RAFFLES INSTITUTION
H2 Mathematics 9758
2025 Year 6 Term 2 Revision (Summary)

Topic: Hypothesis Testing

Summary for Hypothesis Testing

Performing a Hypothesis Test

Step 1	Understand the given question and write down the **null hypothesis** H_0 and the **alternative hypothesis** H_1
Step 2	Write down the **level of significance** α (usually given in the question)
Step 3	Decide on the **test statistic** to be used and determine its **distribution**
Step 4	Use the GC to calculate the ***p*-value**
Step 5	**Reject** H_0 if p-value $\le \alpha$, OR **Do not reject** H_0 if p-value $> \alpha$ Write down the **conclusion** in the context of the question

Example:

From records, the IQ of university students is normally distributed with a mean of 118. Students from ABC University claim that the mean IQ of the students in their university is greater than 118. A sample of 50 students was tested and the mean IQ was found to be 121.

Test, at 5% significance level, whether the sample provides significant evidence to support the students' claim, assuming that

(a) the IQ of university students is normally distributed with standard deviation, σ, where $\sigma = 12$,
(b) the IQ of university students is normally distributed and an unbiased estimate of the population variance, calculated based on the sample of 50 students, is 97.454,
(c) an unbiased estimate of the population variance for the IQ of university students, calculated based on the sample of 50 students, is 97.454.

Solution

Let X be the IQ and μ be the mean IQ of a student in ABC University.

Remember to always define X and any notations used if they are not defined in the question.

Step 1: To test $\mathrm{H}_0 : \mu = 118$ vs $\mathrm{H}_1 : \mu > 118$

Read question carefully to decide >, < or ≠.

Step 2: Perform a $\underbrace{\text{1-tail test}}$ at 5% level of significance.

If H_1 involves > or <, it is a 1-tail test
If H_1 involves $\neq$, it is a 2-tail test.

For part a: Sample from a Normal population of known variance

Step 3: Under H_0, $\bar{X} \sim \mathrm{N}\left(\mu_0, \dfrac{\sigma^2}{n}\right)$, where $\mu_0 = 118$ and $\sigma = 12$.

From the sample, $\bar{x} = 121$, $n = 50$.

Step 4: Using a z-test, p-value $= \underbrace{\mathrm{P}\left(\bar{X} \geq 121\right)} = 0.0385$ (3 s.f.)

$$\begin{array}{|l|}\hline "\mathrm{H}_1 : \mu > \mu_0" \Rightarrow \mathrm{P}\left(\bar{X} \geq \bar{x}\right) \\ "\mathrm{H}_1 : \mu < \mu_0" \Rightarrow \mathrm{P}\left(\bar{X} \leq \bar{x}\right) \\ "\mathrm{H}_1 : \mu \neq \mu_0" \Rightarrow 2\mathrm{P}\left(\bar{X} \leq \bar{x}\right) \text{ if } \bar{x} < \mu_0 \\ \qquad\qquad\qquad \text{or } 2\mathrm{P}\left(\bar{X} \geq \bar{x}\right) \text{ if } \bar{x} > \mu_0 \\ \hline \end{array}$$

Step 5: Since p-value $= 0.0385 < 0.05$, we **reject** H_0 and conclude that there is **sufficient** evidence, at 5% level of significance, to support the claim that the mean IQ of students in ABC University is greater than 118.

<u>If test is performed at 1% level of significance,</u>

Step 5: Since p-value $= 0.0385 > 0.01$, we **do not reject** H_0 and conclude that there is **insufficient** evidence, **at 1% level** of significance, to support the claim that the mean IQ of students in ABC University is greater than 118.

[Notice the only difference are the phrases in bold, the end of the sentence is to describe the alternative hypothesis H_1.]

For part b: Large sample from a Normal population of unknown variance

Step 3: Under H_0, $\bar{X} \sim \mathrm{N}\left(\mu_0, \dfrac{s^2}{n}\right)$ **approximately**, with $\mu_0 = 118$.

From the sample, $\bar{x} = 121$, $s = \sqrt{97.454}$

Step 4: Using a z-test, p-value $= \mathrm{P}\left(\bar{X} \geq 121\right) = 0.0158$ (3 s.f.)

Step 5: Since p-value $= 0.0158 < 0.05$, we **reject** H_0 and conclude that there is **sufficient** evidence, at 5% level of significance, to support the claim that the mean IQ of students in ABC University is greater than 118.

For part c: Large sample from a non-Normal population of unknown variance

Step 3: Under H_0, **since** $n = 50$ **is large**, $\bar{X} \sim \mathrm{N}\left(\mu_0, \frac{s^2}{n}\right)$ **approximately, by Central Limit Theorem**, with $\mu_0 = 118$.

From the sample, $\bar{x} = 121$, $s = \sqrt{97.454}$

Step 4 and Step 5 are identical to those in part b.

General Tips

- Take note of the nature of the "claim" in the question and the respective conclusion e.g. if the claim is "the mean IQ is equal to 118 or at least 118 or at most 118", then when $\mathrm{H}_0 : \mu = 118$ is rejected, there is "sufficient evidence at … to conclude that the claim is **invalid**"; whereas if the claim is "the mean IQ is higher or lower than 118", then when $\mathrm{H}_0 : \mu = 118$ is rejected, there is "sufficient evidence at … to conclude that the claim is **valid**".
- If H_0 is rejected at 5% level of significance for a certain *p*-value, H_0 will also be rejected at 10% level of significance or any level higher than 5%. On the contrary, H_0 may or may not be rejected at 1% or any level lower than 5%.

Definitions

1. The **level of significance** (or significance level) of a hypothesis test, denoted by α, is defined as the **probability of rejecting** H_0 **when** H_0 **is true** e.g. "5% level of significance" means "there is a 0.05 probability of wrongly concluding that the mean IQ of the students is more than 118 when in fact it is 118".
2. The p-value $= \mathrm{P}(\bar{X} \geq 121)$ in this context refers to the probability of getting a sample with average IQ at least 121. This is also the value we will put down on our script if question asked for the smallest level of significance at which H_0 can be rejected in favour of H_1.

Important Cases where conclusion is given and we are asked to find:

Case 1: Level of significance $\alpha\%$

Carry on as if we are performing a test. After finding the *p*-value:

If given H_0 is rejected, *p*-value $\le \frac{\alpha}{100}$; if given H_0 is not rejected, *p*-value $> \frac{\alpha}{100}$.

Case 2: Sample mean $\bar{x}$ (No standardization required)

Example: For a test at 5% level of significance and under H_0, $\bar{X} \sim \mathrm{N}\left(\mu_0, \frac{\sigma^2}{n}\right)$, where $\mu_0 = 118$ and $\sigma = 12$. From the sample, $n = 50$.

If given H_0 **is rejected**, use: [GC invNorm: key in standard deviation $= \frac{12}{\sqrt{50}}$, not 12]

<table>
<tr><th>1-tail $\left(\text{e.g. } \mathrm{H}_1: \mu > 118\right)$</th><th>2-tail $\left(\text{e.g. } \mathrm{H}_1: \mu \ne 118\right)$</th></tr>
<tr><td>p-value ≤ 0.05
$\mathrm{P}\left(\bar{X} \ge \bar{x}\right) \le 0.05$
Using GC, $\mathrm{P}\left(\bar{X} \ge 120.791\right) = 0.05$
$\bar{x} \ge 121$</td><td>p-value ≤ 0.05
where $p-\text{value} = \begin{cases} 2\mathrm{P}\left(\bar{X} \ge \bar{x}\right), & \text{if } \bar{x} \ge 118 \\ 2\mathrm{P}\left(\bar{X} \le \bar{x}\right), & \text{if } \bar{x} < 118 \end{cases}$
H_0 is rejected $\Rightarrow$
$\begin{cases} \mathrm{P}\left(\bar{X} \ge \bar{x}\right) \le 0.025, & \text{if } \bar{x} \ge 118 \\ \mathrm{P}\left(\bar{X} \le \bar{x}\right) \le 0.025, & \text{if } \bar{x} < 118 \end{cases}$
$\vdots$</td></tr>
</table>

Case 3: μ_0 or n or σ^2 or s^2

All steps are similar to that of Case 2 above except we have to standardize in order to solve for the unknown parameter.

Example: (to find μ_0)

1-tail $\left(\text{e.g. } \mathrm{H}_1: \mu > \mu_0;\ \text{from sample, } \bar{x} = 121\right)$
p-value$=\mathrm{P}\left(\bar{X} \ge 121\right) \le 0.05$ $\mathrm{P}\left(Z \ge \frac{121-\mu_0}{\sqrt{\frac{12^2}{50}}}\right) \le 0.05$ Using GC, $\mathrm{P}\left(Z \ge 1.6449\right) = 0.05$ $\frac{121-\mu_0}{\sqrt{\frac{12^2}{50}}} \ge 1.6449 \quad \cdots$