RAFFLES INSTITUTION
H2 Mathematics (9758)
2025 Year 6

Term 2 W4 Math Focus: Discrete Random Variables, Binomial Distribution

Questions

1 JPJC BT2 9758/2021/02/Q9

A biased four-sided die has the numbers '1' to '4' labelled on its faces. When the die is thrown, the number shown on the die is the score, X, which follows the probability distribution:

$$\mathrm{P}(X=x)=\begin{cases} kx & , \ x=1,2 \\ \dfrac{1}{x}-\dfrac{1}{4}k & , \ x=3,4 \end{cases}$$

where k is a non-zero positive constant.

(i) Show that the value of k is $\dfrac{1}{6}$. [2]

In a game, Evan tosses two such dice. The probability distribution of Y, which is the sum of the scores shown on the dice, is given in the following table:

y	2	3	4	5	6	7	8
$\mathrm{P}(Y=y)$	a	$\frac{1}{9}$	$\frac{5}{24}$	b	$\frac{43}{192}$	$\frac{35}{288}$	$\frac{25}{576}$

(ii) Find the values of a and b. [3]

(iii) If Y is an odd number, Evan wins \$$Y$. If Y is an even number, Evan loses \$$Y$. Let W be Evan's winnings after one game. Find the expectation and variance of W. [3]

Solution:

(i) Total probability = 1

$$k(1)+k(2)+\frac{1}{3}-\frac{1}{4}k+\frac{1}{4}-\frac{1}{4}k=1$$

$$\frac{5}{2}k=\frac{5}{12}$$

$$k=\frac{1}{6}$$

x	1	2	3	4
$\mathrm{P}(X=x)$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{7}{24}$	$\frac{5}{24}$

(ii)

Sum	**1 (1/6)**	**2 (1/3)**	**3 (7/24)**	**4 (5/24)**
1 (1/6)	2	3	4	5
2 (1/3)	3	4	5	6
3 (7/24)	4	5	6	7
4 (5/24)	5	6	7	8

Note: The outcomes are not equally likely to happen given that it is not a fair die.

$$a = \mathrm{P}(Y=2) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36} \qquad b = \mathrm{P}(Y=5) = \frac{1}{6}\times\frac{5}{24}\times 2 + \frac{1}{3}\times\frac{7}{24}\times 2 = \frac{19}{72}$$

Alternatively,

$$b = \mathrm{P}(Y=5) = 1 - \mathrm{P}(Y \neq 5) = \frac{19}{72}$$

(iii)

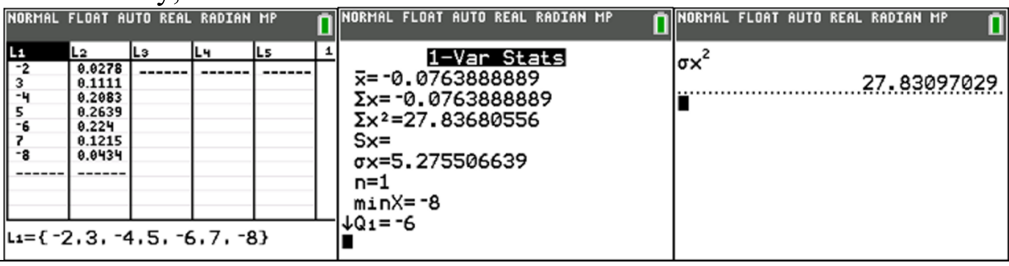
y	2	3	4	5	6	7	8
w	-2	3	-4	5	-6	7	-8
$\mathrm{P}(W=w)$	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{24}$	$\frac{19}{72}$	$\frac{43}{192}$	$\frac{35}{288}$	$\frac{25}{576}$

$$\mathrm{E}(W) = 3\times\frac{1}{9} + 5\times\frac{19}{72} + 7\times\frac{35}{288} - 2\times\frac{1}{36} - 4\times\frac{5}{24} - 6\times\frac{43}{192} - 8\times\frac{25}{576}$$
$$= -\frac{11}{144}$$
$$= -0.08 \text{ (to 2 d.p.)}$$

$$\mathrm{E}(W^2) = 9\times\frac{1}{9} + 25\times\frac{19}{72} + 49\times\frac{35}{288} + 4\times\frac{1}{36} + 16\times\frac{5}{24} + 36\times\frac{43}{192} + 64\times\frac{25}{576}$$
$$= \frac{8017}{288}$$

$$\mathrm{Var}(W) = \mathrm{E}(W^2) - [\mathrm{E}(W)]^2 = \frac{8017}{288} - \left(-\frac{11}{144}\right)^2 = 27.8$$

Alternatively, use 1-Var stats:



2 SAJC BT2 8865/2021/10

A glass vial is an important primary packaging component for pharmaceutical drugs. Vaccines, and most injectable drugs, need to be packaged in these sterile glass vials. On average, it was found that 5% of glass vials shipped by a certain pharmaceutical company are defective. The glass vials are packed in boxes. Each box consists of 50 glass vials.

(i) State, in context, two assumptions needed for the number of defective vials in a box to be well modelled by a binomial distribution. [2]

Assume now that the number of defective vials in a box of shipment follows a binomial distribution.

(ii) Find the most likely number of defective glass vials in a box. [2]

A box of glass vials is rejected if there are more than 5 defective glass vials.

(iii) Find the probability that a box is rejected. [1]

(iv) 2 boxes of glass vials are randomly chosen. Find the probability that 1 box is rejected. [2]

(v) n boxes of glass vials are to be chosen. Find the least value of n such that the probability of at least 20 boxes that are not rejected is greater than 0.95. [3]

A sample of 50 boxes of glass vials is taken. The sample is rated as 'substandard' if 4 or more boxes are rejected. If none of the boxes are rejected, the sample is rated as 'excellent'. Otherwise, the sample is rated as 'fair'.

(vi) Find the probability that 2 boxes are rejected if the sample is rated as 'fair'. [3]

Solution:

(i)	Whether a randomly chosen glass vial is defective or not is independent of any other glass vials. The probability that a glass vial is defective is constant at 0.05 for every glass vial.
(ii)	Let X be the random variable "number of defective vials in a box of shipment containing 50 glass vials". $X \sim \mathrm{B}(50, 0.05)$ Using GC, $\mathrm{P}(X = 1) = 0.2025$ $\mathrm{P}(X = 2) = 0.2611$ $\mathrm{P}(X = 3) = 0.2199$ Therefore, the most likely number of defective glass vials in a box is 2.
(iii)	$\mathrm{P}(\text{a box is rejected})$ $= \mathrm{P}(X > 5)$ $= 1 - \mathrm{P}(X \leq 5)$ $= 0.037776$ $= 0.0378 \text{ (to 3 s.f.)}$
(iv)	$\mathrm{P}(\text{1 box is rejected out of 2})$ $= 2 \times \mathrm{P}(X > 5) \times \mathrm{P}(X \leq 5)$ $= 2(1 - \mathrm{P}(X \leq 5))\mathrm{P}(X \leq 5)$ $= 0.072698$ $= 0.0727 \text{ (to 3 s.f.)}$
(v)	Let Y be the number of boxes that are not rejected out of n. $Y \sim \mathrm{B}(n, 0.96222)$

	$P(Y \geq 20) > 0.95$ $1 - P(Y \leq 19) > 0.95$ $P(Y \leq 19) < 0.05$ Using GC, n : $P(Y \leq 19)$ 21 : $0.1873 > 0.05$ 22 : $0.0486 < 0.05$ Thus, least n is 22.
(vi)	Let W be the number of boxes that are rejected out of 50. $W \sim B(50, 0.037776)$ P(2 boxes are rejected | sample is rated as fair) $= \dfrac{\text{P(2 boxes are rejected and sample is rated as fair)}}{\text{P(sample is rated as fair)}}$ $= \dfrac{P(W = 2 \ \cap \ 1 \leq W \leq 3)}{P(1 \leq W \leq 3)}$ $= \dfrac{P(W=2)}{P(W=1)+P(W=2)+P(W=3)}$ $= 0.375$ (to 3 s.f.)

n	$P(Y \leq 19)$
21	$0.1873 > 0.05$
22	$0.0486 < 0.05$

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