TAMPINES MERIDIAN JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION

CANDIDATE NAME: ____

CIVICS GROUP: ____

H2 MATHEMATICS 9758/01

Paper 1

16 SEPTEMBER 2025

3 hours

Additional materials: Printed Answer Booklet

List of Formulae and Results (MF27)

READ THESE INSTRUCTIONS FIRST

Answer **all** the questions.

Write your answers in the spaces provided in the Printed Answer Booklet. Follow the instructions on the front cover of the Printed Answer Booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

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[Turn Over

1 **(a)** On the same diagram, sketch the graphs of $y=|x-a|$ and $y=\dfrac{1}{|x-a|}$, where a is a real constant such that $a>1$. You should show clearly the equations of any asymptotes and axial intercepts of both graphs. [3]

(b) Hence, or otherwise, solve the inequality $|x-a|>\dfrac{1}{|x-a|}$. [3]

2 A curve C has equation $y^2 = x + \mathrm{e}^{2x}y$.

(a) Show that $\left(2y-\mathrm{e}^{2x}\right)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1+2y\mathrm{e}^{2x}$. [2]

(b) Find the equations of the tangents to C at the points where $x=0$. [4]

3 Find the following integrals.

(a) $\displaystyle\int \frac{x^2-3x-1}{x^2-4x+1}\,\mathrm{d}x$, [3]

(b) $\displaystyle\int x\ln(kx)\,\mathrm{d}x$, where k is a real constant, [2]

(c) $\displaystyle\int \frac{\sin\theta}{2\cos 2\theta+1}\,\mathrm{d}\theta$, using the substitution $x=\cos\theta$. [4]

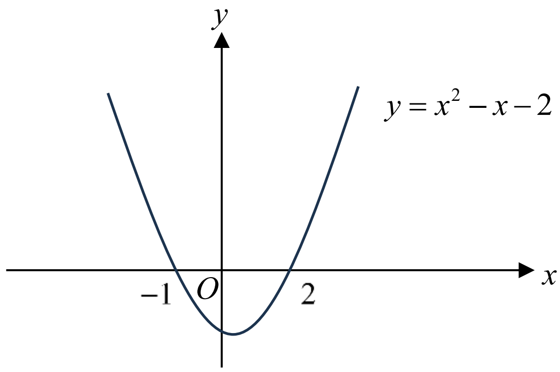
4 An arithmetic series has first term a and common difference d, where a and d are non-zero real constants. A convergent geometric series has first term b and common ratio r, where $b>0$ and r is a non-zero real constant. The sum of the first n terms of the geometric series is denoted by S.

It is given that the 7th, 10th and 11th terms of the arithmetic series are equal to the 3rd, 6th and 10th terms of the geometric series respectively.

(a) Show that r satisfies the equation $3r^7-4r^3+1=0$. Solve this equation, giving your answer correct to 4 decimal places. [4]

(b) A new series is formed by taking the even numbered terms of the geometric series. Find the smallest value of n for which S differs from the sum to infinity of the new series by at least $1.78b$. [4]

5



The diagram shows the graph of $y = x^2 - x - 2$. The two roots of the equation $x^2 - x - 2 = 0$ are -1 and 2.

A sequence of real numbers $x_1, x_2, x_3, ...$ satisfies the recurrence relation

$$x_{n+1} = x_n^{\ 2} - 2$$

for $n \geq 1$.

(a) It is given that as $n \to \infty$, $x_n \to L$. Show that L can take values -1 or 2. [2]

(b) Hence or otherwise, determine the behaviour of the sequence when $x_1 = -1$. [1]

(c) By considering $x_{n+1} - x_n$, prove that $x_{n+1} < x_n$ if $-1 < x_n < 2$. [2]

6 [It is given that the volume of a circular cone with base radius r and height h is $\frac{1}{3}\pi r^2 h$.]

A container is made in the shape of an inverted open right circular cone. The height of the container is 6 metres, and the radius is 2 metres. The container is initially empty. Water is poured into the container while, at the same time, water is leaking from a hole at the bottom of the container at a rate of $0.03 \text{ m}^3/\text{s}$, resulting in the depth of water in the container to be increasing at a rate of 0.1 m/s. At time t seconds after the start, the depth of water in the container is h metres.

(a) Find, in terms of h, an expression for the volume of water in the container at time t seconds after the start. [2]

(b) Find the rate at which water is being poured into the container when the depth of water in the container is 3 metres. [4]

7 The function f is such that

$$\mathrm{f}: x \mapsto \frac{1}{2}\sqrt{36-(x-3)^2} \quad \text{for} \quad x \in \mathbb{R}, \quad k \leq x < 9,$$

where k is a real constant.

(a) State the least value of k for which the function f^{-1} exists. [1]

For the rest of the question, take the value of k as the value found in part **(a)**.

(b) Find $\mathrm{f}^{-1}(x)$. [3]

(c) Sketch, on the same diagram, the graphs of $y=\mathrm{f}(x)$, $y=\mathrm{f}^{-1}(x)$ and $y=\mathrm{f}^{-1}\mathrm{f}(x)$, showing clearly the relationship between the three graphs. [3]

The function g is such that

$$\mathrm{g}: x \mapsto |4x-1|(2x-5) \quad \text{for} \quad x \in \mathbb{R}, x > 0.$$

(d) Show that the composite function gf exists. [2]

(e) Find the range of gf. [2]

8 A curve C has parametric equations

$$x = 2t+3,\ y = 5-4t^2,$$

for all real values of t.

The line N is the normal to C at the point where $t = -\frac{1}{2}$.

(a) Find the cartesian equation of N. [3]

(b) Find the cartesian equation of C. [2]

(c) The region R is bounded by C, N and the y-axis. Without using a calculator, find the exact volume of the solid generated when the region R is rotated through 2π radians about the y-axis. [5]

9 It is given that $-2+2\text{i}$ is a root of the equation

$$z^3+az^2+bz-16\sqrt{2}=0,$$

where a and b are real numbers.

(a) Find the values of a and b and the other two roots. Leave your answers in the exact form. [5]

(b) In an Argand diagram with origin O, the three roots are represented by points A, B and C where A represents $-2+2\text{i}$ and C represents the real root. Label these points on an Argand diagram, indicating clearly the modulus and argument of each root. State also a geometrical relationship between A and B. [3]

(c) Hence, prove that

$$\tan\frac{3\pi}{8}=1+\sqrt{2}.$$ [3]

10 A drone flies in a straight line towards a rooftop for inspection. The drone's flight path is modelled by the line $l:\mathbf{r}=\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-2\\1\end{pmatrix}$ where $\lambda\in\mathbb{R}$, and the rooftop is modelled by the plane $p:2x+y-z=7$.

(a) On Day 1, the drone flies towards the rooftop using the flight path. Upon reaching the rooftop, the drone flies in a new flight path, m. The acute angle between the original flight path and the normal to the rooftop is the same as the acute angle between the new flight path and the same normal to the rooftop. Both flight paths and the normal lie on the same plane. Find the cosine of the angle between the original flight path and the new flight path in an exact non-trigonometrical form. [3]

(b) On Day 2, the drone starts to descend from point A with coordinates $(2,1,0)$ towards the rooftop using the shortest possible path. Find the exact coordinates of the point where the drone lands on the rooftop. [4]

(c) After landing, the drone glides on the rooftop in another new flight path, s, that is perpendicular to the flight path l. Find a cartesian equation of the flight path s. [3]

A second rooftop is built parallel to the first rooftop p such that the perpendicular distance between these two rooftops is d units. Find two possible cartesian equations of the second rooftop in terms of d. [3]

11 An architect designs a flower bed for a garden. The flower bed is enclosed by the curve C with equation $\left(\frac{x+y}{2}\right)^2+\left(\frac{y-x}{4}\right)^2=1$.

The region in the first quadrant bounded by C, the x-axis and the lines $x=0$ and $x=0.5$, is to be planted with tulips.

(a) By finding the Maclaurin series expansion of y up to and including the term in x^2, where $y>0$, find an approximation for the area to be planted with tulips. [7]

The curve C is obtained by rotating an ellipse S about the origin in the x-y plane. It is known that the curve C can be expressed by the following parametric equations

$$x=\cos t-2\sin t,\ y=\cos t+2\sin t,$$

where $-\pi<t\leq\pi$.

(b) Let L be the distance between the origin and a point on C. By using differentiation, determine the exact maximum and minimum values of L. You do not need to show that these values are maximum and minimum. [5]

(c) Write down a possible cartesian equation of S. Hence, determine the total area of the flower bed. [3]

End of Paper