

1. Numbers, Prime Factorisation, HCF, LCM

numbers - irrational numbers, $\sqrt{2}$, $\sqrt[3]{5}$, π , $-\frac{\pi}{4}$

rational numbers, $\frac{1}{8}$, $-3\frac{1}{2}$, 99 , $\frac{\sqrt{16}}{\sqrt{25}}$, $\sqrt{9}$, $\sqrt[3]{27}$

decimal / fractions

integers - negative integers, -4 , -3 , -2 , -1

positive
 $1\frac{2}{3}$, $\frac{3}{4}$, 0.4

negative
 -0.9 , $-8\frac{1}{3}$, $-\frac{7}{8}$

whole numbers - zero (not positive or negative)

positive integers
 $1, 2, 3, 4$

prime factorisation - index notation

e.g.
$$\begin{array}{r} 2 \overline{) 20} \\ 2 \overline{) 10} \\ 5 \overline{) 5} \\ 1 \end{array} \quad 20 = 2^2 \times 5$$

HCF: e.g. $60 = 2^2 \times 3 \times 5$
 $120 = 2^3 \times 3 \times 5$
 $285 = 3 \times 5 \times 19$
HCF = 3×5

} takes what every number has

LCM: e.g. $6 = 2 \times 3$
 $12 = 2^3 \times 3$
 $21 = 3 \times 7$
LCM = $2^3 \times 3 \times 7$

} takes highest power

2. Maps and Scales

e.g. map: actual

$$50\text{cm}^2 : 200\text{km}^2$$

$$1\text{cm}^2 : 4\text{km}^2$$

$$1\text{cm} : 2\text{km}$$

$$1\text{cm} : 2000\text{m}$$

$$1\text{cm} : 200000\text{cm}$$

$$\text{answer} = 1 : 200000$$

3. Rate and ratio, direct and inverse proportion

direct proportion: $y = kx$
 $k = y/x$

inverse proportion: $yx = k$
 $y = k/x$

4. Algebra - Expansion and factorisation

expansion $\left\{ \begin{array}{l} (a+b)^2 = a^2 + 2ab + b^2 \\ (a-b)^2 = a^2 - 2ab + b^2 \\ (a+b)(a-b) = a^2 - b^2 \end{array} \right.$

factorisation $\left\{ \begin{array}{l} \text{grouping: } 2x + 2y - ax - ay \\ \quad = 2(x+y) - a(x+y) \\ \quad = (2-a)(x+y) \\ \text{common factor: } -ax - ay \\ \quad = -a(x+y) \end{array} \right.$

5. Polygons, triangle & quadrilaterals

ext. Δ s of a polygon: sum of ext. $\Delta = 360^\circ$

n-sided regular polygon: $360^\circ \div n$

int. Δ s of a polygon: sum of int. $\Delta = (n-2) \times 180^\circ$

angle properties:

- alt. Δ s
- vert. opp. Δ s
- corr. Δ s
- $\angle$ sum of Δ
- int. Δ s
- Δ s on a straight line

6. Percentage

discount = original price - selling price

% discount = $\frac{\text{discount}}{\text{orig. price}} \times 100\%$

profit = selling price - cost price

loss = cost price - selling price

simple interest:

$$I = \frac{PRT}{100}$$

compound interest:

$$A = P \left(1 + \frac{r}{100}\right)^n$$

Hire - purchase:

loan = price - downpayment

total interest: loan $\times$ rate $\times$ time
 $\rightarrow$ years

monthly instalment: loan of total interest / no. of mths

7. Misleading charts / graphs

- misuse of averages (mean, median and mode)
- incorrect graphs / visualisations
 - $\hookrightarrow$ 3D pie charts $\rightarrow$ may not add up to 100%
 - $\hookrightarrow$ inconsistent vertical / horizontal axis (e.g. 12 to 50)
- incorrect scaling
- ignoring missing data

8. Speed / time

distance-time graphs:

constant speed: dist. / time

speed-time graphs:

gradient > 0 : acceleration

gradient < 0 : deceleration / retardation

9. Indices & standard form

9 laws

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $a^0 = 1$
7. $a^{-n} = \frac{1}{a^n}$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$
9. $a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}}$

standard form

$$A \times 10^n \rightarrow 0 < A < 10$$

deci 10^{-1}

centi 10^{-2}

milli 10^{-3}

micro 10^{-6}

nano 10^{-9}

pico 10^{-12}

kilo 10^3

mega 10^6

giga 10^9

tera 10^{12}

million 10^6

billion 10^9

trillion 10^{12}

10. Solving quadratic eqns.

quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

completing the square:

$$\begin{aligned} \text{e.g. } f(x) &= 2x^2 - 4x + 9 \\ &= 2 \left[x^2 - 2x + \frac{5}{2} \right] \quad \text{* make sure coefficient} \\ &= 2 \left[(x-1)^2 - 1^2 + \frac{5}{2} \right] \quad \text{of } x^2 \text{ is 1} \\ &= 2 \left[(x-1)^2 + \frac{3}{2} \right] \\ &= 2(x-1) + 3 \quad \text{X} \end{aligned}$$

11. Quadratic Functions and Graphs

Quadratic Functions:
(x-h) (x-k)

1. U or $\wedge$
2. x-intercepts (let $y=0$)
3. y-intercepts (let $x=0$)
4. turning point (line of symmetry)
5. sketch! :)

12. Coordinate Geometry

length of line segment: $\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

gradient of straight line: $\frac{y_2 - y_1}{x_2 - x_1}$

eqn. of line: $y = mx + c$

parallel lines = same gradient

collinear points = same gradient

perpendicular = $m_1 \times m_2 = -1$

crosses x-axis: $y=0$

crosses y-axis: $x=0$

13. Trigonometry

$$\sin Q = \sin (180^\circ - Q)$$

$$\cos Q = -\cos (180^\circ - Q)$$

$$\tan Q = -\tan (180^\circ - Q)$$

area of Δ : $\frac{1}{2} ab \sin C$ [included $\angle$]

* bearings must be 3 digits!

e.g. 028°

sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A \text{ (find length)}$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \text{ (find angle)}$$

14. Congruence and similarity

congruent Δ s:

1. SSS
2. SAS
3. AAS
4. RHS

similar Δ s:

1. AA similarity
2. SSS similarity
3. SAS similarity

similar plane figures:

1. area:

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

2. volume:

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

15. Mensuration

<u>figure</u>	<u>volume</u>	<u>surface area</u>
cylinder	$\pi r^2 h$	$2\pi r^2 + 2\pi rh$
cone	$\frac{1}{3} \pi r^2 h$	$\pi r^2 + \pi rl$
pyramid	$\frac{1}{3} \times \text{base area} \times h$	base area + 4 sides
sphere	$\frac{4}{3} \pi r^3$	$4\pi r^2$
cuboid	$l \times b \times h$	$2(lb + lh + bh)$
cube	l^3	$6l^2$
hemisphere	$\frac{2}{3} \pi r^3$	$3\pi r^2$
prism	$b \times h$	cross-sectional area $\times h$

16. Arc length, area sector, radian measure

length of minor arc: $\frac{\theta}{360^\circ} \times 2\pi r$

area of sector: $\frac{\theta}{360^\circ} \times \pi r^2$

in radian measure

length of arc: $s = r\theta$

area of sector: $A = \frac{1}{2} r^2 \theta$

18. Sets and Venn diagrams

empty set: $\emptyset, \{\}$

universal set: $\mathcal{U}$

$a \in Q$ means that a is an element of Q

$b \notin Q$ means that b is not an element of Q

$n(Q)$ = number of elements in set Q

$A \subseteq B$ = A is a subset of B , $A = B$

$A \subset B$ = A is a proper subset of B , but $A \neq B$

$A \cap B$ = A intersects B

$A \cup B$ = union of A and B

19. Mean, median and mode

mean: sum of data / no. of data (average)

mode: highest frequency

median: middle value,

$\frac{n+1}{2}$ for even numbers

e.g. $\underbrace{2 \ 5 \ 6 \ 7 \ 8}_{\text{lower quartile}} \quad \uparrow \quad \underbrace{9 \ 14 \ 16 \ 20 \ 21 \ 30}_{\text{upper quartile}}$
median

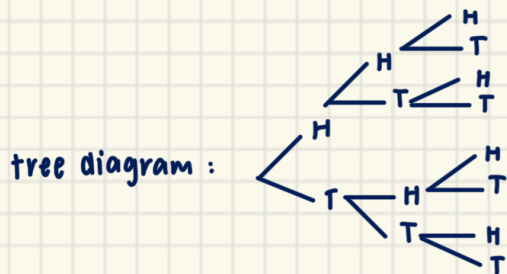
range = highest number - lowest number

interquartile range (IQR): median of upper quartile - median of lower quartile

mean: $\frac{\sum fx}{f}$

standard deviation: $\sqrt{\frac{\sum fx^2}{f}}$

20. Probability



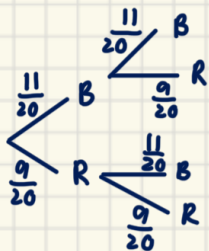
poker cards: 52 cards

- Ace / Queen / King / Prince ... = 4 each
- Each set (spade / club / heart / diamond) = 13 each

multiplication law

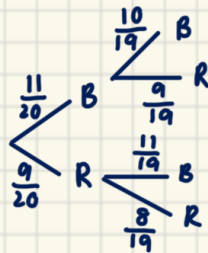
20 balls: 11B, 9R

"places it back in"



$$P(2B) = \frac{11}{20} \times \frac{11}{20} \\ = \frac{121}{400}$$

no placing back in



$$P(2B) = \frac{11}{20} \times \frac{10}{19} \\ = \frac{11}{38}$$

21. Matrices

e.g. $3B - 2A$

$$= 3 \begin{pmatrix} 1 & 9 \\ 2 & 5 \end{pmatrix} - 2 \begin{pmatrix} 3 & 4 \\ 5 & 9 \end{pmatrix} \\ = \begin{pmatrix} 3 & 27 \\ 6 & 15 \end{pmatrix} - \begin{pmatrix} 6 & 8 \\ 10 & 18 \end{pmatrix} \\ = \begin{pmatrix} -3 & 19 \\ -4 & -3 \end{pmatrix}$$

e.g. $\begin{pmatrix} -1 & 2 \\ 8 & 5 \\ 3 & -7 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 4 \end{pmatrix}$

3×2 2×2

$$= \begin{pmatrix} -4 & 5 \\ 11 & 44 \\ 13 & -19 \end{pmatrix}$$

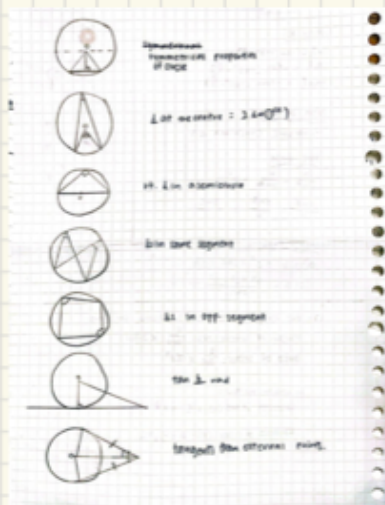
3×2

Qn: what does the elements of A represent?

It represents the total [number / cost / ...] for / to [] and [] respectively

Can be names / item, depending on question

22. Circle properties



23. Vectors ✕

direction of line segment: direction of vector

length of line segment: magnitude of vector

→ vector is usually denoted by $\overrightarrow{AB}$ or $\underline{a}$

column vector: $\begin{pmatrix} x \\ y \end{pmatrix}$, where x is the horizontal displacement and y is the vertical displacement

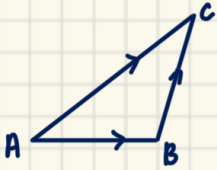
position vector: $\begin{pmatrix} x \\ y \end{pmatrix}$, where x is the x -coordinate and y is the y -coordinate

gradient of vector: y/x

1. $-\overrightarrow{AB} = \overrightarrow{BA}$

2. $-\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$

operation of vectors:



addition of vectors:

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

subtraction of vectors:

$$\overrightarrow{AB} = \overrightarrow{AC} + \underline{(-\overrightarrow{BC})}$$

$$\downarrow$$
$$\overrightarrow{CB} = -\overrightarrow{BC}$$

scalar multiplication of vectors:

$$k \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} kx \\ ky \end{pmatrix}, \text{ [where } k \text{ is a scalar]} \quad \text{* need to write in exams!}$$

magnitude of vector:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{x^2 + y^2}$$