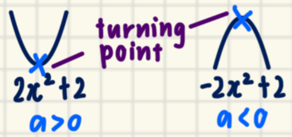
1. Functions and graphs and quadratic equations



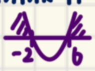
$b^2-4ac<0$
↳ above:
- coefficient of $x^2>0$
- min. value is positive

↳ below:
- coefficient of $x^2<0$
- min. value is negative

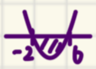
nature of roots:
$b^2-4ac\geq0$ real roots
$b^2-4ac>0$ 2 real and distinct roots
$b^2-4ac<0$ non-real roots
$b^2-4ac=0$ 2 real and equal roots

intersection:
$b^2-4ac>0$ line cuts curve at 2 distinct roots
$b^2-4ac=0$ line cuts curve at 1 point (tangent to curve)
$b^2-4ac<0$ line does not intersect curve

$(p-6)(p+2)>0$ → think it as if it is pointing outwards!



$(p-6)(p+2)<0$



→ think it as it is facing inside / pointing inside

2. Surds

law1 : $\sqrt{ab}=\sqrt{a}\times\sqrt{b}$
law2 : $\sqrt{\frac{a}{b}}=\sqrt{a}\div\sqrt{b}$
$=\frac{\sqrt{a}}{\sqrt{b}}$

law 3 : $\sqrt{a}\times\sqrt{a}=a$
law 4 : $m\sqrt{a}\pm n\sqrt{a}=(m\pm n)\sqrt{a}$

rationalising denominators:

1. $\frac{n}{\sqrt{a}}\frac{\times\sqrt{a}}{\times\sqrt{a}}$
2. $\frac{n}{\sqrt{a}+b}\frac{\times\sqrt{a}-b}{\times\sqrt{a}-b}$

3. Polynomials and partial fractions

dividend = divisor x quotient + remainder

e.g. long division

$$
\begin{array}{r l}
 & 2x^2 + 3x - 3 \\
4x^2 - x + 1 \,\big)\!\! & \overline{8x^4 + 10x^3 - 13x^2 + 0x + 2} \\
 & -(8x^4 - 2x^3 + 2x^2) \downarrow \\
\hline
 & 12x^3 - 15x^2 + 0x \\
 & -(12x^3 - 3x^2 + 3x) \downarrow \\
\hline
 & -12x^2 - 3x + 2 \\
 & -(-12x^2 + 3x - 3) \\
\hline
 & -6x + 5
\end{array}
$$

$\therefore\ 8x^4 + 10x^3 - 13x^2 + 2 = (4x^2 - x + 1)(2x^2 + 3x - 3) - 6x + 5$

cubic expressions:
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

partial fractions:

e.g. $\dfrac{2x+3}{x^2+x-12} = \dfrac{A}{x+4} + \dfrac{B}{x-3}$ $\qquad \dfrac{2x^2+3}{(x-2)(x+1)^2} = \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2}$

$\dfrac{2x+3}{(2x-5)^2} = \dfrac{A}{2x-5} + \dfrac{B}{(2x-5)^2}$ $\qquad \dfrac{14-11x}{(3x-2)(x^2+4)} = \dfrac{A}{3x-2} + \dfrac{Bx+C}{x^2+4}$

note: if numerator's highest power > denominator's highest power, do long division!!

4. Exponential and logarithmic equations

$2^x = 3$
$x = \log_2 3$

$\lg a + \lg b = \lg(ab)$

$\lg a - \lg b = \lg\left(\frac{a}{b}\right)$

$a = \log_b c$
$b^a = c$

$\lg a^b = b \lg a$
$\ln e = \log_e e$
$= 1$

$\log_a a = 1$
$\log_a 1 = 0$

change of base:
$\log_a b = \dfrac{\log_c b}{\log_c a}$

5. Binomial Theorem

general term:
$T_{r+1} = \binom{n}{r}(a^{n-r})(b^r)$

term independent of x:
$x^{18-3r} = x^0$
$18 - 3r = 0$
$3r = 18$
$r = 6$

6. Coordinate Geometry

length of line: $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

midpoint: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

note: $\perp$ bisector cuts through midpoint
parallel lines = same gradient

area of quadrilaterals: $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$
(anti-clockwise)

red subtract blue!

7. Circles

standard form: $(x-a)^2 + (y-b)^2 = r^2$

centre : (a, b)

radius: r

general formula: $x^2 + y^2 + 2gx + 2fy + c = 0$

centre: $(-g, -f)$

radius: $\sqrt{g^2 + f^2 - c}$

8. Trigonometric functions, trigonometric identities and equations

sin + cos − tan −	S	A	sin + cos + tan +
sin − cos − tan +	T	C	sin − cos + tan −

	30°	45°	60°
sin	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tan	$1/\sqrt{3}$	1	$\sqrt{3}$

$y = a\sin bx + c \;/\; a\cos bx + c \;/\; a\tan bx + c$

period $= \frac{360}{b} \Big/ \frac{2\pi}{b}$

max. value $= a + c$

amplitude $= a$

↳ always positive

min. value $= -a + c$

$\tan Q = \frac{\sin Q}{\cos Q}$

$\cot Q = \frac{1}{\tan Q} = \frac{\cos Q}{\sin Q}$

$\operatorname{cosec} Q = \frac{1}{\sin Q}$

$\sec Q = \frac{1}{\cos Q}$

$\sin(90^\circ - Q) = \cos Q$

$\cos(90^\circ - Q) = \sin Q$

$\tan(90^\circ - Q) = \cot Q$

trigonometric identities

$\sin^2 Q + \cos^2 Q = 1$

$1 + \tan^2 Q = \sec^2 Q$

$1 + \cos^2 Q = \operatorname{cosec}^2 Q$

double angle formulae

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

R- formula:

$a\sin Q \pm b\cos Q = R\sin(Q \pm \alpha)$

$a\cos Q \pm b\sin Q = R\cos(Q \mp \alpha)$

where $\tan\alpha = \frac{b}{a}$ and $R = \sqrt{a^2 + b^2}$

addition formula:

$\sin(a \pm b) = \sin a\cos b \pm \cos a\sin b$

$\cos(a \pm b) = \cos a\cos b \mp \sin a\sin b$

$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a\tan b}$

9. Differentiation

$\frac{d}{dx}(k) = 0$

power rule:
e.g. x^{20}
$= 20x^{19}$

chain rule:
e.g. $y = (4x^2-3)^9$
$= 9(4x^2-3)(8x)$

product rule:

$y = (x^3-2)(6-x)^4$
$= [4(6-x)^3(-1)(x^3-2)] + (3x^2)(6-x)^4$
$= -4(6-x)^3(x^3-2) + (3x^2)(6-x)^4$
$= (6-x)^3[-4(x^3-2) + (3x^2)(6-x)]$
$= (6-x)^3[-4x^3+8+18x^2-3x^3]$
$= (6-x)^3[-7x^3+18x^2+8]$

quotient rule:
* keep bottom, differentiate top!

$\frac{d}{dx}\sin x = \cos x$ $\quad$ $\frac{d}{dx}\sin^n x = n\sin^{n-1}x\cos x$

$\frac{d}{dx}\cos x = -\sin x$ $\quad$ $\frac{d}{dx}\cos^n x = -n\cos^{n-1}x\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\quad$ $\frac{d}{dx}\tan^n x = n\tan^{n-1}x\sec^2 x$

$\frac{d}{dx}e^x = e^x$ $\quad$ $\frac{d}{dx}\ln x = \frac{1}{x}$

$\frac{d}{dx}e^{f(x)} = f'(x)e^{f(x)}$ $\quad$ $\frac{d}{dx} = \ln f(x) = \frac{f'(x)}{f(x)}$

$\frac{d}{dx}e^{ax+b} = ae^{ax+b}$ $\quad$ $\frac{d}{dx} = \ln(ax+b) = \frac{a}{ax+b}$

10. Tangents and normal

grad. of tangent $= \frac{dy}{dx}$

grad. of normal $= -1 \div \frac{dy}{dx}$

stat. pt., $\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} > 0$, min. value
$\frac{d^2y}{dx^2} < 0$, max. value

11. Integration

$\int f(x)\,dx = F(x) + c$ $\quad$ $\int k f(x)\,dx = k\int f(x)\,dx$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ $\quad$ $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\int x^0\,dx = x + c$ $\quad$ $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int \cos x\,dx = \sin x + c$ $\quad$ $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c$

$\int \sin x\,dx = -\cos x + c$ $\quad$ $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c$

$\int \sec^2 x\,dx = \tan x + c$ $\quad$ $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + c$

$\int e^x\,dx = e^x + c$ $\quad$ $\int \frac{1}{x}\,dx = \ln x + c$

$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + c$ $\quad$ $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + c$

$\int \frac{f'(x)}{f(x)}\,dx = \ln[f(x)] + c$

$\int_a^b f(x)\,dx = [F(x)]_a^b$

big number on top!

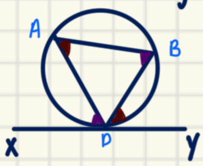
12. Kinematics

$$\begin{tikzcd}
\text{displacement}, s \arrow[r, bend left, "v = \frac{ds}{dt}"] & \text{velocity}, v \arrow[l, bend left, "s = \int v\,dt"] \arrow[r, bend left, "a = \frac{dv}{dt}"] & \text{acceleration}, a \arrow[l, bend left, "v = \int a\,dt"]
\end{tikzcd}$$

initial, $t=0$
instantaneously at rest, $v=0$
at fixed point, $s=0$
max. or min. displacement, $v=0$
max. or min. velocity, $a=0$

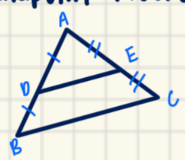
13. Plane Geometry

alternate segment theorem:



∠BAD = ∠BDY
∠ABD = ∠ADX
(∠s in alt. segment theorem)

midpoint theorem:



* prove they are similar △s, if not specified!