RIVER VALLEY HIGH SCHOOL
2025 JC2 Preliminary Examination
Higher 2

NAME	

CLASS							**INDEX NUMBER**	

MATHEMATICS **9758/01**

Paper 1 **17 Sep 2025**

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **6** printed pages and **2** blank pages.

1 The function f is defined by $\mathrm{f}(x) = ax^3 + bx^2 + cx + d$ where a, b, c and d are real numbers. Given that $4+\mathrm{i}$ and -1 are roots of $\mathrm{f}(x) = 0$, find b, c and d in terms of a. [4]

2 Without using a calculator, solve the inequality $\dfrac{9}{1-x^2} < \dfrac{x+5}{x+1}$. [3]

Hence solve $\dfrac{9}{1-\mathrm{e}^{2x}} < \dfrac{\mathrm{e}^x+5}{\mathrm{e}^x+1}$. [3]

3 **Do not use a calculator in answering this question.**
Find the roots of the equation $z^2 - (1+2\mathrm{i})z + 1 + 7\mathrm{i} = 0$, giving your answers in the cartesian form $a + \mathrm{i}b$. [6]

4 It is given that $\displaystyle\sum_{r=1}^{n} \frac{1}{r(r+1)} = 1 - \frac{1}{n+1}$.

(a) Find $\displaystyle\sum_{r=1}^{N} \frac{1}{(r+1)(r+2)}$ in terms of N. [3]

(b) It is given that

$$8\sum_{r=k+1}^{\infty} \frac{1}{r(r+1)} = \sum_{r=1}^{k} \frac{1}{r(r+1)}.$$

Find the value of k. [3]

5 (a) Find $\displaystyle\int \frac{x}{\sqrt{16-x^4}}\,\mathrm{d}x$. [2]

(b) Find $\displaystyle\int \frac{x}{2}\sin^{-1}\left(\frac{x^2}{4}\right)\mathrm{d}x$. [5]

6 For any vectors $\mathbf{m}$ and $\mathbf{n}$, explain why $\mathbf{m}\cdot(\mathbf{m}\times\mathbf{n})=0$. [1]

With respect to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively. O, A, B and C are non-coplanar. The point M is the mid-point of AC and p denotes the plane OAB.

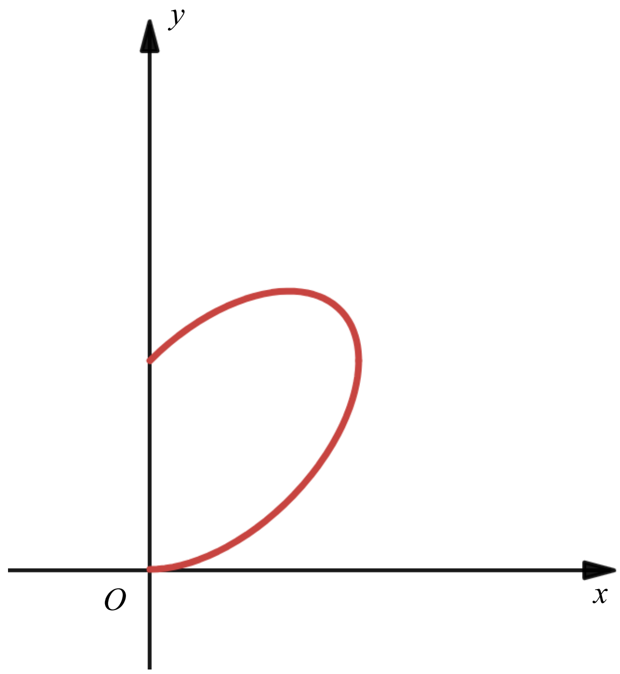
(a) The point R is such that MR is perpendicular to p. Show that R lies on a line ℓ with equation $\mathbf{r}=k(\mathbf{a}+\mathbf{c})+\lambda(\mathbf{a}\times\mathbf{b})$, $\lambda\in\mathbb{R}$, where k is a constant to be determined. [2]

(b) Given that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors perpendicular to each other, $\mathbf{a}\cdot\mathbf{c}=-2$ and $\mathbf{b}\cdot\mathbf{c}=4$, find in terms of $\mathbf{a}$ and $\mathbf{b}$, the position vector of the point of intersection between ℓ and p. [3]

7 A curve C has equation $x^2+y^2-4y=xy$, where $x\geq 0$.

(a) Show that $(2y-x-4)\dfrac{\mathrm{d}y}{\mathrm{d}x}=y-2x$. [2]

The diagram below shows the curve C.



M is a point on C with coordinates (x, y) and N is a fixed point $(3,0)$. The area of triangle OMN is denoted by A.

(b) Find A in terms of y. [1]

(c) Show that $\dfrac{\mathrm{d}A}{\mathrm{d}x}=\dfrac{3}{2}\left(\dfrac{y-2x}{2y-x-4}\right)$. [2]

(d) Hence find the exact value of x for which A is a maximum.
(You do not need to show that A is a maximum for the value of x found.) [2]

8 It is given that $(1+9x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=3y$, and the curve $y=\mathrm{f}(x)$ passes through the y-axis at $(0,\mathrm{e}^2)$.

(a) Show that $(1+9x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(18x-3)\frac{\mathrm{d}y}{\mathrm{d}x}=0$. [2]

(b) Find the Maclaurin series for y, up to and including the term in x^3, giving the exact coefficients for each term. [4]

(c) Given that $\ln y = 2+\tan^{-1}3x$, find the Maclaurin series for $\mathrm{e}^{\tan^{-1}3x}$, up to and including the term in x^3. [2]

9 A curve C has equation $y=ax+b+\frac{b-2a}{x+2}$, where a and b are real constants such that $a>0$, $a\neq\frac{1}{2}b$ and $x\neq-2$.

(a) Given that C has stationary points, use differentiation to find the relationship between a and b. [3]

It is now given that $a=1$ and $b=3$.

(b) Prove algebraically that y cannot lie between -1 and 3. [3]

(c) Sketch C, stating the equations of any asymptotes and the coordinates of any axial intercepts and turning points. [2]

10 The function f is defined by $\mathrm{f}: x \mapsto \dfrac{1}{(x+2)^2}$, $x \in \mathbb{R}, x \neq -2$.

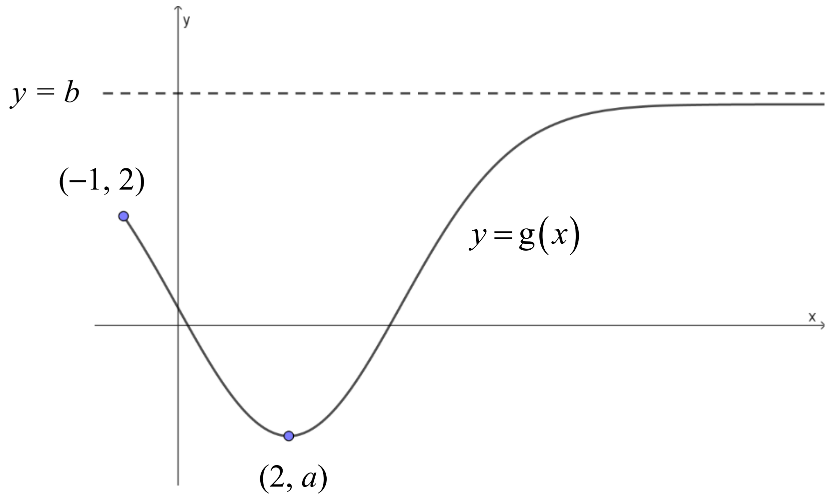
The domain of f is further restricted to be $x > a$, where a is an integer.

(a) State the least value of a such that the function f^{-1} exists. [1]

(b) Hence find $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(c) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram, giving the equations of any asymptotes and the coordinates of the points where the curves meet the axes. [2]

The function g is defined on the domain $[-1, \infty)$. The graph of $y = \mathrm{g}(x)$ has minimum point at (2, a) and the equation of its asymptote is $y = b$ as shown below.



(d) Determine if gf exist. If it exists, find its range. [3]

11 The curve C has cartesian equation $y = \sqrt{4 - 2x^2}$.

(a) Sketch the curve C, labelling the exact coordinates of any axial intercepts. [2]

(b) Find the equation of the tangent to C at the point $P\left(1, \sqrt{2}\right)$, leaving your answer in exact form. [2]

(c) Using the substitution $x = \sqrt{2}\sin\theta$, find the exact value of $\int_1^{\sqrt{2}} \sqrt{4 - 2x^2}\, \mathrm{d}x$. [4]

(d) Hence, find the exact value of the area of the region bounded by C, the tangent to C at P, and the x-axis. [2]

12 The curve C is defined by the parametric equations $x = 6t - 5$ and $y = 2t^2 + 1$, where $t \geq \frac{5}{6}$.

(a) Sketch the curve C, labelling the exact coordinates of any axial intercept(s). [1]

(b) Find the equation of the normal to C at the point P where $t = 1$. [2]

(c) Find the acute angle between the normal to C at the point P where $t = 1$ and the tangent to C at the point M where $t = 3$. [2]

(d) The curve C is translated 5 units in the positive x-direction and translated 3 units in the negative y-direction, to form the curve D. Find the equation of D in parametric form. [2]

The curve E is defined by the parametric equations $x = 7u$ and $y = \frac{9}{u}$, where $u \neq 0$.

(e) Show that at the point of intersection of the curves C and E, $6t^3 - 5t^2 + 3t - 34 = 0$. Deduce that there is only one point of intersection and find the coordinates of this point. [4]

13 A cafe roasts its own coffee beans and packages them to be sold. The amount of roasted coffee beans remaining in the cafe at time t days is denoted by x kg. The cafe produces roasted coffee beans at a fixed rate of 10 kg/day, and sells the roasted coffee beans at a rate proportional to x^2 kg/day.

(a) Given that there is no change in the amount of roasted coffee beans remaining in the cafe when there is 5 kg of beans remaining, show that $\frac{dx}{dt} = 10 - \frac{2}{5}x^2$. Find the general solution of this differential equation. (You do not have to make x the subject.) [3]

Due to improvement in roasting capabilities of the cafe, it is now able to produce roasted coffee beans at the rate of $3x$ kg/day, and sells the roasted coffee beans at a rate proportional to x^2 kg/day.

(b) Given that amount of roasted coffee beans is increasing at the rate of 6 kg/day when the amount of roasted coffee beans remaining in the cafe is 6 kg, write down the modified differential equation relating $\frac{dx}{dt}$ and x. [2]

(c) Given that the initial amount of roasted coffee beans in the cafe is 13.5 kg, solve the differential equation in part (b), expressing x in terms of t.
Deduce the amount of roasted coffee beans remaining in the cafe in the long run, and sketch the curve of x against t. [6]

(d) Based on the curve in part (c), explain in context why this might be favourable for the cafe owner. [1]

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