RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Term 2 W9 Revision`: Hypothesis Testing

Questions

1 DHS JC2 Mid-Year CT 9758/2018/02/Q1

The government of a particular country reported that the mean monthly water consumption of households is 17.9 m^3. This is followed by a nationwide water conservation campaign to encourage households to reduce their water consumption. Subsequently after the campaign, the water consumption $x \text{ m}^3$ of a random sample of 50 households is recorded and their data are summarised as follows.

$$\sum(x-15)=103, \quad \sum(x-15)^2=599.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% significance level whether the campaign is effective.

State, giving a reason, whether any assumption about the distribution of the monthly water consumption of households is needed in order for the test to be valid. [4]

(iii) Explain the meaning of '5% significance level' in the context of the question. [1]

Solution :

(i)	An unbiased estimate for the population mean is $\bar{x}=\dfrac{\sum(x-15)}{50}+15=\dfrac{103}{50}+15=17.06$ An unbiased estimate for the population variance is $s^2=\dfrac{1}{50-1}\left[\Sigma(x-15)^2-\dfrac{\left(\sum(x-15)\right)^2}{50}\right]$ $=\dfrac{1}{49}\left[599-\dfrac{103^2}{50}\right]$ $=7.8943$ (to 5 s.f.) $=7.89$ (to 3 s.f.)
(ii)	Let X represent the water consumption (in cubic metres) of a household with population mean μ. To test H_0: $\mu=17.9$ vs H_1: $\mu<17.9$ Perform 1-tail test at 5% significance level

	Under H_0, $\overline{X} \sim N\left(17.9, \frac{7.8943}{50}\right)$ approximately, by Central Limit Theorem, since $n = 50$ is large. From the sample, $\overline{x} = 17.06$. Using z-test, p-value = $P\left(\overline{X} \le 17.06\right) = 0.0173$ (3 s.f.) Since p-value <0.05, we reject H_0 and conclude that there is sufficient evidence at 5% significance level that the campaign was effective. No assumptions on the distribution of X are required as the sample size is large, by Central Limit Theorem, the distribution of the sample mean ($\overline{X}$) is approximately normal.
(iii)	The 5% significance level means that there is a probability of 0.05 that we conclude that the mean water consumption of households has decreased, when it is actually unchanged at 17.9 m^3.

2 **RI Prelim 9758/2020/02/Q7**

In a factory, machines pack sugar into bags of 1 kg each on average, with variance σ^2 kg^2. The manufacturer is concerned that the machines are putting too much sugar into the bags and decides to carry out a hypothesis test. A random sample of 8 bags are selected and their total mass is 8.4 kg.

(i) Stating a necessary assumption, carry out a test of the manufacturer's concern at the 5% significance level if $\sigma = 0.08$. [5]

(ii) Use an algebraic method to calculate the range of values of σ^2 for which the null hypothesis would not be rejected at the 5% significance level. [3]

Solution :

(i)	Let X be the mass of a bag of sugar in kg, and μ be the population mean mass of sugar in a 1 kg bag. The necessary assumption is X follows a normal distribution. $H_0: \quad \mu = 1$ $H_1: \quad \mu > 1$ Perform a 1-tail test at 5% significance level. Under H_0, $\overline{X} \sim N\left(1, \frac{0.08^2}{8}\right)$ $\overline{x} = \frac{8.4}{8} = 1.05$ Using a z-test, p-value $= P\left(\overline{X} \ge 1.05\right) = 0.0385 < 0.05$ Since p-value< 0.05, we reject H_0 and conclude that there is sufficient evidence, at the 5% significance level, to support the manufacturer's concern.

(ii)	Under H_0, $\bar{X} \sim N\left(1, \frac{\sigma^2}{8}\right)$ Since H_0 is not rejected at 5% significance level, $p\text{-value} > 0.05$ $P\left(\bar{X} \geq 1.05\right) > 0.05$ $P\left(Z \geq \frac{1.05-1}{\sqrt{\frac{\sigma^2}{8}}}\right) > 0.05$ Since $P\left(Z \geq 1.6449\right) = 0.05$, $0.05\sqrt{\frac{8}{\sigma^2}} < 1.6449$ $\sigma^2 > 8\left(\frac{0.05}{1.64485}\right)^2$ $\sigma^2 > 0.0073918$ $\sigma^2 > 0.00739$ (3sf)

3 ACJC Prelim 9758/2019/02/Q7 modified

A concert promoter claims that the mean price of a ticket to a pop concert is \$200. A media company took a large random sample of n tickets, and found that the average ticket price for the sample is \$206. If the standard deviation of ticket price is known to be \$32.25, find the minimum value of n such that there is sufficient evidence at the 4% level of significance to reject the concert promoter's claim. [4]

Solution :

Let \$$X$ be the price of a pop concert ticket and μ be the population mean of X.

$H_0 : \mu = 200$ vs $H_1 : \mu \neq 200$

Perform a 2-tail test at 4% significance level

Under H_0, $\overline{X} \sim N\left(200, \frac{32.25^2}{n}\right)$ approximately by Central Limit Theorem since n is large

From the sample, $\overline{x} = 206$

To reject H₀, $p\text{-value} = 2P\left(\overline{X} \geq 206\right) \leq 0.04$

$$P\left(\overline{X} \geq 206\right) \leq 0.02$$

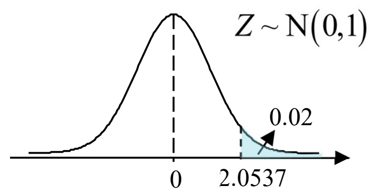
$$P\left(Z \geq \frac{206-200}{32.25/\sqrt{n}}\right) \leq 0.02$$

Since $P\left(Z \geq 2.0537\right) = 0.02$,

$$\frac{6\sqrt{n}}{32.25} > 2.0537$$

$$n > 121.85$$

Minimum value of n is 122.



Alternatively,

$P\left(\overline{X} \geq 206\right) \leq 0.02$

Using GC table of values,

n	$P\left(\overline{X} \geq 206\right)$
121	0.0204 > 0.02
122	0.0199 < 0.02

Minimum value of n is 122.