RAFFLES INSTITUTION
H2 Mathematics 9758
2025 Year 6 Term 2 Revision (Summary)

Topic: Normal Distribution

Summary for Normal Distribution

Note that Normal random variable is a special continuous random variable. For continuous random variable, probability is calculated using the area under its probability density function.

In H2 Math syllabus, we just need to know how to use GC to evaluate these probabilities.

We also need to be aware that

$\mathrm{P}\left(X < x\right) \neq \mathrm{P}\left(X \leq x\right)$ for discrete random variable (Binomial included)

BUT

$\mathrm{P}\left(X < x\right) = \mathrm{P}\left(X \leq x\right)$ for continuous random variable

Normal Distribution

If a continuous random variable X follows a normal distribution, we write $X \sim \mathrm{N}(\mu, \sigma^2)$,

where $\mathrm{E}(X) = \mu$ and $\mathrm{Var}(X) = \sigma^2$.

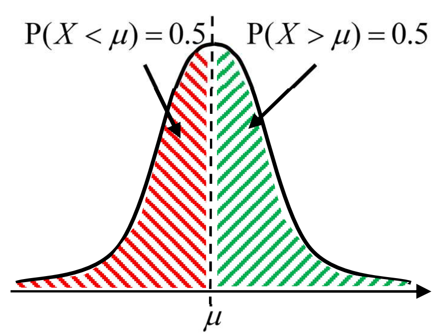
Properties of a Normal Curve

Let $X \sim \mathrm{N}(\mu, \sigma^2)$.

(1) It is symmetrical about the line $x = \mu$.

(2) The mean, median and mode are all equal to μ.

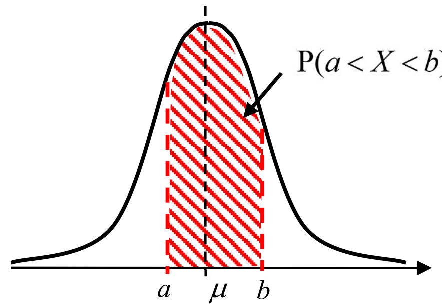
(3) It approaches the $x-$axis as $x \to \pm\infty$.



(4) Area under the graph gives the probabilities,

i.e., $\mathrm{P}(a < X < b) = \int_a^b \mathrm{f}(x)\,\mathrm{d}x$

where $y = \mathrm{f}(x)$ represents the probability density function of the normal curve.



Hence $\mathrm{P}(a < X < b)$ is given by the area under the graph from $x = a$ to $x = b$.

(5) Total area under the curve is 1.

(6) $\mathrm{P}(\mu - \sigma < X < \mu + \sigma) \approx 0.68$

$\mathrm{P}(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.95$

$\mathrm{P}(\mu - 3\sigma < X < \mu + 3\sigma) \approx 0.997$

i.e., approximately 68%, 95% and 99.7% of the values drawn from a normal distribution lies within 1, 2 and 3 standard deviations of the mean respectively.

Standard Normal Distribution

Let $X \sim \mathrm{N}(\mu, \sigma^2)$. The random variable Z, which is the standard normal variable, is defined by $Z = \dfrac{X - \mu}{\sigma}$. The standard normal distribution is $Z \sim \mathrm{N}(0,1)$.

The process of converting $X \sim \mathrm{N}(\mu, \sigma^2)$ into $Z \sim \mathrm{N}(0,1)$ is known as **standardization**.

$$\mathrm{P}(X \le x) = \mathrm{P}\left(\frac{X - \mu}{\sigma} \le \frac{x - \mu}{\sigma}\right) = \mathrm{P}\left(Z \le \frac{x - \mu}{\sigma}\right)$$

Remarks: Standardization is usually applied when there are unknown parameter(s) and we can’t use the GC commands normalcdf(or invNorm(directly.

Using the Properties of Expectation and Variance of Random Variables, we have the following results for independent Normal Random Variables

Let $X \sim \mathrm{N}(\mu_1, {\sigma_1}^2)$ and $Y \sim \mathrm{N}(\mu_2, {\sigma_2}^2)$ be **independent** random variables and a and b be constants. We have

(1) $X \pm Y \sim \mathrm{N}(\mu_1 \pm \mu_2, {\sigma_1}^2 + {\sigma_2}^2)$

(2) $aX \pm b \sim \mathrm{N}(a\mu_1 \pm b, a^2{\sigma_1}^2)$

(3) $aX \pm bY \sim \mathrm{N}(a\mu_1 \pm b\mu_2, a^2{\sigma_1}^2 + b^2{\sigma_2}^2)$