H

TEMASEK JUNIOR COLLEGE

2025 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS 9758/01

1 Sep 2025

3 hours

Additional Materials: Printed Answer Booklet
List of Formulae (MF27)

READ THESE INSTRUCTIONS FIRST

Answer all questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages and **2** blank pages.

1 The sequence $\{u_n\}$ is a geometric sequence with first term a and common ratio $\frac{3}{2}$, where $a > 0$.

Another sequence $\{v_n\}$ is defined by $v_n = \frac{9}{u_n}$ for all positive integers n.

(a) Show that $\{v_n\}$ is also a geometric sequence. [2]

(b) Given that $v_2 = u_2$, find the value of a. [2]

2 The curve C has equation

$$xy = 1 + (x - y)^3.$$

Find the gradient of the tangent to C at the point A where $y = 0$. [4]

3 A curve C has equation $y = \frac{a}{2bx - x^2}$, where $a, b > 0$.

(a) Describe the transformation that maps the graph of C onto the graph of $y = \frac{a}{b^2 - x^2}$. [2]

(b) Given that $a = 1$ and $b = 2$, sketch C stating the equations of any asymptotes and coordinates of any stationary points and of the points where the curve crosses the axes. [2]

4 In the triangle ABC, $AC = 1$, angle $BAC = \frac{\pi}{3}$ radians and angle $ABC = \left(\frac{\pi}{6} + \theta\right)$ radians.

(a) Show that $BC = \frac{\sqrt{3}}{\cos\theta + \sqrt{3}\sin\theta}$. [2]

(b) Given that θ is sufficiently small such that θ^3 and higher powers of θ may be neglected, show that

$$BC \approx \sqrt{3}\left(1 + a\theta + b\theta^2\right)$$

where a and b are constants to be determined. [3]

5 **(a)** Without using a calculator, solve the inequality $\frac{2x - 25}{x^2 - 2x - 3} \le 2$. [4]

(b) Hence solve the inequality $\frac{2|x+1| - 25}{(x+1)^2 - 2|x+1| - 3} \le 2$. [3]

6 The points A and B lie on a circle with center O and radius λ unit. With reference to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point X on the line segment AB is such that $AX : XB = 1 : 3$ and the point Y is the foot of perpendicular of X on OB.

(a) Find the position vector of X. [1]

It is given that the acute angle AOB is $\dfrac{\pi}{6}$.

(b) Find $\mathbf{a}\cdot\mathbf{b}$ in terms of λ. [1]

(c) Show that the position vector of Y is $\dfrac{2+3\sqrt{3}}{8}\mathbf{b}$. [2]

(d) Hence find the exact area of ΔOXY in term of λ. [4]

7 Mabel and Janice decided to start a 5-year savings plan beginning in January 2026.

Mabel saves using a piggy bank. At the start of January 2026, she deposits \$101. Each subsequent month, she increases her deposit by \$1—so she deposits \$101 in January, \$102 in February, \$103 in March, and so on, until \$112 in December. At the start of each new year, she resets her monthly deposit to \$101 in January and repeats the same pattern through December. She continues this routine from 2026 to 2030, inclusive.

Janice, on the other hand, deposits \$100 at the start of every month into a bank account that earns 0.3% interest per month, with interest calculated and added into the account at the end of each month.

(a) Show that Janice will have more money in her savings account than Mabel has in her piggy bank at the end of December 2030. [5]

(b) Find the month and year when Janice's savings first exceed Mabel's savings. [4]

8 **(a)** The complex numbers z and w satisfy the following equations.

$$2z + |w| = 2 - 4\mathrm{i}$$
$$\mathrm{i}z - w = 2\mathrm{i}$$

Find z and w, giving your answers in the form $a + \mathrm{i}b$, where a and b are real numbers.

[5]

(b) It is given that $z_1 = 2 - \mathrm{i}$ and $z_3 = -3 + 2\mathrm{i}$. On an Argand diagram, mark the points A and C representing z_1 and z_3 respectively. [1]

The points B and D on the Argand diagram represent complex numbers z_2 and z_4 respectively. Given that $ABCD$ is a square, labelled in an anti-clockwise direction, find z_2 and z_4. [4]

9 A curve C is defined parametrically by the equations

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t \neq 0.$$

(a) Sketch the curve C, showing clearly the coordinates of the axial intercepts. [2]

(b) Use differentiation to find the values of t for which the tangents to the curve are parallel to the y-axis. [3]

(c) Show that the equation of normal at the point where $t = 2$ is given by $y = -\frac{3}{5}x + 3$. [3]

(d) The normal at the point where $t = 2$ cuts the curve C again at the point Q. Determine the exact coordinates of Q. [3]

10 **(a)** Differentiate $\sqrt{1-x^2}$ with respect to x.

Hence evaluate $\int_{-1}^{1} \cos^{-1} x \, dx$, giving your answer in exact form. [5]

(b) The finite region R is bounded by the curve $y = \cos^{-1}(x-3)$, where $2 \leq x \leq 4$, the line $y = \pi$ and the axes.

(i) Describe a geometrical transformation which will map the graph of $y = \cos^{-1}(x-3)$ onto the graph of $y = \cos^{-1} x$. [1]

(ii) Hence or otherwise, find the area of R in exact form. [2]

(c) Find the exact value of the volume generated when R is rotated completely about the y-axis. [4]

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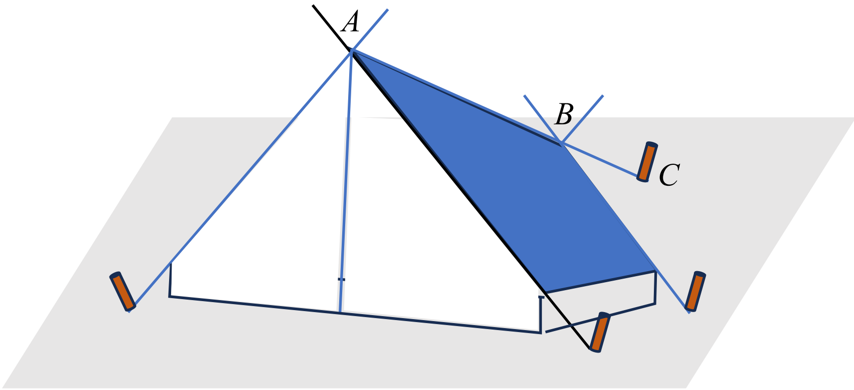

At a campsite, an A-frame tent is pitched on level ground. Its roof is formed by two slanted polyester sheets lying on two intersecting planes,

$$\pi_1 : \quad x + y + \alpha z = 6$$

$$\pi_2 : \; 2x - y + z = 4$$

where α is a positive real constant. The intersection of π_1 and π_2 forms a central ridge in the form of a line AB. This ridge is extended with a rope, which is secured to the ground at the point C for stability. The ground is assumed to be the horizontal xy-plane.

(a) Find a vector parallel to the central ridge, giving your answer in terms of α. [2]

(b) If C lies on AB produced, show that the coordinates of C is $\left(\frac{10}{3}, \frac{8}{3}, 0\right)$. [3]

Hence, or otherwise, write down a vector equation of line AB in term of α. [1]

(c) If the angle between the two slanted polyester sheets is 60°, find the value of α. [3]

It is now given that $\alpha = 1$.

(d) A string is tied between two hooks inside the tent to hang decorations. One hook is fixed at the point D (2, 3, 1) which lies on π_1. The other hook is located at the point E which lies on π_2 such that the string DE is perpendicular to π_2. Find the coordinates of E. [3]

12 **(a)** Show, by means of the substitution $w = x^2 y$, that the differential equation

$$2y + x\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^4 y^2}{\sqrt{x^2+1}}$$

can be reduced to the form $\dfrac{\mathrm{d}w}{\mathrm{d}x} = \dfrac{w^2 x}{\sqrt{x^2+1}}$. [2]

Hence find the general solution of the differential equation $2y + x\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x^4 y^2}{\sqrt{x^2+1}}$, leaving your answer in the form $y = \mathrm{f}(x)$. [3]

(b) At a durian plantation, mature durians are susceptible to pests' infection. The spread of pests at the plantation resulted in durians being classified into two categories, either they are bad durians that have been infected by the pests or good durians that have not been infected. It is given that x denotes the number of bad durians, in thousands, in a fixed population size P, in thousands, where $P > 4$. The rate of spread of infection, $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, where t represents the time taken in days, can be modelled as being proportional to the product of the number of bad durians and good durians that have not been infected. It is given that $x = 4$ and $\dfrac{\mathrm{d}x}{\mathrm{d}t} = P - 4$ when $t = 0$.

(i) Write down the differential equation involving $\dfrac{\mathrm{d}x}{\mathrm{d}t}$, x and P [2]

(ii) Solve the differential equation in part **(i)** and find x in terms of P and t. [5]

(iii) By using an appropriate graph, explain in context, the long-term implications if no additional measures are carried out to control the spread of the pests' infection. [2]

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