RAFFLES INSTITUTION
H2 Mathematics 9758
2025 Year 6 Term 4 Post-Prelim Revision
Paper 1 (Source: A level questions)

Total number of marks: 100 **3 hour**

Please attempt this paper ***in one sitting under exam conditions within 3 hours before the lesson on 17 October 2025.***

1 The graph of $y = \dfrac{x+1}{ax^2+bx+c}$, where a, b and c are non-zero constants, has an asymptote at $x = -\dfrac{1}{2}$. The graph also has a turning point at $\left(-2, -\dfrac{1}{9}\right)$. Find the values of a, b and c. [4]

2 Find the exact equation of the tangent to the curve

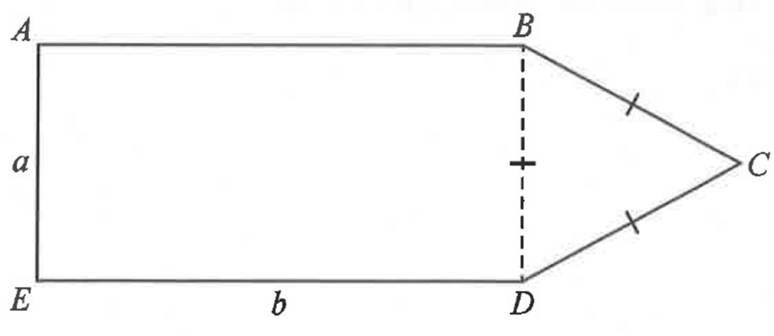
$$\ln y = (11-5x)^2$$

at the point where $x = 2$. [5]

3 **(a)** Without using a calculator, solve the inequality $\dfrac{4}{x+2} \geq \dfrac{x-3}{x}$. [4]

(b) Hence, solve the inequality $\dfrac{4}{|x|+2} \geq \dfrac{|x|-3}{|x|}$. [2]

4



A gardener designs a flower bed *ABCDE* in the shape of a rectangle with an equilateral triangle on one of the shorter sides. Side *AE* is of length a m and side *ED* is of length b m (see diagram). The total perimeter of the flower bed is 20 m.

Find the maximum possible area of the flower bed, showing that it is a maximum value. Give your answer correct to 4 significant figures. [6]

5 Vectors **a** and **b** are such that $\mathbf{a}.\mathbf{b}=-1$. It is also given that $(\mathbf{a}\times\mathbf{b}+\mathbf{a})$ is perpendicular to $(\mathbf{a}\times\mathbf{b}+\mathbf{b})$.

(a) Show that $|\mathbf{a}\times\mathbf{b}|=1$ [3]

(b) Hence find the angle between the direction of **a** and direction of **b**. [3]

6 **(b)** Given that $n\neq 0$, show that $\int x\cos nx\,\mathrm{d}x=\dfrac{x\sin nx}{n}+\dfrac{\cos nx}{n^2}+c$, where c is an arbitrary constant. [3]

(c) Using the result in part **(b)** show that, for all positive integers, n , the value of $\int_0^{\pi} x\cos nx\ \mathrm{d}x$ can be expressed as $\dfrac{k}{n^2}$, where the possible value(s) of k are to be determined [2]

(d) Using the result in part **(b)** find the exact value of $\int_0^{\frac{\pi}{2}}|x\cos 2x|\ \mathrm{d}x$. [3]

7 **(a)** Use double angle formula, show that $\cos^4\theta=\dfrac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$. [2]

(b) The region R lies in the first quadrant and is bounded by the curve $y^4=(9-x^2)^3$, the x-axis and the lines $x=1.5$ and $x=3$. R is rotated about the x-axis through 2π radians. Using the substitution $x=3\sin\theta$, find the exact volume generated. [6]

8 **(a)** The complex number $-2+\mathrm{i}$ is denoted by z and the complex number $1-3\mathrm{i}$ is denoted by w.

Without using a calculator, evaluate $wz-\dfrac{w}{z}$. . Give your answer in the form $c+d\,\mathrm{i}$, where c and d are real numbers. [4]

(b) The equation $z^3+az^2+bz+c=0$, where a, b and c are constants, has roots 3 and w where w is a complex number.

(i) State a condition on a, b and c for the third root to be w^*. [1]

(ii) Given that the condition in part **(bi)** holds, and that $w=-1+2\mathrm{i}$, find the values of a, b and c. [3]

9 It is given that $y = \cos\left(1 - \mathrm{e}^{2x}\right)$.

(a) Show that $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = k\left(\dfrac{\mathrm{d}y}{\mathrm{d}x} - 2y\mathrm{e}^{4x}\right)$, where k is a constant to be found. [3]

(b) By differentiation of the result in part **(a)**, find the first three non-zero terms of the Maclaurin expansion of $\cos\left(1 - \mathrm{e}^{2x}\right)$. [4]

(c) The first two non-zero terms of the Maclaurin expansion of $\cos\left(1 - \mathrm{e}^{2x}\right)$ are equal to the first two non-zero terms of the series expansion of $\dfrac{1}{\sqrt{a + bx^2}}$, where a and b are constants.

Using standard series from the List of Formulae (MF26), find the values of a and b. [2]

10 The line l_1, contains the point A with coordinates $(-3, 1, 2)$ and is parallel to the vector $\begin{pmatrix} 2 \\ 1 \\ a \end{pmatrix}$ where a is a constant. The line l_2 has equation $\dfrac{x+2}{3} = \dfrac{y-1}{2} = z - 5$. It is given that l_1 and l_2 cross at the point B.

(a) Find the value of a and the coordinates of B. [5]

(b) The plane π_1 contains the point A and is perpendicular to l_2.

(i) Find the shortest distance from B to π_1. [3]

(ii) Hence find the acute angle between l_1 and π_1. [2]

(c) The plane π_2 is perpendicular to π_1 and contains l_1. Find a cartesian equation of π_2. [3]

11 **(a)** Using the substitution $u=\sqrt{4x+1}$, show that $\int \frac{\sqrt{4x+1}}{x-2}\,\mathrm{d}x=\int \frac{2u^2}{u^2-9}\,\mathrm{d}u$. [3]

(b) The region R lies in the first quadrant and is bounded by the curve $y=\frac{\sqrt{4x+1}}{x-2}$ and the lines $x = 6$, $x = 12$ and $y = 0.5$.

(i) Sketch the graph of $y=\frac{\sqrt{4x+1}}{x-2}$ for $x\geq 0$, giving the coordinates of any intercepts with the axes and the equations of any asymptotes. Shade the region R on your sketch. [3]

(ii) Find the exact area of R, giving your answer in the form $a+b\ln c$. [5]

(iii) Find the volume of the solid generated when R is rotated through 2π radians about the x-axis. Give your answer correct to 2 decimal places. [3]

12 The mass of a person depends both on daily rate of energy intake and on daily rate of energy expenditure.

In this question, mass is in kg, time is in days, and energy intake and energy expenditure are measured in Calories per day.

The rate of change of a person's mass with respect to time is proportional to the difference between energy intake and energy expenditure.

Andrew has a mass of M kg and his energy intake is fixed at C Calories per day. For every kg of his mass, he expends 30 Calories per day.

(a) Show that $\frac{\mathrm{d}M}{\mathrm{d}t}=k\left(C-30M\right)$, where t is time and k is a constant. [1]

Andrew's initial mass is 110 kg.

(b) Find the energy intake such that he maintains his mass at 110 kg. [1]

As part of a health plan, Andrew fixes his energy intake at 80% of the value found in part **(b)**.

(c) By solving the differential equations in part **(a)**, show that Andrew's mass while he is on the plan satisfies the equation $M=88+22\mathrm{e}^{-30kt}$. [4]

Andrew's mass after 75 days on the plan is 100 kg.

(d) Find the number of additional days required on the plan for Andrew's mass to fall below 96 kg. [4]

(e) **(i)** Sketch a graph of Andrew's mass while on this plan. Explain why Andrew cannot achieve a mass of 80 kg using this plan. [2]

(ii) State the range of possible values of energy intake for which Andrew could achieve a mass of 80 kg. [1]