RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Term 2 Week 6 Math Focus
Topic: Normal Distribution

1 EJC Prelim 9758/2020/02/Q11(i), (ii), (iii) [modified]

The masses, in pounds, of apples and pears have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Apple	0.33	σ
Pear	0.45	0.08

(i) The probability of a randomly chosen apple weighing less than t pounds and that weighing between t and 0.35 pounds is 0.34 each. Find the values of σ and t. [4]

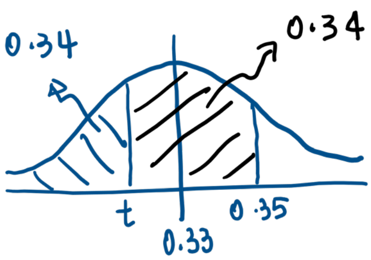
(ii) Three pears are chosen at random. Find the probability that one of them weighs less than the population mean mass and each of the other two pears has a mass within one standard deviation of the population mean mass. [2]

The apples cost \$2.20 per pound and the pears cost \$3.10 per pound.

(iii) Find the probability that the total cost of 3 randomly chosen apples and 2 randomly chosen pears is more than \$5. [3]

(i) Let X be the mass of an apple (in pounds).

$X \sim \mathrm{N}\left(0.33, \sigma^2\right)$



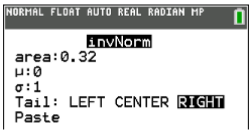
$\mathrm{P}(X > 0.35) = 1 - 0.34 - 0.34 = 0.32$

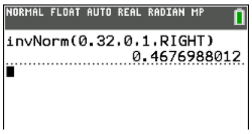
$\mathrm{P}\left(Z > \dfrac{0.35 - 0.33}{\sigma}\right) = 0.32$

From GC, $\mathrm{P}(Z > 0.46770) = 0.32$

$\therefore \dfrac{0.02}{\sigma} = 0.46770$

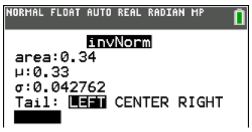
$\sigma \approx 0.042762 = 0.0428$ (3 s.f)

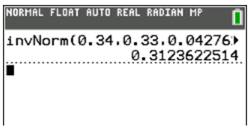




$\mathrm{P}(X < t) = 0.34$

From GC, $t = 0.312$ (3 s.f)





	Alternatively, $\mathrm{P}(t < X < 0.35) = 0.34$ $\mathrm{P}(X < 0.35) - \mathrm{P}(X < t) = 0.34$ $\mathrm{P}(X < 0.35) = 0.34 + 0.34 = 0.68$ $\mathrm{P}\left(Z < \frac{0.35 - 0.33}{\sigma}\right) = 0.68$ From GC, $\frac{0.35 - 0.33}{\sigma} = 0.46770$ $\sigma = 0.042762 = 0.0428$ (3s.f.)
(ii)	Let Y be the mass of a pear (in pounds). $Y \sim \mathrm{N}\left(0.45, 0.08^2\right)$ Required probability $= {}^3C_1\left(\mathrm{P}(Y < 0.45)\right)\left[\mathrm{P}(0.45 - 0.08 < Y < 0.45 + 0.08)\right]^2$ $= {}^3C_1(0.5)\left[\mathrm{P}(0.37 < Y < 0.53)\right]^2$ $= 0.699$ (3 s.f)
(iii)	Let $T = 2.2(X_1 + X_2 + X_3) + 3.1(Y_1 + Y_2)$ $\mathrm{E}(T) = 2.2[3\mathrm{E}(X)] + 3.1[2\mathrm{E}(Y)] = 2.2 \times 3 \times 0.33 + 3.1 \times 2 \times 0.45 = 4.968$ $\mathrm{Var}(T) = 2.2^2[3\mathrm{E}(X)] + 3.1^2[2\mathrm{E}(Y)] = 2.2^2 \times 3 \times 0.042762^2 + 3.1^2 \times 2 \times 0.08^2 = 0.14956$ $T \sim \mathrm{N}(4.968, 0.14956)$ $\mathrm{P}(T > 5) = 0.467$ (3 s.f)

2 EJC Prelim 9758/2021/02/Q10 (modified)

In this question, you should state the parameters of any distributions that you use.

A stationery factory manufactures pens for sale. The diameters (in mm) of the pens have distribution $\mathrm{N}(10, 0.003)$. The pens are packed in sets of 24 into a box. Within the box, the pens are laid out side by side. The widths of the boxes (in mm) are normally distributed with mean 240.5 mm and variance 0.02 mm^2.

(i) Find the probability that a random sample of 24 pens would fit into a randomly selected box. [3]

A pen is considered "defective" if its diameter is not within 0.1mm of the population mean.

(ii) Find the probability that a randomly selected pen is defective. [2]

Pens are produced and inspected in batches. For each batch, a sample of 12 pens is randomly selected and checked.

- If there are fewer than 2 defective pens in this sample of 12, the batch passes the inspection.
- If there are exactly 2 defective pens in this sample, a second sample of 12 pens is randomly selected and checked. If there are no defective pens in the second sample, the batch passes the inspection.
- Otherwise, the batch does not pass the inspection.

(iii) Show that the probability of a batch of pens passing the inspection is 0.871, correct to 3 significant figures. [3]

(iv) Find the probability that not more than 3 defective pens were found in a batch of pens during the inspection process, given that the batch of pens did not pass the inspection. [3]

(i)	Let Y be the width (in mm) of a box. Let W be the width (in mm) of a pen. $Y \sim N(240.5, 0.02) \qquad W \sim N(10, 0.003)$ Let $C = W_1 + W_2 + ...W_{24} - Y$ $\mathrm{E}(C) = 24[\mathrm{E}(W)] - \mathrm{E}(Y) = 24(10) - 240.5 = -0.5$ $\mathrm{Var}(C) = 24[\mathrm{Var}(W)] + \mathrm{Var}(Y) = 24(0.003) + 0.02 = 0.092$ $C \sim N(-0.5, 0.092)$ $P(\text{pens fit into box}) = P(W_1 + W_2 + ...W_{24} < Y)$ $= P(C < 0)$ $= 0.95037$ $= 0.950 \text{ (to 3 sf)}$

(ii)	Let W be the width (in mm) of a pen. $W \sim N(10, 0.003)$ $P(\text{pen is defective}) = 1 - P(9.9 < W < 10.1) = 1 - 0.93211 = 0.067889 = 0.0679$ (to 3sf)
(iii)	Let X be the number of defective pens in a box of 12. $X \sim B(12, 0.067889)$ $P(\text{batch passes inspection})$ $= P(X \le 1) + P(X = 2)P(X = 0)$ $= 0.806087 + (0.150598)(0.430142)$ $= 0.870865 = 0.871$ (to 3 sf)
(iv)	$P(\le 3 \text{ defective pens found} \mid \text{batch did not pass inspection})$ $= \dfrac{P(X=2)P(X=1) + P(X=3)}{P(\text{batch did not pass inspection})}$ $= \dfrac{(0.150598)(0.375945) + 0.036562}{1 - 0.870865}$ $= \dfrac{0.0931785}{0.129135} = 0.722$ (to 3 sf)

3 HCI JC2 Prelim 9758/2019/02/Q7

A cafe sells sandwiches in 2 sizes, "footlong" and "6-inch". The lengths in inches of "footlong" loaves have the distribution $\mathrm{N}(12.2, 0.04)$ and the lengths in inches of "6-inch" loaves have the distribution $\mathrm{N}(6.1, 0.02)$.

(i) Is a randomly chosen "footlong" loaf more likely to be less than 12 inches in length or a randomly chosen "6-inch" loaf more likely to be less than 6 inches in length? [2]

(ii) Find the probability that two randomly chosen "6-inch" loaves have total length more than one randomly chosen "footlong" loaf. [2]

Sue buys a "6-inch" sandwich 3 times a week.

(iii) Find the probability that Sue gets at most one sandwich that is less than 6 inches in length in a randomly chosen week. [2]

(iv) Given that Sue gets more than four sandwiches that are less than 6 inches in length in a randomly chosen 4-week period, find the probability that she gets exactly one such sandwich in the first week. [3]

(i)	Let X and Y denote the length (in inches) of a "footlong" and a "6-inch" loaf respectively. $X \sim \mathrm{N}(12.2, 0.04)$, $Y \sim \mathrm{N}(6.1, 0.02)$ $\mathrm{P}(Y < 6) \approx 0.23975 = 0.240(3\text{sf})$ $\mathrm{P}(X < 12) \approx 0.15866 = 0.159(3\text{sf}) < \mathrm{P}(Y < 6)$ $\therefore$ A "6-inch" loaf is more likely to be less than 6 inches.
(ii)	$\mathrm{E}(Y_1 + Y_2 - X) = 2\mathrm{E}(Y) - \mathrm{E}(X) = 2(6.1) - 12.2 = 0$ $\mathrm{Var}(Y_1 + Y_2 - X) = 2\mathrm{Var}(Y) + \mathrm{Var}(X) = 2(0.02) + 0.04 = 0.08$ $Y_1 + Y_2 - X \sim \mathrm{N}(0, 0.08)$ $\mathrm{P}(Y_1 + Y_2 > X) = \mathrm{P}(Y_1 + Y_2 - X > 0) = 0.500$ (3 s.f)
(iii)	Let A be the number of "6-inch" sandwiches less than 6-inches in length, out of 3 bought in a week. $A \sim \mathrm{B}(3, 0.23975)$ $\mathrm{P}(A \le 1) = 0.855(3\text{ s.f})$
(iv)	Let C and D denote the number of "6-inch" sandwiches less than 6-inches in length in a 3-week and 4-week period respectively. $C \sim \mathrm{B}(9, 0.23975)$ and $D \sim \mathrm{B}(12, 0.23975)$

	$\mathrm{P}\left(A=1 \mid D>4\right)=\dfrac{\mathrm{P}(A=1)\mathrm{P}(C>3)}{\mathrm{P}(D>4)}$ $=\dfrac{\mathrm{P}(A=1)\left[1-\mathrm{P}(C\le 3)\right]}{1-\mathrm{P}(D\le 4)}$ $\approx\dfrac{0.41571\times 0.14710}{0.13721}$ $=0.446\left(3\text{ s.f}\right)$