-\mathrm{i})(z^2+(4-\mathrm{i})z+3-3\mathrm{i})=0$ $z=1+\mathrm{i}$ or $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $(z+3)(z+1-\mathrm{i})=0$ $z=-3$ or $z=-1+\mathrm{i}$ OR $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $z=\dfrac{-(4-\mathrm{i})\pm\sqrt{(4-\mathrm{i})^2-4(1)(3-3\mathrm{i})}}{2}$ $z=\dfrac{-4+\mathrm{i}\pm(2+\mathrm{i})}{2}$ $z=-3$ or $z=-1+\mathrm{i}$ Therefore, the other roots are -3 and $-1+\mathrm{i}$.	
	Method 2: Since $1+\mathrm{i}$ is a root,	

	$(1+\mathrm{i})^3+(3-a)(1+\mathrm{i})^2-(2+6\mathrm{i})(1+\mathrm{i})-6=0$ $-2+2\mathrm{i}+(3-a)(2\mathrm{i})-(-4+8\mathrm{i})-6=0$ $(3-a)2\mathrm{i}=4+6\mathrm{i}$ $3-a=\dfrac{4+6\mathrm{i}}{2\mathrm{i}}$ $3-a=3-2\mathrm{i}$ $a=2\mathrm{i}$ $\therefore z^3+(3-2\mathrm{i})z^2-(2+6\mathrm{i})z-6=(z-1-\mathrm{i})(z^2+Bz+3-3\mathrm{i})$ Comparing coefficients, $z^2:\quad 3-2\mathrm{i}=B-1-\mathrm{i}$ $B=4-\mathrm{i}$ $z^3+(3-a)z^2-(2+6\mathrm{i})z-6=0$ $\Rightarrow (z-1-\mathrm{i})(z^2+(4-\mathrm{i})z+3-3\mathrm{i})=0$ $z=1+\mathrm{i}$ or $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $(z+3)(z+1-\mathrm{i})=0$ $z=-3$ or $z=-1+\mathrm{i}$ OR $z^2+(4-\mathrm{i})z+3-3\mathrm{i}=0$ $z=\dfrac{-(4-\mathrm{i})\pm\sqrt{(4-\mathrm{i})^2-4(1)(3-3\mathrm{i})}}{2}$ $z=\dfrac{-4+\mathrm{i}\pm(2+\mathrm{i})}{2}$ $z=-3$ or $z=-1+\mathrm{i}$ Therefore, the other roots are -3 and $-1+\mathrm{i}$.	
3(a)	$BC=2$ E is the midpoint of BC. $\therefore BE=EC=1$ By Pythagoras' theorem, $AE=\sqrt{2^2-1^2}=\sqrt{3}$ In ΔAED, $\tan\left(\dfrac{\pi}{4}+x\right)=\dfrac{\sqrt{3}}{ED}$ $ED=\dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4}+x\right)}$	

	$\therefore BD = BE + ED = 1 + \dfrac{\sqrt{3}}{\tan\left(\frac{\pi}{4}+x\right)}$ (shown)	
(b)	$BD = 1 + \dfrac{\sqrt{3}}{\tan\left(\frac{\pi}{4}+x\right)}$ $= 1 + \dfrac{\sqrt{3}}{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}$ $= 1 + \dfrac{\sqrt{3}}{\dfrac{1+\tan x}{1-\tan x}}$ Since $\tan x \approx x$, $BD \approx 1+\sqrt{3}(1-x)(1+x)^{-1}$ $=1+\sqrt{3}(1-x)(1-x+x^2)$ $=1+\sqrt{3}(1-x+x^2-x+x^2)$ $=1+\sqrt{3}-2\sqrt{3}x+2\sqrt{3}x^2$ where $a=1+\sqrt{3},\ \ b=-2\sqrt{3},\ c=2\sqrt{3}$	
4(i)	Note that from $23u_{r+1}=19u_r+16$ $\therefore\ u_{r+1}=\dfrac{19}{23}u_r+\dfrac{16}{23}$ From GC, $u_{10}=3.6417$	
(ii)	As $r\to\infty$, $u_r \to l$ and $u_{r+1}\to l$: $23l=19l+16$ $4l=16$ $l=4$	
(iii)	When u_r exceeds 99.9% of l: $u_r > \dfrac{99.9}{100}l$ $u_r > \dfrac{99.9}{100}(4)$ $u_r > 3.996$	

	$\therefore\ u_{r+1} = \dfrac{19}{23}u_r + \dfrac{16}{23}$ By GC, <table><tr><td>r</td><td>u_r</td></tr><tr><td>33</td><td>3.9956 < 3.996</td></tr><tr><td>34</td><td>3.9963 > 3.996</td></tr><tr><td>35</td><td>3.9970 > 3.996</td></tr></table> $\therefore$ Smallest $r = 34$	
(iv)	From $S_n = \sum_{r=1}^{n}(u_r - c)$ where $u_r = 4 - 2\left(\frac{19}{23}\right)^{n-1}$, $S_n = \sum_{k=1}^{n}\left[4 - 2\left(\frac{19}{23}\right)^{k-1} - c\right]$ $= \sum_{k=1}^{n}(4-c) - 2\sum_{k=1}^{n}\left(\frac{19}{23}\right)^{k-1}$ $= (4-c)n - 2\left[1 + \frac{19}{23} + \left(\frac{19}{23}\right)^2 + \ldots + \left(\frac{19}{23}\right)^{n-1}\right]$ $= (4-c)n - 2 \times \dfrac{1\left[1-\left(\frac{19}{23}\right)^n\right]}{1-\frac{19}{23}}$ $= (4-c)n - \frac{23}{2}\left[1-\left(\frac{19}{23}\right)^n\right]$ Hence, $A = 4 - c$ and $B = -\frac{23}{2}$	
(v)	$S_n = (4-c)n - \frac{23}{2}\left[1-\left(\frac{19}{23}\right)^n\right]$ As $n \to \infty$, $\left(\frac{19}{23}\right)^n \to 0$ and $(4-c)n \to \infty$ if $c \neq 4$. Hence if S_n diverges if $c \neq 4$	

5(i)

$$x=\tan\theta-\cot\theta,\ y=\tan\theta+\cot\theta,\ \ 0<\theta<\frac{\pi}{2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta+\operatorname{cosec}^2\theta \qquad\qquad \frac{\mathrm{d}y}{\mathrm{d}\theta}=\sec^2\theta-\operatorname{cosec}^2\theta$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\div\frac{\mathrm{d}x}{\mathrm{d}\theta}$$

$$=\frac{\sec^2\theta-\operatorname{cosec}^2\theta}{\sec^2\theta+\operatorname{cosec}^2\theta}$$

$$=\frac{\dfrac{1}{\cos^2\theta}-\dfrac{1}{\sin^2\theta}}{\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}}$$

$$=\frac{\sin^2\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta}$$

$$=\sin^2\theta-\cos^2\theta$$

$$=-\cos 2\theta \text{ (Shown)}$$

<u>Alternative method</u>

$$x=\tan\theta-\cot\theta,\ y=\tan\theta+\cot\theta,\ \ 0<\theta<\frac{\pi}{2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta}=\sec^2\theta+\operatorname{cosec}^2\theta \qquad\qquad \frac{\mathrm{d}y}{\mathrm{d}\theta}=\sec^2\theta-\operatorname{cosec}^2\theta$$

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sec^2\theta-\operatorname{cosec}^2\theta}{\sec^2\theta+\operatorname{cosec}^2\theta}$$

$$=\frac{\sec^2\theta\left(1-\dfrac{\operatorname{cosec}^2\theta}{\sec^2\theta}\right)}{\sec^2\theta\left(1-\dfrac{\operatorname{cosec}^2\theta}{\sec^2\theta}\right)}$$

$$=\frac{\left(1-\cot^2\theta\right)}{1+\cot^2\theta}$$

$$=\frac{1-\cot^2\theta}{\operatorname{cosec}^2\theta}$$

$$=\frac{1}{\operatorname{cosec}^2\theta}-\cos^2\theta$$

$$=\sin^2\theta-\cos^2\theta$$

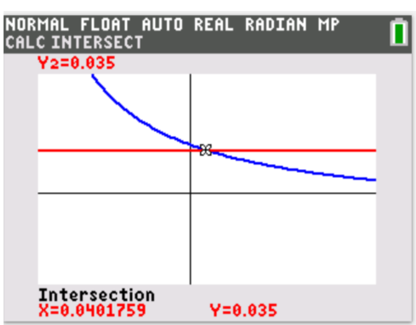
$$=-\left(\cos^2\theta-\sin^2\theta\right)$$

$$=-\cos 2\theta \text{ (Shown)}$$

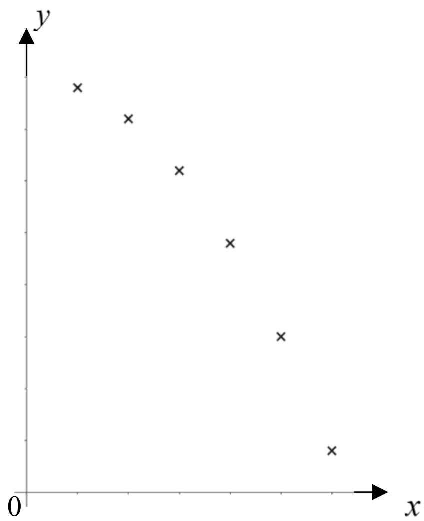
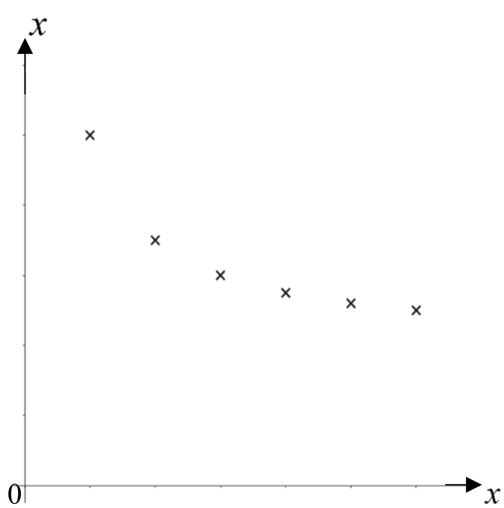
(ii)	At $\theta = p$, Equation of $L: y-(\tan p+\cot p) = -\cos 2p(x-(\tan p-\cot p))$ $y+(\cos 2p)x = \tan p+\cot p+\cos 2p\tan p-\cos 2p\cot p$ $y+(\cos 2p)x = \tan p(1+\cos 2p)+\cot p(1-\cos 2p)$ $y+(\cos 2p)x = \tan p(2\cos^2 p)+\cot p(2\sin^2 p)$ $y+(\cos 2p)x = \dfrac{\sin p}{\cos p}(2\cos^2 p)+\dfrac{\cos p}{\sin p}(2\sin^2 p)$ $y+(\cos 2p)x = 2\sin p\cos p+2\sin p\cos p$ $y+(\cos 2p)x = 2\sin 2p$ (shown)	
(iii)	Given $\dfrac{\mathrm{dp}}{\mathrm{d}t} = 0.2$ When $y=0$, $\left(\cos 2p\right)x = 2\sin 2p$ $x = 2\tan 2p$ Note: Since $0<\theta<\dfrac{\pi}{4} \Rightarrow\ 0<p<\dfrac{\pi}{4} \Rightarrow\ 0<2p<\dfrac{\pi}{2}$ $\tan 2p>0$ $Q\left(2\tan 2p, 0\right)$ When $x=0$, $y=2\sin 2p$ Note: Since $0<\theta<\dfrac{\pi}{4} \Rightarrow\ 0<p<\dfrac{\pi}{4} \Rightarrow\ 0<2p<\dfrac{\pi}{2}$ $\sin 2p>0$ $R\left(0, 2\sin 2p\right)$ Area of OQR, $A = \dfrac{1}{2}\left(2\sin 2p\right)\left(2\tan 2p\right) = 2\tan 2p\sin 2p$ $\dfrac{\mathrm{d}A}{\mathrm{d}p} = 2\left(\tan 2p(2\cos 2p)+(\sin 2p)2\sec^2 2p\right)$ $=4\left(\sin 2p+\tan 2p\sec 2p\right)$ At $p=\dfrac{\pi}{6}$, $\dfrac{\mathrm{d}A}{\mathrm{d}p} = 4\left(\sin\left(\dfrac{\pi}{3}\right)+\tan\left(\dfrac{\pi}{3}\right)\sec\left(\dfrac{\pi}{3}\right)\right) = 4\left(\dfrac{\sqrt{3}}{2}+2\sqrt{3}\right) = 4\left(\dfrac{5}{2}\right)\sqrt{3} = 10\sqrt{3}$ $\dfrac{\mathrm{d}A}{\mathrm{dt}} = \left(\dfrac{\mathrm{d}A}{\mathrm{d}p}\right)\left(\dfrac{\mathrm{d}p}{\mathrm{d}t}\right) = 10\sqrt{3}\times 0.2 = 2\sqrt{3}$ units^2/s	

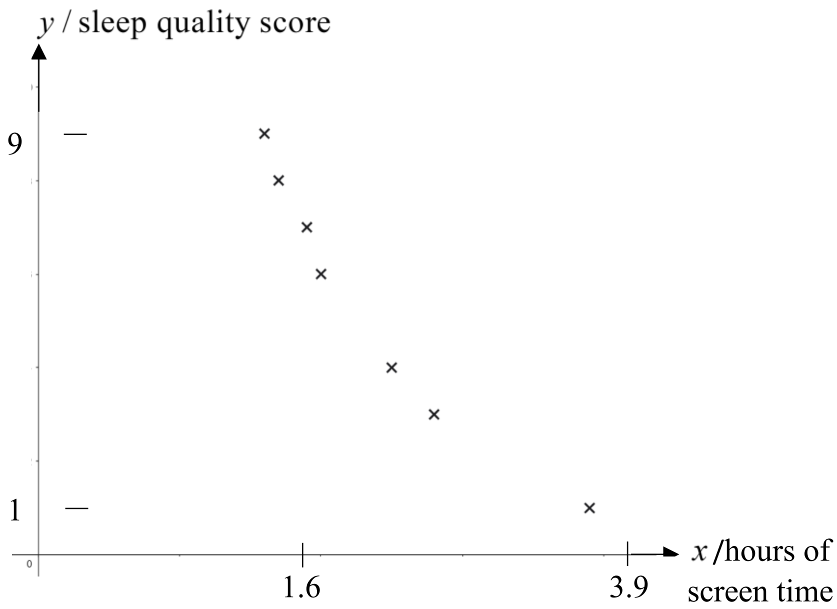
6(a)(i)	Case 1: Two married couples are in the same taxi Number of ways = 2 Case 2: Two married couples are in different taxis Number of ways = ${}^{2}C_{1} \times {}^{4}C_{2} \times {}^{2}C_{2} = 12$ Total number of ways = 2 + 12 = 14	
(ii)	The two married couples must sit at the back in different taxis. Number of ways to sit the two married couples = $2 \times (2 \times 2) \times (2 \times 2) = 32$ Number of ways to sit the 4 singles = 4! or ${}^{4}C_{2} \times 2! \times 2! = 24$ Required number of ways $= 2 \times (2 \times 2) \times (2 \times 2) \times 4! = 768$	
7 (a)	If A and B are independent, $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$ Since $B \subset A$, $\mathrm{P}(A \cap B) = \mathrm{P}(B)$ and since $A \subset S$, $\mathrm{P}(A) \neq 1$ So $\mathrm{P}(A \cap B) = \mathrm{P}(B) \neq \mathrm{P}(A)\mathrm{P}(B)$. Hence A and B are not independent. OR A and B are not independent since the occurrence of B implies the occurrence of A. (or the non-occurrence of A implies the non-occurrence of B) OR From the Venn Diagram, $B \subset A \subset S$. Hence $P(A \mid B) = 1 \neq P(A)$ as seen from the Venn Diagram. OR From the Venn Diagram, $B \subset A \subset S$. Hence $P(B \mid A) \neq P(B)$ as the reduced sample space $A \neq S$ as seen from the Venn Diagram. $P(A \mid B) = 1$	
7(b)(i)	$P(X = 2)$ $= P(\text{'2' appears once and '0' appears twice or '1' appears twice and '0' appears once})$ $= \frac{1}{2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{3!}{2!} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{6} \times \frac{3!}{2!}$ $= \frac{7}{72}$	
(ii)	$P(X = 6)$ $= P(\text{'2' appears 3 times})$ $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ $= \frac{1}{8}$	

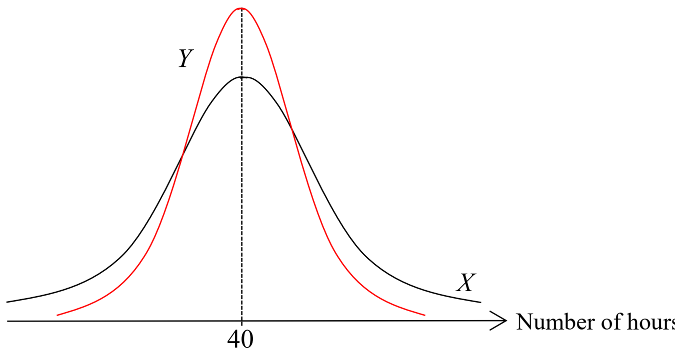
	$P(X=5)$ $=P(\text{'2' appears twice and '1' appears once})$ $=\frac{1}{2}\times\frac{1}{2}\times\frac{1}{3}\times\frac{3!}{2!}$ $=\frac{1}{4}$ $P(X=3)$ $=P(\text{'2','1','0' all appear once or '1' appear 3 times})$ $=\frac{1}{2}\times\frac{1}{3}\times\frac{1}{6}\times 3!+\frac{1}{3}\times\frac{1}{3}\times\frac{1}{3}$ $=\frac{11}{54}$ Or Complement Method $P(X=3)=1-\frac{1}{216}-\frac{1}{36}-\frac{7}{72}-\frac{7}{24}-\frac{1}{4}-\frac{1}{8}=\frac{11}{54}$	
(iii)	Required Probability $=\mathrm{P}(X=0)\times[\mathrm{P}(X>2)]^2\times[\mathrm{P}(X=1\text{ or }2)]^2\times\frac{5!}{2!2!}$ $=\frac{1}{216}\times\left(1-\frac{1}{216}-\frac{1}{36}-\frac{7}{72}\right)^2\times\left(\frac{1}{36}+\frac{7}{72}\right)^2\times\frac{5!}{2!2!}$ $=0.00164$ (3 s.f.)	
8(i)	Assumptions: Whether a toy is faulty or not is independent of other toys. The probability of a toy being faulty is constant at 0.02 for each toy.	
(ii)	Let X be the random variable denoting the number of faulty toys in 15 blind boxes. $X\sim\mathrm{B}(15,0.02)$ $\mathrm{P}(X\geq 3)=1-\mathrm{P}(X\leq 2)=0.00304=0.00304$ (3 s.f.)	
(iii)	**Method 1:** Let Y be the random variable denoting the number of faulty toys in 13 blind boxes. $Y\sim\mathrm{B}(13, 0.02)$ Required probability $=\mathrm{P}(Y=1)\times(0.02)$ $=0.204026\times(0.02)$ $=0.00408$ (3 s.f.)	

	Method 2: Required probability $= {}^{13}C_1(0.02)(0.98)^{12} \times (0.02) = 0.00408\,(3\,\text{s.f.})$	
(iv)	Let K be the random variable denoting the number of faulty keychains out of 3. $K \sim \mathrm{B}(3\ ,p)$ $\mathrm{P}(K=1)$ $= {}^{3}C_1 p^1(1-p)^2$ $= 3p(1-p)^2$	
(v)	Let A be the number of faulty toys in 2 blind boxes. $A \sim \mathrm{B}\ (2, 0.02)$ $K \sim \mathrm{B}\ (3, p)$ P(1 faulty toy and no faulty keychains | at most 1 faulty toy in the pack) = 0.035 $\dfrac{\mathrm{P}(A=1)\cdot\mathrm{P}(K=0)}{\mathrm{P}(A=0)\cdot\mathrm{P}(K=0)+\mathrm{P}(A=1)\cdot\mathrm{P}(K=0)+\mathrm{P}(A=0)\cdot\mathrm{P}(K=1)} = 0.035$ $\dfrac{0.0392\cdot(1-p)^3}{0.9604\cdot(1-p)^3+0.0392\cdot(1-p)^3+0.9604\cdot{}^{3}C_1 p(1-p)^2} = 0.035$ **Method 1: Using GC** From GC,  Since $0 < p \le 1$, $p = 0.0401759 = 0.0401$ (3 s. f.) **Method 2: Algebraic approach**	

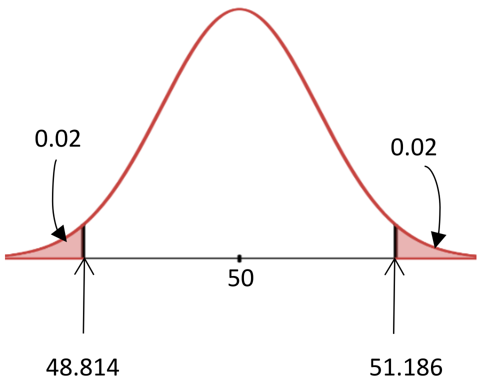
	$\dfrac{0.0392(1-p)^3}{0.9604(1-p)^3+0.0392(1-p)^3+0.9604\times {}^3C_1 p(1-p)^2}=0.035$ $\dfrac{0.0392(1-p)}{0.9604(1-p)+0.0392(1-p)+0.9604(3p)}=0.035$ $0.0392(1-p)=0.033614(1-p)+0.001372(1-p)+0.100842p$ $0.004214-0.105056p=0$ $p=\dfrac{43}{1072}=0.0401\ \text{(3 s.f.)}$	

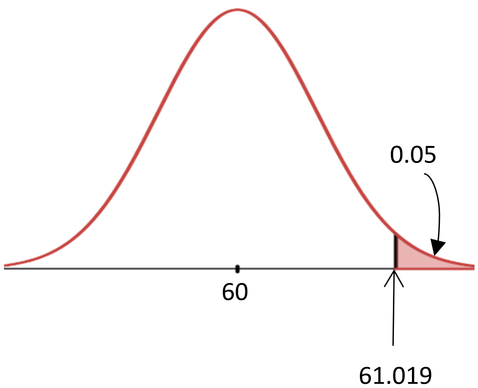
9(i)	(A): $y = a + bx^2$, where $a > 0$ and $b < 0$  (B): $y = c + \frac{d}{x}$ where $c > 0$ and $d > 0$ 	

(ii)		
(iii)	From the scatter diagram, as x increases, y decreases at a decreasing rate. Case (B) is the most appropriate model. $r = 0.99868 = 0.999$ (3 s.f.)	
(iv)	Case (B): $y = -4.6636 + \dfrac{21.716}{x}$ $y = -4.66 + \dfrac{21.7}{x}$ (3 s.f.) When x=3, $y = -4.6636 + \dfrac{21.716}{3} = 2.5751$ Sleep quality score =3 (nearest integer) Yes, the estimate is reliable as the $r = 0.999$ is close to 1 and x=3 is within the data range.	
(v)	Since $t = 60x$, $y = -4.6636 + \dfrac{21.716(60)}{60x}$ $y = -4.6636 + \dfrac{1303.0}{t}$ $y = -4.66 + \dfrac{1300}{t}$ (3 s.f.)	

	No, the r value does not differ/ stays the same as it is unaffected by change of units/ scaling.	
(vi)	I disagree with Alvin because the outcome of the study determines how sleep quality is linearly correlated to the amount of screen time and does not imply that the increased amount of the screen time causes poorer sleep quality.	
10 (a)(i)		
(ii)	$\dfrac{t-40}{\alpha}=1.8$ $t=40+1.8\alpha$	
(iii)	required probability $=\left[\mathrm{P}\left(Z>1.8\right)\right]^2=\left(0.035930\right)^2=0.00129$	
(iv)	$Y\sim N(40,\beta^2)$ $\mathrm{P}(Y>42)\geq 0.25$ $\mathrm{P}(\dfrac{Y-40}{\beta}>\dfrac{42-40}{\beta})\geq 0.25$ $\mathrm{P}(Z>\dfrac{2}{\beta})\geq 0.25$ $\dfrac{2}{\beta}\leq 0.67449$ $\beta\geq 2.9652=2.97$	
(b) (i)	Let S be the random variable denoting the mass, in kg, of a small bag of rice. $S\sim \mathrm{N}(1,\ 0.16^2)$ $\mathrm{P}(S>1.25)=0.059085$ Let W be the number of small bags of rice with a mass greater than 1.25 kg out of 300 bags. $W\sim B(300,0.059085)$ $E(W)=300\times 0.059085=17.7$ Expected number of small bags of rice with a mass greater than 1.25 kg is 17.7	

(b)(ii)	Let L be the random variable denoting the mass, in kg, of a big bag of rice. Since $n = 30$ is large enough, by Central Limit Theorem, $\overline{L} \sim \mathrm{N}\left(5, \dfrac{0.5^2}{30}\right)$ approximately . Let $W = S_1 + S_2 + \cdots + S_5 - \overline{L}$. $\mathrm{E}(W) = 5\mathrm{E}(S) - \mathrm{E}\left(\overline{L}\right)$ $= 5(1) - 5$ $= 0$ $\mathrm{Var}(W) = 5\mathrm{Var}(S) + \mathrm{Var}\left(\overline{L}\right)$ $= 5(0.16^2) + \dfrac{0.5^2}{30}$ $= \dfrac{409}{3000}$ or 0.13633 (5sf) $\therefore W \sim \mathrm{N}\left(0, \dfrac{409}{3000}\right)$ approximately Required Prob. $= \mathrm{P}\left(W > 0.25\right)$ $= 0.249$ (3sf)	
11 (i)	Let X mg be the Vitamin C content in a randomly chosen bottle of orange juice and μ mg be the population mean vitamin C content in the bottles of orange juice. $\mathrm{H}_0 : \mu = 50$ $\mathrm{H}_1 : \mu \neq 50$ Test at 4 % level of significance. Under H_0, $\overline{X} \sim \mathrm{N}\left(50, \dfrac{2^2}{12}\right)$.	

	Critical Region:  Reject H_0 if $\bar{x} \le 48.814$ or $\bar{x} \ge 51.186$ i.e. $\bar{x} \le 48.8$ or $\bar{x} \ge 51.2$ (3sf)	
(ii)	In the earlier test, the distribution of the Vitamin C content in the bottles of juice was known to be normally distributed with a known variance. Therefore, regardless of the sample size, the sample mean Vitamin C content in the bottles of juice will still be normally distributed. However, the distribution of the new "Vitamin Boost" Vitamin C content is unknown. Hence, a larger sample size of at least 30 needs to be taken so that Central Limit Theorem can be applied to approximate the distribution of the sample mean Vitamin C content of the new juice line to a normal distribution.	
(iii)	Unbiased estimate of population mean, $\bar{y} = \frac{1865}{30} = 62.167 \text{ (5sf)} = 62.2 \text{ (3sf)}$ Unbiased estimate of population variance, $s^2 = \frac{1}{29}\left[116275 - \frac{1865^2}{30}\right] = 11.523 \text{ (5sf)} = 11.5 \text{ (3sf)}$	
(iv)	Let Y mg be the Vitamin C content in a randomly chosen bottle of "Vitamin Boost" Orange Juice, and let μ_Y mg be the population mean Vitamin C content in the "Vitamin Boost" Orange Juice bottles. $H_0: \quad \mu_Y = 60$ $H_1: \quad \mu_Y > 60$ Test at 5% level of significance. Under H_0, since $n = 30$ is large enough, by the Central Limit Theorem, $\bar{Y} \sim N\left(60, \frac{11.523}{30}\right)$ approximately.	

	Method 1: p-value Method $p\text{-value} = \mathrm{P}\left(\overline{Y} \geq 62.167\right)$ $= 0.000236 \quad \text{or} \quad 2.36\times10^{-4}\ (3\text{sf})$ < 0.05 $\therefore$ Reject H_0. Therefore, there is sufficient evidence at 5% level of significance to conclude that the mean vitamin C content in the bottles of the new juice line is greater than 60 mg. Method 2: Critical Value method Critical Region:  Reject H_0 if $\overline{y} \geq 61.019$ Since $\overline{y} = 62.167 > 61.019$, Reject H_0. Therefore, there is sufficient evidence at 5% level of significance to conclude that the mean vitamin C content in the bottles of the new juice line is greater than 60 mg.	