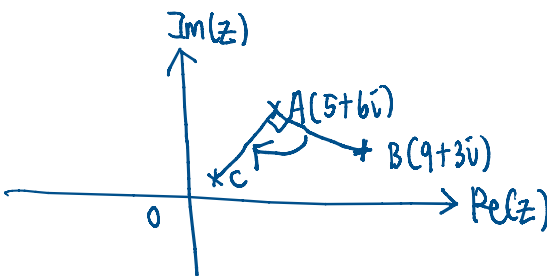


2025 H2MA Prelim Paper 2

1	Solution [5] Sequences	
(a) (i)	For $N = 4$, the sequence takes on a <u>constant value</u> with <u>$u_n = 4$</u> for all $n \geq 1$.	
(a) (ii)	For $N = 3$, $u_1 = 3$ $u_2 = 3(3) - 8 = 1$ $u_3 = 3(-5) - 8 = -23$ $\dots$ The sequence <u>decreases</u> and <u>approaches negative infinity</u> .	
(b)	<u>Method 1</u> $u_5 = 409$ Using $u_{n+1} = 3u_n - 8$, $3u_4 - 8 = 409 \Rightarrow u_4 = 139$ $3u_3 - 8 = 139 \Rightarrow u_3 = 49$ $3u_2 - 8 = 49 \Rightarrow u_2 = 19$ $3u_1 - 8 = 19 \Rightarrow u_1 = 9 = N$	
	<u>Method 2</u> $u_1 = N$ $u_2 = 3N - 8$ $u_3 = 9N - 32$ $u_4 = 27N - 104$ $u_5 = 81N - 320$ $81N - 320 = 409$ $N = 9$	
(c)	From part (a), when $u_1 = 4$, $u_n = 4$ for $n \geq 1$ $v_n = (-1)^n (u_n + k)$ $v_1 = -(4 + k)$ $v_2 = (4 + k)$ $v_3 = -(4 + k)$ $\vdots$	

	For $v_n = (-1)^n (u_n + k)$ to be convergent, then $k = -4$.	
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2	Solution [5] Complex Numbers	
(a)	The line segment AC can be obtained by rotating the line segment AB, 90° in a clockwise direction about A. $z_C - z_A = -i(z_B - z_A)$ $z_C = z_A - i(z_B - z_A)$ $z_C = (5 + 6i) - i[(9 + 3i) - (5 + 6i)]$ $= (5 + 6i) - i[4 - 3i]$ $= 2 + 2i$ 	
(b)	Method 1 $ABDC$ is a parallelogram Midpoint of AD = Midpoint of BC $\frac{z_A + z_D}{2} = \frac{z_B + z_C}{2}$ $(5 + 6i) + z_D = (9 + 3i) + (2 + 2i)$ $z_D = 11 + 5i - (5 + 6i)$ $z_D = 6 - i$ Method 2 $\overrightarrow{BD} = \overrightarrow{AC}$ $z_D - z_B = z_C - z_A$ $z_D = z_B + z_C - z_A$ $z_D = 6 - i$ Method 3 BD can be obtained by rotating the line segment BA, 90° in an anti-clockwise direction about B.	

$$iz_{BA} = z_{BD}$$

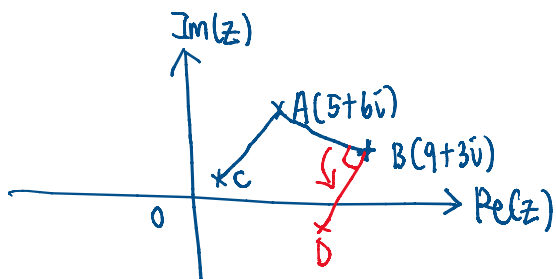
$$i(z_B - z_A) = z_D - z_B$$

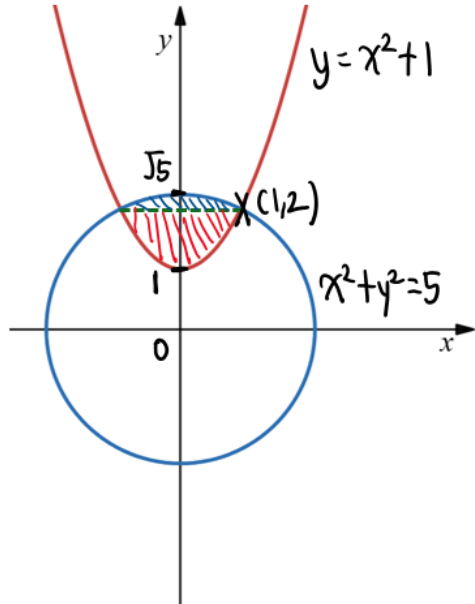
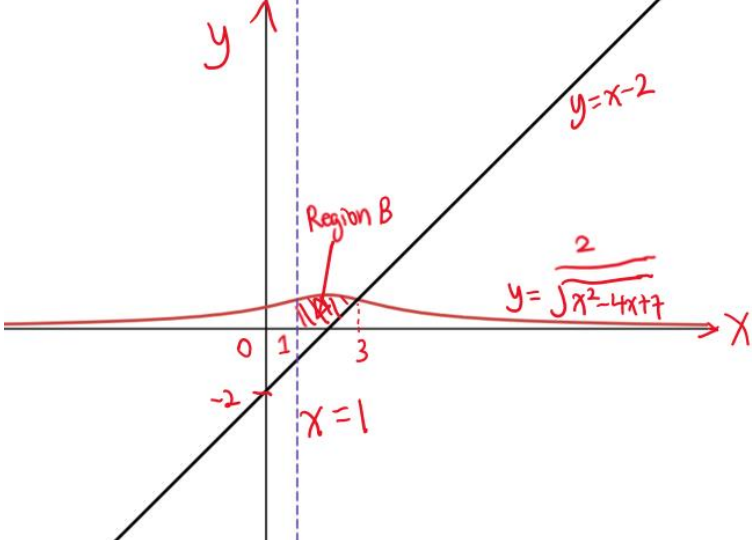
$$i(5 + 6i - 9 - 3i) = z_D - 9 - 3i$$

$$i(3i - 4) = z_D - 9 - 3i$$

$$z_D = 9 + 3i + i(3i - 4)$$

$$z_D = 6 - i$$



3	Solution [8] Volume of revolution	
(a)	 Volume required $= \pi \int_1^2 y - 1 \, dy + \pi \int_2^{\sqrt{5}} 5 - y^2 \, dy$ $= 1.95 \text{ units}^3$	
(b)	 x-coordinate of point of intersection between $y = \frac{2}{\sqrt{x^2 - 4x + 7}} \text{ and } y = x - 2 \text{ is } x = 3 \text{ (from GC)}$	

	Volume required $= \pi \int_1^3 \left(\frac{2}{\sqrt{x^2 - 4x + 7}} \right)^2 dx - \pi \int_2^3 (x-2)^2 dx$ $\text{OR } \pi \int_1^3 \left(\frac{2}{\sqrt{x^2 - 4x + 7}} \right)^2 dx - \frac{1}{3} \pi (1)^2 (1)$ $= \pi \int_1^3 \frac{4}{x^2 - 4x + 7} dx - \pi \int_2^3 x^2 - 4x + 4 dx$ $= 4\pi \int_1^3 \frac{1}{(x-2)^2 - 4 + 7} dx - \pi \left[\frac{x^3}{3} - 2x^2 + 4x \right]_2^3$ $= 4\pi \int_1^3 \frac{1}{(x-2)^2 + (\sqrt{3})^2} dx - \pi \left[\frac{x^3}{3} - 2x^2 + 4x \right]_2^3$ $= 4\pi \left[\frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{x-2}{\sqrt{3}} \right) \right]_1^3 - \frac{1}{3} \pi$ $= \frac{4}{\sqrt{3}} \pi \left[\tan^{-1} \left(\frac{1}{\sqrt{3}} \right) - \tan^{-1} \left(\frac{-1}{\sqrt{3}} \right) \right] - \frac{1}{3} \pi$ $= \frac{4}{\sqrt{3}} \pi \left[\frac{\pi}{6} + \frac{\pi}{6} \right] - \frac{1}{3} \pi$ $= \frac{4}{\sqrt{3}} \pi \left(\frac{\pi}{3} \right) - \frac{1}{3} \pi$ $= \frac{4}{9} \sqrt{3} \pi^2 - \frac{1}{3} \pi \text{ units}^3$ $a = \frac{4}{9}, b = -\frac{1}{3}$	
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4	Solution [10] AP GP	
(a)	Let u_n be the dose on the n^{th} day. $u_n = 0.75 + 0.5(n-1)$ $S_n = \frac{n}{2}(2(0.75) + (n-1)(0.5))$ $10 \leq \frac{n}{2}\left(1 + \frac{n}{2}\right) \quad \text{-----} (*)$ $\frac{1}{4}n^2 + \frac{1}{2}n \geq 10$ $n^2 + 2n \geq 40$ $(n+1)^2 \geq 41$ $n \geq -1 + \sqrt{41} \text{ or } n \leq -1 - \sqrt{41}$ (rej as no. of days cannot be -ve) $n \geq 5.40312$ Therefore, it takes 6 days. Alternatively, When $n = 5$, $\frac{1}{4}n^2 + \frac{1}{2}n = 8.75 < 10$ When $n = 6$, $\frac{1}{4}n^2 + \frac{1}{2}n = 12 \geq 10$ Therefore, it takes 6 days. $u_6 = 0.75 + 0.5(5) = 3.25$ The dose on the 6th day is 3.25 units.	

(b) (i)	Increase on n^{th} day $= S_n - S_{n-1}$ $= \frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} \right) - \frac{3}{20} \left(\frac{5^{n-1} - 4^{n-1}}{5^{n-2}} \right)$ $= \frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} - \frac{5^{n-1} - 4^{n-1}}{5^{n-2}} \right)$ $= \frac{3}{20} \left(\frac{1}{5^{n-2}} \right) \left(5^{n-1} - \frac{4}{5} (4^{n-1}) - 5^{n-1} + 4^{n-1} \right)$ $= \frac{3}{20} \left(\frac{1}{5^{n-2}} \right) \left(\frac{1}{5} \right) (4^{n-1})$ $= \frac{3}{20} \left(\frac{4}{5} \right)^{n-1}$ which is of the form ar^{n-1}, where $a = \frac{3}{20}$ and $r = \frac{4}{5}$ which is a GP. <u>Alternative:</u> Consider $\frac{u_{n+1}}{u_n} = \frac{\frac{3}{20} \left(\frac{4}{5} \right)^{n+1-1}}{\frac{3}{20} \left(\frac{4}{5} \right)^{n-1}} = \frac{4}{5}$ which is a constant Since consecutive terms have a common ratio $r = \frac{4}{5}$, the increase in dosage follows a GP.	
(b) (ii)	<u>Method 1</u> Long term dosage $= \lim_{n \rightarrow \infty} \frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} \right)$ $= \frac{3}{1 - \frac{4}{5}}$ $= \frac{3}{4}$ In the long term, the total dosage approaches 0.75 units. <u>Note:</u> After n days, the total dosage of the medicine is given by	

	$\frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} \right).$ Let S_n denote the total dosage of the new medicine after n days. Then $S_n = \frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} \right).$ Let v_n the change in dosage on nth day found in part (b)(i) where $v_n = S_n - S_{n-1}$ Since v_n is a geometric progression, we can use $S_\infty = \frac{a}{1-r}$, the sum to infinity formula to find the long term total dosage.	
	Method 2 $\begin{aligned} & \frac{3}{20} \left(\frac{5^n - 4^n}{5^{n-1}} \right) \\ &= \frac{3}{20} (5) \left(\frac{5^n - 4^n}{5^n} \right) \\ &= \frac{3}{20} (5) \left(1 - \left(\frac{4}{5} \right)^n \right) \end{aligned}$ As $n \rightarrow \infty$, $\left(\frac{4}{5} \right)^n \rightarrow 0$ Total dosage in the long term $\begin{aligned} &= \frac{3}{20} (5) (1 - 0) \\ &= \frac{3}{4} \end{aligned}$	
(c)	If the dosage increases as an AP, then it will continue to <u>increase to</u> (positive) <u>infinity</u>.	

5	Solution [12] 3D Vectors	
(a)	Since $\begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} = -2 - 3 + 5 = 0$, the satellite's path is parallel to Earth's Orbital plane. Since $\begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = 3 - 2 + 15 = 16 \neq 0$, the satellite does not sit on Earth's Orbital plane. Therefore, the satellite does not cross Earth's Orbital Plane. <u>Alternatively</u> Earth: $\mathbf{r} \bullet \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = 0 \text{ --- (1)}$ Satellite: $\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R} \text{ --- (2)}$ Sub (2) into (1): $\left[\begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \right] \bullet \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix} = 0 \text{ --- (3)}$ $16 + 0\lambda = 0$ There is no consistent value of λ for eqn (3) Therefore, the satellite does not cross Earth's Orbital Plane.	

(b)	$l_E : \mathbf{r} = \begin{pmatrix} 15 \\ 20 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix}, \mu \in \mathbb{R}$ $l_A : \frac{x-10}{2} = y+2 = \frac{0.25-z}{k}$ $\mathbf{r} = \begin{pmatrix} 10 \\ -2 \\ 0.25 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -k \end{pmatrix}, \text{ where } t \in \mathbb{R}$ Consider $\begin{pmatrix} 10+2t \\ -2+t \\ 0.25-tk \end{pmatrix} = \begin{pmatrix} 15+4\mu \\ 20-\mu \\ -1-\mu \end{pmatrix}$ $2t-4\mu=5$ $t+\mu=22$ $\mu-tk=-1.25$ From GC, $t=15.5, \mu=6.5, tk=7.75$ $\Rightarrow k = \frac{7.75}{15.5} = 0.5$ Therefore, the paths intersect when $k=0.5$. Therefore, for the paths to not intersect, $k \neq 0.5, k \in \mathbb{R}^+.$	
(c)	Let θ be the acute angle between the paths of the Earth and the asteroid. $\cos \theta = \frac{\begin{vmatrix} 2 \\ 1 \\ -3 \end{vmatrix} \bullet \begin{vmatrix} 4 \\ -1 \\ -1 \end{vmatrix}}{\sqrt{2^2+1^2+(-3)^2} \sqrt{4^2+(-1)^2+(-1)^2}}$ $= \frac{ 8-1+3 }{\sqrt{14}\sqrt{18}}$ $= \frac{10}{\sqrt{14}\sqrt{18}}$ $\theta = \cos^{-1}\left(\frac{10}{\sqrt{14}\sqrt{18}}\right) = 51.0^\circ \text{ (1 d.p.)}$	

(d) (i)	$\text{Distance} = \left \begin{pmatrix} 12 \\ 18 \\ -3 \end{pmatrix} - \begin{pmatrix} 12 \\ -1 \\ -2.75 \end{pmatrix} \right $ $= \left \begin{pmatrix} 0 \\ 19 \\ -0.25 \end{pmatrix} \right $ $= \sqrt{19^2 + (-0.25)^2}$ $= 19.0 \text{ million km (3 s.f.)}$	
(d) (ii)	The shortest distance between the paths will be when the vector connecting the two points on the paths is perpendicular to each of the two paths. $\begin{pmatrix} 0 \\ 19 \\ -0.25 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = 19.75 \neq 0 \text{ --- (1)}$ OR $\begin{pmatrix} 0 \\ 19 \\ -0.25 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} = -18.75 \neq 0 \text{ --- (2)}$ Since the vector is not perpendicular to one of the paths, the distance found is not the shortest distance between the paths. To show that $\left \begin{pmatrix} 0 \\ 19 \\ -0.25 \end{pmatrix} \right$ is not the shortest distance, it suffices to show either (1) or (2).	

6	Solution [4] Permutations and Combinations	
(a)	Number of ways $(5-1)! = 24$	
(b)	<u>Case 1:</u> One person gets 3 more slices $\binom{3}{1} \times \left(\frac{5!}{3!}\right) = 60$ <u>Case 2:</u> Two people get 2 more slices $\binom{3}{2} \times \left(\frac{5!}{2!2!}\right) = 90$ Total Number of ways = 150 Alternatively <u>Case 1:</u> One person gets 3 more slices $\binom{3}{1} \binom{5}{3} \binom{2}{1} = 60$ <u>Case 2:</u> Two people get 2 more slices $\binom{3}{2} \binom{5}{2} \binom{3}{2} = 90$ Total Number of ways = 150	

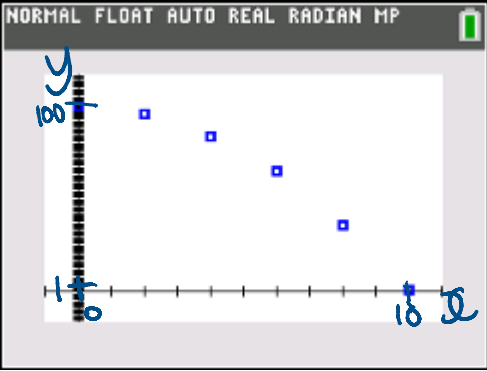
7	Solution [5] DRV															
(a)	Let X be the r.v. denoting the score. <table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$P(X=x)$</td><td>$\frac{1}{18}$</td><td>$\frac{1}{18}$</td><td>$\frac{2}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{2}{9}$</td></tr></table> $P(X=1)=\left(\frac{1}{6}+\frac{1}{6}\right)\left(\frac{1}{6}\right)=\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{1}{18}$ $P(X=2)=\left(\frac{1}{6}+\frac{1}{6}\right)\left(\frac{1}{6}\right)=\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{1}{18}$ $P(X=3)=\frac{1}{6}+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{4}{18}$ or $\frac{2}{9}$ $P(X=4)=\frac{1}{6}+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{4}{18}$ or $\frac{2}{9}$ $P(X=5)=\frac{1}{6}+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{4}{18}$ or $\frac{2}{9}$ $P(X=6)=\frac{1}{6}+\left(\frac{1}{3}\right)\left(\frac{1}{6}\right)=\frac{4}{18}$ or $\frac{2}{9}$	x	1	2	3	4	5	6	$P(X=x)$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	
x	1	2	3	4	5	6										
$P(X=x)$	$\frac{1}{18}$	$\frac{1}{18}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{2}{9}$										
(b)	$E(X)=(1)\left(\frac{1}{18}\right)+(2)\left(\frac{1}{18}\right)+(3)\left(\frac{4}{18}\right)+4\left(\frac{4}{18}\right)$ $+ (5)\left(\frac{4}{18}\right)+(6)\left(\frac{4}{18}\right)$ $=\frac{1+2+12+16+20+24}{18}$ $=\frac{75}{18}=\frac{25}{6}$ $\frac{25}{6}$ is the average score when the game is played infinitely many (or a large number of) times.															

(c)	$E(X^2) = \frac{349}{18}$ $E(X) = \frac{25}{6}$ $\text{Var}(X) = \frac{349}{18} - \left(\frac{25}{6}\right)^2 = \frac{73}{36}$	
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8	Solution [7] Probability	
(a)	$P(W, G, G) = \frac{3!}{2!} \left(\frac{1}{10} \right) \left(\frac{3}{10} \right)^2 = \frac{27}{1000}$	
(b)	$P(\text{White ball drawn} \mid \text{Red ball on 2nd attempt})$ $= \frac{P(\text{White ball drawn} \cap \text{Red ball on 2nd attempt})}{P(\text{Red ball on 2nd attempt})} \quad \text{---(*)}$ $= \frac{P(WRW') + P(RRW) + P(BRW) + P(GRW)}{P(R'R) + P(RR)}$ $= \frac{\left[\left(\frac{1}{10} \right) \left(\frac{4}{10} \right) \left(\frac{8}{9} \right) + \left(\frac{4}{10} \right) \left(\frac{3}{9} \right) \left(\frac{1}{8} \right) \right.}{\left(\frac{6}{10} \right) \left(\frac{4}{10} \right) + \left(\frac{4}{10} \right) \left(\frac{3}{9} \right)}$ $\left. + \left(\frac{2}{10} \right) \left(\frac{4}{10} \right) \left(\frac{1}{9} \right) + \left(\frac{3}{10} \right) \left(\frac{4}{10} \right) \left(\frac{1}{9} \right) \right]$ $= \frac{67/900}{28/75}$ $= \frac{67}{336}$	
(c)	$P(A) = \frac{1}{10}$ $P(B) = P(RW) + P(R'W) = \left(\frac{4}{10} \right) \left(\frac{1}{9} \right) + \left(\frac{6}{10} \right) \left(\frac{1}{10} \right) = \frac{47}{450}$ $P(A \cap B) = P(WW) = \left(\frac{1}{10} \right) \left(\frac{1}{10} \right) = \frac{1}{100}$ $P(A \cap B) \neq P(A)P(B)$ Therefore, A and B are not independent. Alternative: $P(A) = \frac{1}{10}$ $P(B) = P(RW) + P(R'W) = \left(\frac{4}{10} \right) \left(\frac{1}{9} \right) + \left(\frac{6}{10} \right) \left(\frac{1}{10} \right) = \frac{47}{450}$ $P(B \mid A) = \left(\frac{1}{10} \times \frac{1}{10} \right) \div \left(\frac{1}{10} \right) \neq \frac{47}{450}$ $P(B \mid A) \neq P(B)$ Therefore, A and B are not independent.	

9	Solution [8] Binomial Distribution	
(a)	Assume that the <u>probability in which a lens is defective is constant</u> throughout the entire population of lenses produced. This may not be valid, since machine parts degrade gradually which affects the manufacturing process and thus <u>probability of a lens being defective changes over time</u>.	
OR	Assume that the <u>event in which a lens is defective is independent to all other lenses</u>. This may not be appropriate, since it is possible that a <u>defective lens could cause other lens to be defective</u> downstream (e.g. a defective lens occurring during the process of production causes misalignment in subsequent lenses and thus causes more defective lenses).	
(b)	Let X denote the number of defective lenses, out of 100. $X \sim B\left(100, \frac{1}{20}\right)$ $P(X = 3) = 0.13958$ Let Y be the random variable for the number of samples that has exactly 3 defective lenses, out of 30. $Y \sim B(30, 0.13958)$ $P(Y > 10)$ $= 1 - P(Y \leq 10)$ $= 0.00163393$ $= 0.00163$	

(c)	$ \begin{aligned} & P(X_1 + X_2 = 10 \mid X_1 = 3 \text{ or } X_2 = 3 \text{ but not both}) \\ &= \frac{P(X_1 + X_2 = 10 \cap (X_1 = 3 \text{ or } X_2 = 3))}{P(X_1 = 3 \text{ or } X_2 = 3 \text{ but not both})} \\ &= \frac{P(X_1 = 3 \cap X_2 = 7) + P(X_2 = 3 \cap X_1 = 7)}{P(X_1 = 3) + P(X_2 = 3) - 2P(X_1 = 3)P(X_2 = 3)} \\ &= \frac{(0.13958)(0.10603) \times 2}{0.13958 + 0.13958 - 2(0.13958)^2} \\ &= 0.12323 \\ &= 0.123 \end{aligned} $ Denominator can be computed via $ \begin{aligned} & P(X_1 = 3 \text{ or } X_2 = 3 \text{ but not both}) \\ &= P(\text{exactly one of } X_1 = 3 \text{ or } X_2 = 3) \\ &= P(Y = 1) = 0.2401883, \end{aligned} $ where Y is the number of samples, out of 2, to have exactly 3 defective lenses, $Y \sim B(2, 0.13958)$	
(d)	$ \frac{X_1 + X_2}{2} \leq 2 \Rightarrow \frac{3 + X_2}{2} \leq 2 \Rightarrow X_2 \leq 1 $ Required Probability $ \begin{aligned} &= P(X_1 \leq 2) + P(X_1 = 3)(X_2 \leq 1) \\ &= 0.11826 + (0.13958)(0.0370815) \\ &= 0.123 \end{aligned} $	

10	Solution [11] Correlation & regression																						
(a)	 From the scatter diagram, <u>when x increases, y decreases at an increasing rate</u>, indicating that model B is not suitable. <u>Model C is also not valid as it is not defined at $x = 10$.</u> Thus Model A is the most suitable.																						
(b)	$r = -0.99995$ $y = -0.99175x^2 + 100.03088$ $y = -0.992x^2 + 100$ (3 s.f)																						
(c)	When $y = 20$, $x = 8.983 \approx 8.98$ m (to 3 s.f) The estimate is reliable as $y = 20$ lies within the input data range $1 \leq y \leq 100$ AND $ r $ is very close to 1.																						
(d)	As residuals may be positive or negative (or zero), they may cancel out if we simply sum the residuals. Squaring the residuals allows us to sum up non-negative numbers for a more accurate assessment of the fit of the model.																						
(d)																							
(ii)	<table><tr><td>x</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td></tr><tr><td>t</td><td>29.1</td><td>58.3</td><td>87.5</td><td>116.6</td><td>145.8</td><td>174.9</td></tr><tr><td>$t = 2.916x$</td><td>29.16</td><td>58.32</td><td>87.48</td><td>116.64</td><td>145.8</td><td>174.96</td></tr></table> By GC, Sum of squares of residuals $= (-0.06)^2 + (-0.02)^2 + (0.02)^2 + (-0.04)^2 + 0^2 + (-0.06)^2$ $= 0.0096$	x	10	20	30	40	50	60	t	29.1	58.3	87.5	116.6	145.8	174.9	$t = 2.916x$	29.16	58.32	87.48	116.64	145.8	174.96	
x	10	20	30	40	50	60																	
t	29.1	58.3	87.5	116.6	145.8	174.9																	
$t = 2.916x$	29.16	58.32	87.48	116.64	145.8	174.96																	
(d)																							
(iii)	The sum of squares of residuals for the line $t = 2.916x + D$ will be less than the sum of squares of residuals for the line $t = 2.916x$ i.e. less than 0.0096.																						

11	Solution [12] Hypothesis testing	
(a)	Each of the 10 000 strings should have an equal probability to be selected as part of the 40 strings to be measured. The selection of a string is independent of the selection of any other strings.	
(b)	Unbiased estimate of population mean, $\bar{x} = \frac{10.316}{40} + a = a + 0.2579 \text{ cm}$ $s^2 = \frac{1}{40-1} \left[41.7 - \frac{(10.316)^2}{40} \right]$ $= 1.0010129 \text{ cm}^2$	
(c)	Let X be the random variable denoting the length of the strings, Let μ be the population mean length of the strings. To test $H_0 : \mu = 100$ against $H_1 : \mu \neq 100$ at 10% level of significance. Test Statistics: Under H_0, $\bar{X} \sim N\left(100, \frac{s^2}{40}\right)$ approximately, $Z = \frac{\bar{X} - 100}{s / \sqrt{40}} \sim N(0,1), \text{ where } s = \sqrt{1.0010129}$ Critical region: Reject H_0 if $z_{cal} < -1.6448536$ or $z_{cal} > 1.6448536$. $z_{cal} < -1.64$ or $z_{cal} > 1.644$ (3 s.f.) Since H_0 is not rejected, $-1.6448536 \leq z_{cal} \leq 1.6448536$	

$$-1.6448536 \leq \frac{\bar{x} - 100}{\frac{\sqrt{1.0010129}}{\sqrt{40}}} \leq 1.6448536$$

$$-0.26020587 \leq \bar{x} - 100 \leq 0.26020587$$

$$99.73979413 \leq \bar{x} \leq 100.26020587$$

$$99.74 \leq \bar{x} \leq 100.26$$

Alternative:

Let X be the random variable denoting the length of the strings,

Let μ be the mean length of the strings.

To test $H_0 : \mu = 100$

against $H_1 : \mu \neq 100$

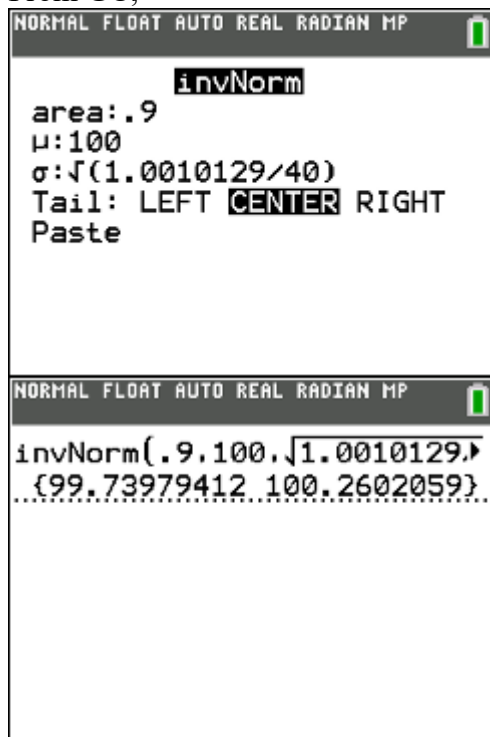
at 10% level of significance.

Test Statistics:

Under H_0 , $\bar{X} \sim N\left(100, \frac{s^2}{40}\right)$ approximately

$$P(a < \bar{X} < b) = 0.9$$

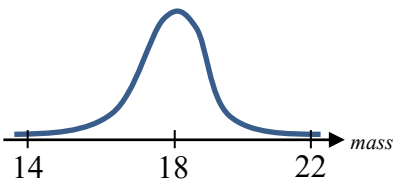
From GC,



$$99.73979413 < \bar{x} < 100.26020587$$

$$99.74 \leq \bar{x} \leq 100.26$$

(d)	As the sample size is large ($n = 40$), Central Limit Theorem can be used to approximate the distribution of $\bar{X}$ to a Normal Distribution. Thus no assumption is required about the distribution of the strings.	
(e)	For the same set of data, $p\text{-value of 1 tail test} = \frac{1}{2} \times p\text{-value of 2 tail test}$ To test $H_0 : \mu = 100$ against $H_1 : \mu < 100$ at 6% level of significance. Test Statistics: Under H_0, $p\text{-value} = \frac{1}{2} \times 0.10286 = 0.05143 < 0.06$, Therefore we reject H_0. There is sufficient evidence that the strings are shorter than 100cm at 6% level of significance.	

12	Solution [12] Normal Dist P2 Q12	
(a)		
(b)	Let D be the random variable denoting the mass of a randomly chosen metal disc. $D \sim N(18, 0.6^2)$ Probability required $= \binom{2}{1} [P(D < 19)][P(D > 19)] \text{ --- } (*)$ $= 0.0910128294$ $\approx 0.0910 \text{ (3 s.f.)}$	
(c)	$P(18 - k < D < 18 + k) < 0.9$ Since $P(17.01308782 < D < 18.98691218) = 0.9$, then $18 + k < 18.98691218$ $k < 0.987$	
(d)	7.65% of the metal bars have masses more than 21 grams. Let B be the random variable denoting the mass of a randomly chosen metal bar. $B \sim N(20, \sigma^2)$ $P(B > 21) = 0.0765$ $P\left(Z > \frac{21 - 20}{\sigma}\right) = 0.0765$ $P\left(Z > \frac{1}{\sigma}\right) = 0.0765$ $\frac{1}{\sigma} = 1.429014729$ $\sigma = 0.6997$ $\sigma = 0.700 \text{ (3.s.f.)}$	
(e)	Let S be the random variable denoting the mass of a randomly chosen metal strip. $S = 0.8B \sim N(0.8 \times 20, 0.8^2 \times 0.5^2)$	

	$S \sim N(16, 0.4^2)$ $P(15.8 < S < 17.2) = 0.690125006$ Let X be the random variable denoting the number of metal strips whose mass lies between 15.8 grams and 17.2 grams, out of 8 bars. $X \sim B(8, 0.690125006)$ $P(X \geq 5)$ $= 1 - P(X \leq 4)$ $= 0.7875986163$ $\approx 0.788 \quad (3.s.f)$	
(f)	Let C be the random variable denoting the mass of a randomly chosen protective case. $C \sim N(0.3 \times 20, 0.3^2 \times 0.5^2)$ $C \sim N(6, 0.15^2)$ $B \sim N(20, 0.5^2)$ Let $Y = B + C$ $Y \sim N(20 + 6, 0.5^2 + 0.15^2)$ $Y \sim N(26, 0.2725)$ Let $T = Y_1 + Y_2 + Y_3 + Y_4$ $\text{Var}(T) = 4 \times 0.2725$ $= 1.09 \quad (3 \text{ s.f})$	
	<u>Common Mistake</u> Let C be the random variable denoting the mass of a randomly chosen protective case. $C = 0.3B$ $B \sim N(20, 0.5^2)$ Let $Y = B + C$ $Y = B + 0.3B = 1.3B$ $Y \sim N(1.3 \times 20, 1.3^2 \times 0.5^2)$ $Y \sim N(26, 0.4225)$ Let $T = Y_1 + Y_2 + Y_3 + Y_4$ $T \sim N(4 \times 26, 4 \times 0.4225)$ $T \sim N(104, 1.69)$	
