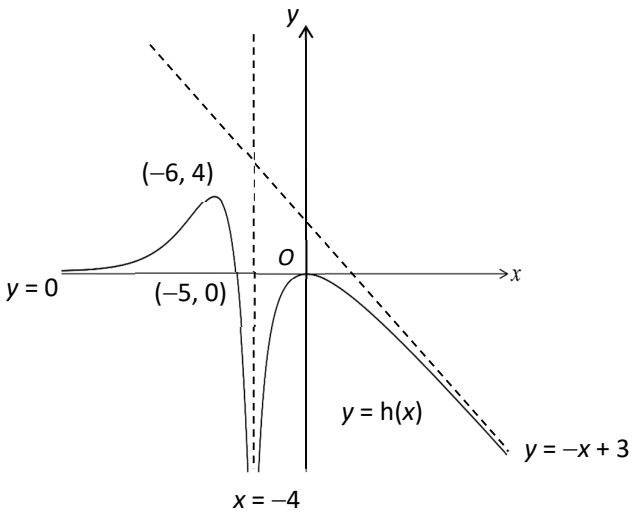
RAFFLES INSTITUTION
H2 Mathematics 9758
2025 Year 6 Term 4 Post-Prelim Revision
Paper 2 (Source: Other JCs' Prelim Qns)

Total number of marks: 100 **3 hour**

Please attempt this paper ***in one sitting under exam conditions within 3 hours before the lesson on 24 October 2025.***

Section A: Pure Mathematics [40 Marks]

1



(a) The diagram shows the curve $y = \mathrm{h}(x)$. The curve has maximum points at $(-6,\ 4)$ and the origin, and crosses the x-axis at $(-5,\ 0)$. The lines $y = 0$, $x = -4$ and $y = -x + 3$ are the horizontal, vertical and oblique asymptotes to the curve respectively.

(i) On the diagram given above, sketch the graph of $(x+6)^2 + (y-9)^2 = r^2$, where r is a positive constant. Find the range of values of r for the equation $(x+6)^2 + (\mathrm{h}(x)-9)^2 = r^2$ to have at least one real root. [3]

(ii) On a separate diagram, sketch the graph of $y = \dfrac{1}{\mathrm{h}(x)}$. [3]

(b) The graph of $y = 10 - |x+1|$ undergoes a sequence of transformations which transform its equation into $y = |x-1|$. Describe and write down the transformations. [3]

2 Referred to the origin O, the points A, B and X are such that $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OX} = \mathbf{a} + 3\mathbf{b}$. It is given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = \sqrt{5}$ and $|\overrightarrow{OX}| = \sqrt{73}$.

(a) By considering the scalar product of $\overrightarrow{OX}$ with itself, or otherwise, find the value of $\mathbf{a}.\mathbf{b}$. [3]

(b) Find the area of triangle OAX. [5]

(c) The variable point V, not necessarily coplanar with A, B and X, has position vector $\mathbf{v}$. Given that $|\mathbf{v} - \mathbf{a}| = |\mathbf{v} - \mathbf{b}|$, describe geometrically the set of all possible positions of the point V. [2]

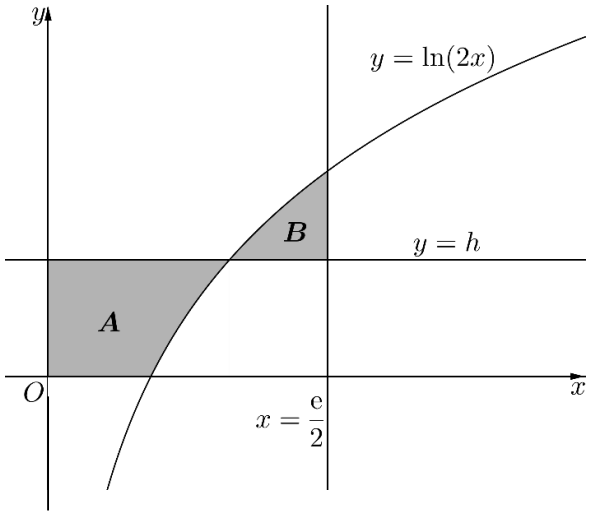
3 **(a)** It is given that $\mathrm{f}(x) = \begin{cases} \sin\left(\frac{x}{2}\right), & \text{for } 0 \le x \le 3\pi, \\ \frac{x}{\pi} - 4, & \text{for } 3\pi < x < 4\pi \end{cases}$

and that $\mathrm{f}(x) = \mathrm{f}(x + 4\pi)$ for all real values of x.

(i) Sketch the graph of $y = \mathrm{f}(x)$ for $-\frac{\pi}{2} \le x \le 6\pi$. [3]

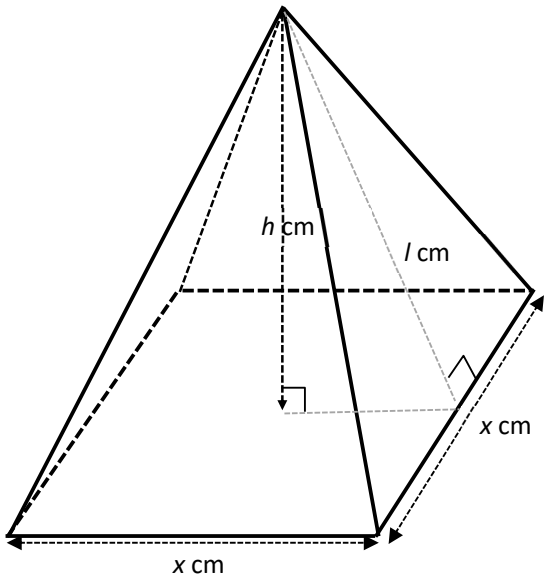
(ii) Find $\int_{-\frac{\pi}{2}}^{6\pi} |\mathrm{f}(x)| \, \mathrm{d}x$, leaving your answer in exact form. [2]

(b)



From the diagram above, the region A is bounded by the curve $y = \ln(2x)$, the line $y = h$, $h \in \mathbb{R}$, the x-axis and the y-axis while the region B is bounded by the curve $y = \ln(2x)$ and the lines $x = \frac{\mathrm{e}}{2}$ and $y = h$. Given that the volumes of the solids generated when A and B are rotated completely about the y-axis are equal, find the exact value of h. [5]

4 [The volume of a right square-based pyramid is $\frac{1}{3} \times \text{base area} \times \text{height}$.]



Jane designs a model in the shape of a right square-based pyramid. The square base has sides x cm. Each of the four lateral faces is a triangle with base x cm and perpendicular height l cm. The four lateral faces converge at the top of the pyramid to form an apex directly above the center of the square base. The vertical height of the pyramid is h cm. The model is assumed to be made of material of negligible thickness.

(a) Form an equation involving x, l and h. [1]

In the design of the model, Jane hopes to fix the total surface area, A cm^2 of the model but maximise the volume, V cm^3 of the model.

(b) Using the result in part **(a)**, show that

$$A = x^2 + 2x\sqrt{h^2 + \frac{x^2}{4}}.$$ [1]

(c) Hence show that

$$V^2 = \frac{Ax^2\left(A - 2x^2\right)}{36}.$$ [2]

(d) Use differentiation to show that the maximum V occurs when $x = \frac{\sqrt{A}}{2}$ and find a simplified expression for the maximum V in terms of A.
(You need not show that your answer gives a maximum.) [4]

(e) Given that V is a maximum, find the angle made by a lateral face and the base of the model, giving your answer to the nearest degree. [3]

Section B: Statistics [60 Marks]

5 In a certain university, there are 110 Co-Curricular Activities (CCAs) clustered into 4 categories. There are 22 Arts and Culture CCAs, 16 Community Service CCAs, 38 Physical Sports CCAs and 34 Special Interest CCAs.

(i) Albert wishes to find out about approaches to training of the Physical Sports CCAs, so he sends a questionnaire to 22 Physical Sports CCAs. Explain whether these 22 Physical Sports CCAs form a sample or a population. [1]

(ii) Benedict wishes to investigate the level of student engagement in CCAs, but does not want to obtain the detailed information necessary from all 110 CCAs. Explain how he should carry out his investigation, and why he should do the investigation in this way. [2]

(iii) Find the number of different possible samples of 16 CCAs, with 4 CCAs chosen from each category. [2]

6 Tetrahedral dice have four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled, and the numbers faced down on the two dice are recorded. The random variable T is defined as the score on the red die multiplied by the score on the blue die.

(i) Find the probability distribution of T. [3]

(ii) Find $\mathrm{E}(T)$ and show that $\mathrm{Var}(T)=\frac{115}{16}$. Show your workings clearly. [2]

(iii) Evaluate $\mathrm{P}(|T-2\mu|>\sigma)$, where $\mu=\mathrm{E}(T)$ and $\sigma^2=\mathrm{Var}(T)$. [3]

7 The events A and B are such that $\mathrm{P}(A)=a$ and $\mathrm{P}(B)=b$. A and B are independent events.

(a) Find an expression for $\mathrm{P}(A'\cap B')$ in terms of a and b, and hence prove that A' and B' are independent events. [2]

It is given that $\mathrm{P}(A'|B')=0.85$ and $\mathrm{P}(B')=0.8$.

(b) Find $\mathrm{P}(A\cap B')$. [2]

For a third event C, it is given that A and C are mutually exclusive and $\mathrm{P}(A'\cap C')=0.52$.

(c) Find $\mathrm{P}(C)$. [1]

(d) Hence find the set of possible values of $\mathrm{P}(A'\cap B'\cap C')$. [3]

8 In this question, you should state the parameters of any distribution you use.

A ceramic shop sells handmade ceramic cups. The mouths of the cups are assumed to be circular in shape. The diameters of the outer circumferences of the top rim of the cups, S, are assumed to follow a normal distribution with mean μ mm and standard deviation σ mm.

(a) It is given that $\mathrm{P}(S < 80.5) = \mathrm{P}(S > 84.5)$ and that the probability of the diameter of the outer circumference of the top rim of a randomly chosen cup being more than 85 mm is 1.15%. Find the value of μ, and show that $\sigma = 1.10$, when corrected to 3 significant figures. [3]

(b) The shop also makes covers of circular shape that can be fitted over the mouths of the cups. The diameter of any randomly chosen cover, C, in mm, follows a normal distribution with mean 83 mm and standard deviation 1.5 mm. A cover would be considered to be well-fitted over the mouth of a cup if the diameter of the cover is larger than that of the outer circumference of the top rim of the cup by not more than 2 mm. Find the probability that a randomly chosen cover is well-fitted over the mouth of a randomly chosen cup. [3]

(c) A cover and a cup are randomly chosen. If the cover is well-fitted over the mouth of the cup, find the probability that the diameter of the cover is larger than that of the cup by more than 1.5 mm. [3]

9 A store owner receives a shipment of stationery items, including notebooks, pens, and correction tapes. Historical data indicates that 1% of the notebooks, 2% of the pens, and 4% of the correction tapes are defective. The quality of notebooks, pens and correction tapes is independent of one another. The store owner decides to sell the stationery in 180 packets, each containing one notebook, two pens and one correction tape. A packet is deemed unsatisfactory if any of the four items is defective.

(i) Show that the probability that a randomly selected packet is unsatisfactory is 0.0872, correct to 3 significant figures. [1]

The number of packets that are unsatisfactory is denoted by X. You may assume that X can be modelled by a binomial distribution.

(ii) Find the probability that there are at least 5 but less than 10 packets that are unsatisfactory. [2]

(iii) Find the least value of r such that the probability that there are more than r packets that are unsatisfactory is at most 0.12. [2]

Before selling the packets of stationery, he decided to select a sample of 9 packets to check for unsatisfactory packets.

(iv) How should the packets be selected? Give a reason for this method of selection. [2]

(v) Find the probability that the ninth packet is the third unsatisfactory packet selected. [2]

10 A refrigerator manufacturer claims that the mean lifespan of refrigerators of a particular model is 12 years. A consumers association representative suspects that the mean lifespan of the refrigerators is actually less than 12 years.

The durations x, in years, of a random sample of 45 refrigerators are summarised as follows.

$$\sum(x-12) = -4.3 \qquad \sum(x-12)^2 = 17.08$$

(a) Calculate unbiased estimates of the population mean and variance of the lifespan of the refrigerators. [2]

(b) State hypotheses that can be used to test if the mean lifespan of the refrigerators is less than 12 years, defining any parameters you use. Test, at the 5% level of significance, whether the mean lifespan of the refrigerators is less than 12 years. [4]

The manufacturer switches to a different coolant and decides to test whether the mean lifespan of the refrigerators has changed from 12 years. He records the durations of a large random sample of n refrigerators and finds that their mean lifespan is 12.4 years and variance is 4.1 square years.

A test at 5% significance level, is carried out on the new random sample. The test shows that there is sufficient evidence that the mean lifespan of refrigerators has changed.

(c) Find the range of possible values of n. [4]

11 **(a)** Observations of 8 pairs of values (u, g), representing the hours of internet usage per week (u) and academic performance (g) in terms of Grade Point Average (GPA), are shown in the table below.

Internet usage (u)	4.0	6.0	8.0	a	12.0	16.0	18.0	20.0
GPA (g)	3.7	3.5	3.4	3.2	3.0	2.7	2.6	2.5

It is known that the equation of the linear regression line of g on u is $g = -0.0765u + 3.99$, find the value of a correct to 1 decimal place. [2]

(b) A researcher is studying the relationship between the battery life (y, in hours) of a new smartphone model and the screen brightness setting (x, in %). The following data was collected from the tests conducted at different brightness levels.

Screen Brightness (x)	10	20	30	40	50	60	70
Battery life (y)	48.2	47.4	45.5	37.3	35.6	31.1	24.3

(i) Draw a scatter diagram for these values. [2]

(ii) One of the values of y appears to be incorrect. Circle this point on your diagram and label it P. [1]

(iii) Explain why a linear model $y = a + bx$ is not a suitable model. [1]

(iv) It is thought that the battery life (y) can be modelled by one of the formulae after removing the point P.

(A) $y = a + bx^2$,

(B) $y = a + b\ln x$,

where a and b are non-zero constants.

Find, correct to 4 decimal places, the product moment correlation coefficient between y and x^2 as well as y and $\ln x$.

Explain clearly which model is a better model for this set of data.

For the case identified, find the equation of a suitable regression line. [3]

(v) Using the regression line found in **(iv)**, estimate the battery life when the screen brightness is set to 80%. [1]

(vi) Comment on the reliability of your answer in part **(v)**. [1]