**ST ANDREW'S JUNIOR COLLEGE**

**General Certificate of Education Advanced Level**

**Higher 2**

**MATHEMATICS 9758/02**

**Paper 2** **16 September 2025 (Tuesday)**

**Preliminary Examination** **3 hours**

**Additional Materials:** **Printed Answer Booklet**

**List of Formulae and Results (MF27)**

---

**READ THESE INSTRUCTIONS FIRST**

Answer **all** questions. Total marks: 100

Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

---

This document consists of **8** printed pages.

## Section A: Pure Mathematics [40 marks]

**1** **(i)** Solve the inequality $3\left|x^2-4\right| \le \left|x+2\right|$. [4]

**(ii)** Hence, solve $\dfrac{3}{x^2}\left|1-4x^2\right| \le \left|\dfrac{1+2x}{x}\right|$. [2]

**2** **(a)** It is given that $\mathrm{f}(x) = px^6 + qx^4 + r$, where *p*, *q*, and *r* are real constants.

**(i)** Show that if $x = \alpha$ is a root of $\mathrm{f}(x) = 0$, then $x = -\alpha$ is also a root. [1]

**(ii)** Given now that $x = 5$ and $x = \beta$ are roots of $\mathrm{f}(x) = 0$, where $\mathrm{Re}(\beta) \ne 0$ and $\mathrm{Im}(\beta) \ne 0$, write down all the remaining roots. [3]

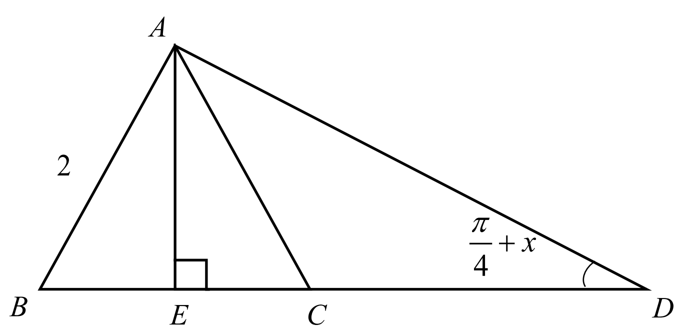
**(b)** The complex number *z* satisfies the equation

$$z^3 + (3-a)z^2 - (2+6\mathrm{i})z - 6 = 0,$$

where *a* is a complex number. It is given that one of the roots is 1 + i.

Find *a* and the other roots of the equation. [5]

**3** The diagram below shows $\Delta ABC$ and $\Delta AED$. $\Delta ABC$ is an equilateral triangle with each side measuring 2 units and $\Delta AED$ is a right-angled triangle with $\angle ADE = \dfrac{\pi}{4} + x$. Points *C* and *E* both lie on the line *BD*.




**(a)** Show that $BD = 1 + \dfrac{\sqrt{3}}{\tan\left(\dfrac{\pi}{4}+x\right)}$. [2]

**(b)** Given that *x* is sufficiently small for $x^3$ and higher powers of *x* to be neglected, show that $BD \approx a + bx + cx^2$ where *a*, *b* and *c* are exact constants to be determined. [4]

**4** A sequence $u_1,\ u_2,\ u_3,\ ...$ is such that $23u_{r+1} = 19u_r + 16,$ for $r > 0$ and $u_1 = 2$.

**(i)** Write down the value of $u_{10}$, giving your answer correct to 4 decimal places. [1]

**(ii)** It is given that as $r \to \infty$, $u_r \to l$. Show that $l = 4$. [1]

**(iii)** Hence, find the smallest integer $r$ for which $u_r$ exceeds 99.9% of $l$. [3]

It is known that the *k*th term of this sequence is given by $u_k = 4 - 2\left(\frac{19}{23}\right)^{k-1}$.

**(iv)** Given that a new series $S_n$ is defined by $S_n = \sum_{k=1}^{n}(u_k - c)$, where *c* is a real constant.

Express $S_n$ in the form of $An + B\left[1 - \left(\frac{19}{23}\right)^n\right]$, where $A$ is a constant in terms of *c* and $B$ is a real number. [3]

**(v)** Determine the possible values of *c* for $S_n$ to diverge as $n \to \infty$. [1]

**5** A curve *C* has parametric equations

$$x = \tan\theta - \cot\theta, \quad y = \tan\theta + \cot\theta, \quad \text{for } 0 < \theta < \frac{\pi}{4}.$$

**(i)** Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\cos 2\theta$. [3]

The line *L* is the tangent to *C* at point *P* with parameter *p*.

**(ii)** Show that the equation of *L* is $y + (\cos 2p)x = 2\sin 2p$. [3]

Let *Q* and *R* be the points where *L* cuts the *x*-axis and *y*-axis respectively.

**(iii)** Given that *p* increases at a constant rate of 0.2 radians per second, find the exact rate of change of the area of $\Delta\, OQR$ when $p = \frac{\pi}{6}$, where *O* is the origin. [4]

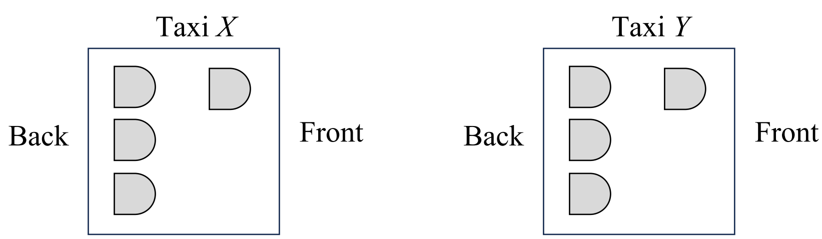
**Section B: Statistics [60 marks]**

**6** A group of 8 tourists – 2 married couples and 4 singles – travels to the airport in two taxis, $X$ and $Y$. Each taxi can take 4 passengers.

The 8 tourists divide themselves into two groups of 4, one group for taxi $X$ and one group for taxi $Y$.

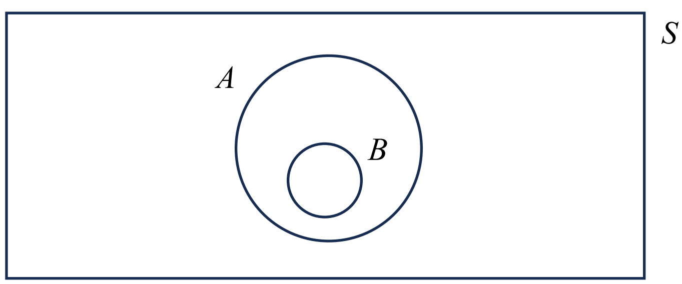
**(i)** Find the number of different ways in which this can be done if each married couple must travel together in the same taxi. [2]

Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram).




**(ii)** Find the number of seating arrangements so that each married couple sit next to each other at the back of the taxi. [3]

**7** **(a)** With reference to the Venn diagram below, $S$ is the universal set and $A$ and $B$ are non-empty proper subsets of $S$. Explain whether $A$ and $B$ are independent, and write down the value of $\mathrm{P}(A \mid B)$.




[2]

**(b)** A fair die has three sides numbered 2, two sides numbered 1 and one side numbered 0. A game is played by throwing this fair die three times and the score, $X$, is the sum of the numbers obtained.

**(i)** Show that $\mathrm{P}(X=2)=\dfrac{7}{72}$ . [2]

**(ii)** The table shows an incomplete probability distribution of $X$. Find the missing probabilities, showing your workings clearly. [3]

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $\mathrm{P}(X=x)$ | $\dfrac{1}{216}$ | $\dfrac{1}{36}$ | $\dfrac{7}{72}$ | | $\dfrac{7}{24}$ | | |

The game is played 5 times.

**(iii)** Find the probability that exactly one of the games obtained a score of 0 and exactly 2 games obtained a score of more than 2. [3]

**8** A company sells toys in blind boxes. Each blind box contains exactly one toy. On average, 2% of such blind boxes contain a faulty toy.

A collector buys 15 such blind boxes at random.

**(i)** State, in context, two assumptions needed for the number of faulty toys he gets to be well modelled by a binomial distribution. [2]

Assume now that the number of faulty toys the collector gets has a binomial distribution.

**(ii)** Find the probability that the collector gets no less than 3 faulty toys. [1]

**(iii)** Find the probability that the fourteenth box is the second box with a faulty toy. [2]

The company also sells keychains. The probability of a keychain being faulty is $p$. The number of faulty keychains follows a binomial distribution. Faults on keychains are independent of faults on toys in blind boxes.

**(iv)** Write down an expression, in terms of $p$, for the probability that in a random sample of 3 keychains, exactly one is faulty. [1]

A surprise pack contains 2 randomly chosen blind boxes and 3 randomly chosen keychains. Given that a randomly selected surprise pack contains at most 1 faulty item, the probability that one of the toys is the only faulty item in the pack is 0.035.

**(v)** Write down an equation satisfied by $p$. Hence, find the value of $p$. [4]

9 **(i)** Sketch a scatter diagram that might be expected when $x$ and $y$ are related approximately as given in each of the cases **(A)** and **(B)** below. In each case your diagram should include 6 points, approximately equally spaced with respect to $x$, and with all $x$- and $y$- values positive. The letters $a$, $b$, $c$ and $d$ represent constants.

**(A)** $y = a + bx^2$, where $a$ is positive and $b$ is negative,

**(B)** $y = c + \dfrac{d}{x}$, where $c$ is positive and $d$ is positive. [2]

Alvin is investigating the relationship between the amount of screen time per day and the sleep quality of youths. He recorded the sleep quality score, $y$, on a scale of 1 to 10 with 1 being the worst and 10 being the best, of particular youths with $x$ hours of screen time per day.

| $x$ | 1.6 | 1.7 | 1.9 | 2.0 | 2.5 | 2.8 | 3.9 |
|---|---|---|---|---|---|---|---|
| $y$ | 9 | 8 | 7 | 6 | 4 | 3 | 1 |

**(ii)** Draw a scatter diagram for these values, labelling the axes. [1]

**(iii)** Explain which of the two cases in part **(i)** is the most appropriate for modelling these values and calculate the product moment correlation coefficient for this case. [2]

**(iv)** Alvin wants to estimate the sleep quality score when the screen time per day is 3 hours. Use the case that you have identified in part **(iii)** to find the equation of a suitable regression line and use your equation to find the required estimate. Explain whether you would expect this estimate to be reliable. [4]

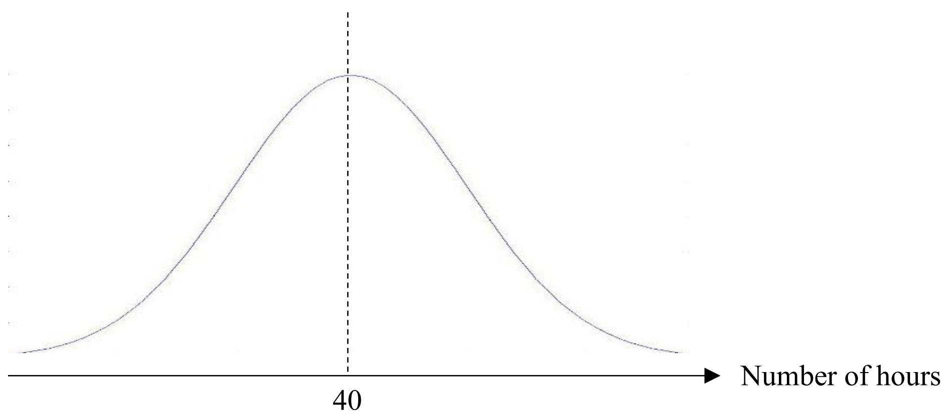
**(v)** Write the equation from part **(iv)** in terms of $y$ and $t$, where $t$ is the amount of screen time per day in minutes. Explain whether the product moment correlation coefficient for this case differs from the one found in part **(iii)**. [3]

**(vi)** Alvin concluded that this study shows that increased amount of screen time per day causes poorer sleep quality. Explain whether you agree with his conclusion. [1]

**10** **In this question, you should state the parameters of any distributions you use.**

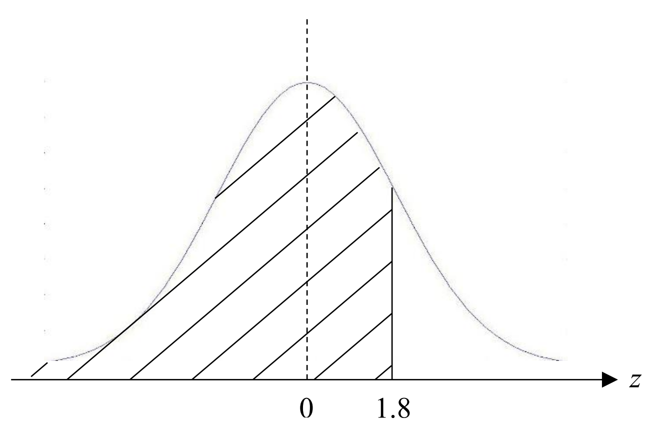
**(a)** The number of hours worked per week by employees of Company $A$ follow a normal distribution with mean 40 hours and standard deviation $\alpha$ hours, while the number of hours worked per week by employees of Company $B$ follow a normal distribution with mean 40 hours and standard deviation $\beta$ hours, where $\alpha > \beta$.

The diagram shows the probability density function of $X$, where $X$ is the random variable denoting the number of hours worked per week by a randomly chosen employee of Company $A$.




**(i)** On the copy of the diagram in the **Printed Answer Booklet**, sketch on the same diagram the probability density function of $Y$, where $Y$ is the random variable denoting the number of hours worked per week by a randomly chosen employee of Company $B$. [1]

The probability that a randomly chosen employee of Company $A$ works less than $t$ hours per week is represented by the shaded area of the standard normal curve in the diagram below.




**(ii)** Write down an equation involving $t$ and $\alpha$. [1]

**(iii)** Two employees of company $A$ are randomly chosen. Find the probability that both employees work more than $t$ hours per week. [1]

**(iv)** Given that the probability that a randomly chosen employee of Company $B$ works more than 42 hours per week is at least 0.25, find the range of values of $\beta$. [2]

**(b)** A shop sells small and large bags of rice. The masses of small bags of rice are normally distributed with mean 1 kg and standard deviation 0.16 kg. The masses of large bags of rice have mean 5 kg and standard deviation 0.5 kg. The masses of the small and large bags of rice are independent of one another.

**(i)** In a random sample of 300 small bags of rice, find the expected number of bags that have a mass greater than 1.25 kg. [2]

**(ii)** Find the probability that the total mass of 5 randomly chosen small bags of rice exceed the mean mass of 30 randomly chosen large bags of rice by more than 0.25 kg. [4]

**11** A beverage company produces bottled orange juice, claiming that the mean vitamin C content in each 250ml bottle is 50 mg. It is known that the vitamin C content is normally distributed with a standard deviation of 2 mg.

Due to feedback from consumers, the quality control manager wants to test, at the 4% level of significance, whether the mean vitamin C content is, in fact, 50 mg. He randomly samples 12 bottles and measures the vitamin C content.

**(i)** State the hypotheses and find the critical region for this test. [3]

Recently, the company launched a new "Vitamin Boost" orange juice line, advertised to contain an average of 60 mg of vitamin C content per 250 ml bottle. After some complaints that the juice is too sour, the manager suspects that the vitamin C content might be higher than advertised. He decides to conduct a hypothesis test to investigate. This time, he randomly samples 30 bottles from the new juice line.

**(ii)** Explain why the manager takes a sample of 30 bottles for this test when he only took a sample of 12 bottles in his earlier test. [2]

The vitamin C content, $y$ mg, in the sample of 30 bottles is summarized as follows:

$$\sum y = 1865, \ \sum y^2 = 116275$$

**(iii)** Calculate unbiased estimates of the population mean and variance for the vitamin C content in the bottles of the new juice line. [2]

**(iv)** At the 5% level of significance, test whether the mean vitamin C content in the bottles of the new juice line is greater than 60 mg. Clearly state your hypotheses and define any symbols you use. [4]