Update: we got A2 for math

CONTENTS

Surds

Always rationalise denominator.
CHECK VALIDITY FOR ALL EQUATIONS especially when there is x^2 or y^2.

Inequalities

Simultaneous inequalities

1) Solve separately
2) Both solutions are related by 'and' not 'or'

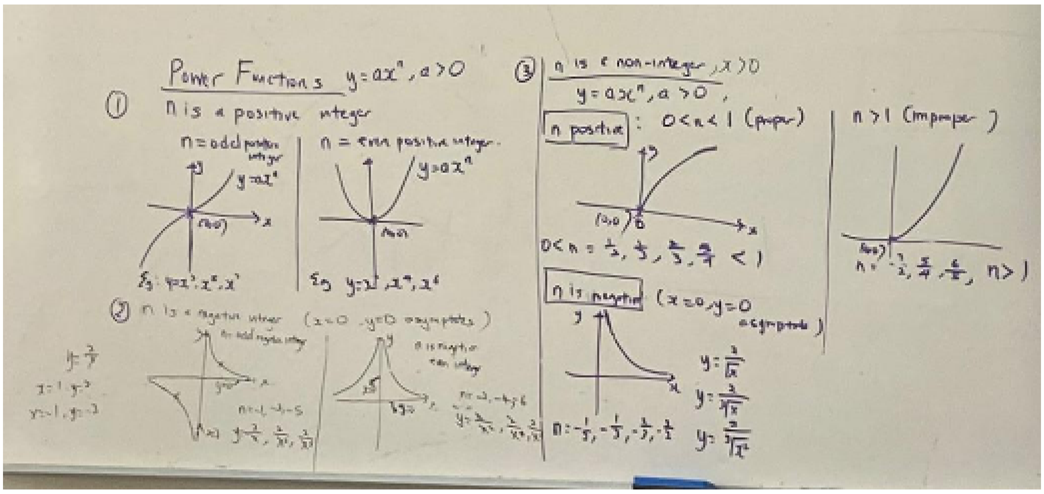
3) Use number line to find common values.
4) Check Validity (LHS=RHS)

Indices and Exponential Functions

Solving

1) $e^{|x|} < 2$
    a) $|x| < \ln 2$
    b) $x < \ln 2$ or $x > -\ln 2$
    c) $-\ln 2 < x < \ln 2$
2) Replacing ln(f(x)), log(f(x)), x^y with k to solve quadratically.

Graphs



Logarithms

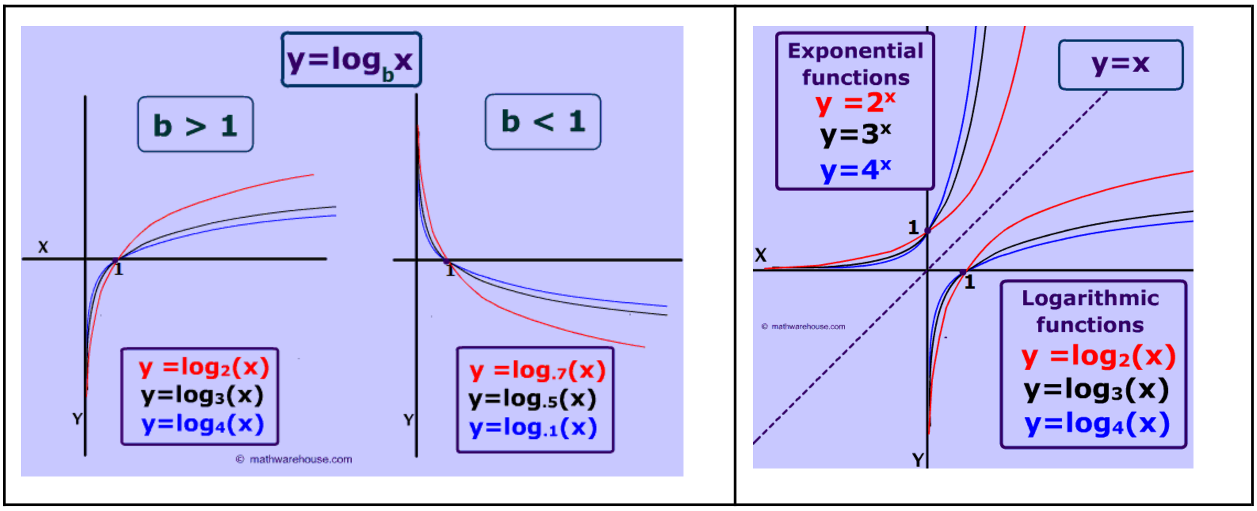
Solving

1) Use properties of log/ln to simplify.
    a) log (ab) = loga + logb
    b) log (a/b) = loga - logb
    c) $\log_a b$ = lgb/lga
    d) alogb = $\log b^a$
2) Equate both sides to a single log/ln
3) Equate one side to log/ln.

Proving not defined

1) Since (expression) is not defined when (denominator) = 0,
2) (expression) is not defined when (solve denominator).
3) Since (equation) is not continuous at (solved value in denominator),
4) It is not possible to evaluate …

Graphs



Discriminant and Roots

Prove there are no real roots.

(Often coupled with asking for proving no other turning points (dy/dx = 0))

1) Prove D < 0

For y to be always increasing/decreasing.

1) Find dy/dx
2) For y to be increasing, dy/dx > 0. For y to be decreasing, dy/dx < 0
   a) $(f(x))^2 > 0$
   b) $e^x > 0$
   c) For dy/dx > 0 or dy/dx < 0, D < 0

To find values where (expressions) have no intersection

1) Equate both into one equation

2) No intersection = no real roots = D < 0

Find range of values of *k* for which f(x) is always positive/negative

1) Convert to ax^2+bx+c
2) Use D < 0, a > x or a < x
    a) To get ak^2+bk+c
3) Solve for range of *k*
4)

Probability

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

If P(A) × P(B) = P(A☹B), A and B are independent events.
If not, they are not independent events.
If P(A☹B) = 0, A and B are mutually exclusive events.

Polynomials and partial fractions

Factor and remainder theorem

1) f(x) leaves a remainder of k when divided by x → f(0) = k
2) (x-1) is a factor of f(x) → f(1) = 0
3) Fully factorise whenever possible

Splitting into partial fractions

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$
	• where $x^2 + bx + c$ cannot be factorised further	

Improper fractions

Long division 🙂

Special methods of substitution

1) Convert to original form either by × or ÷

(ii)

$$4-19\sec\theta+19\sec^2\theta+6\sec^3\theta=0$$

$$\frac{4}{\sec^3\theta}-\frac{19}{\sec^2\theta}+\frac{19}{\sec\theta}+6=0$$

$$4\cos^3\theta-19\cos^2\theta+19\cos\theta+6=0$$

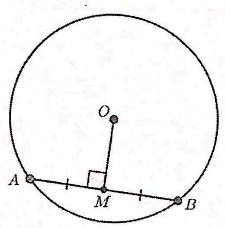
Replace $\cos\theta$ with x,

$$4x^3-19x^2+19x+6=0$$

Circular Measure

Circle properties

Circle Symmetric Property 1: Perpendicular Bisector of a Chord



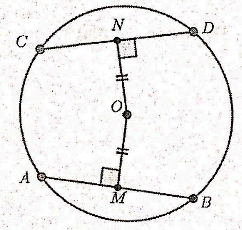
If we denote the midpoint of AB as M, then

$$AM = MB \iff OM \perp AB$$

The perpendicular bisector of chord AB passes through the centre of the circle O.
Note: chord AB must not be the diameter of the circle.

Abbreviated as: $\perp$ bisector of chord

Circle Symmetric Property 2: Equal Chords



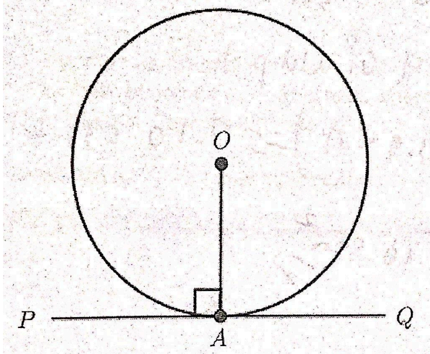
If we denote the midpoint of AB as M and the midpoint of CD as N, then

$$AB = CD \iff OM = ON$$

Equal chords of a circle are equidistant from the centre of the circle.
Conversely, chords equidistant from the centre of a circle are equal in length.

Abbreviated as: equal chords

Circle Symmetric Property 3: Tangent perpendicular to Radius



PQ is the tangent to the circle at A

$\Leftrightarrow$

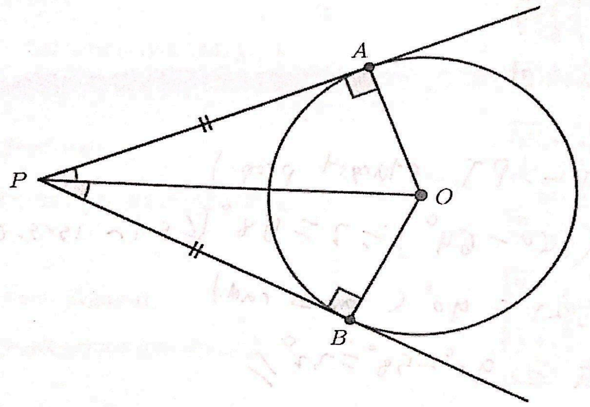
$PQ \perp OA$

(OA is the radius of circle)

The tangent at the point of contact is perpendicular to the radius of the circle.

Abbreviated as: tan $\perp$ rad

Circle Symmetric Property 4: Equal Tangents

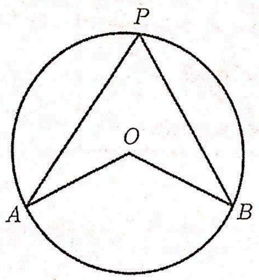


Tangents from an external point are equal (in length), i.e. $PA = PB$.
The line from the centre of a circle to an external point bisects the angle betwee
the two tangents from the external point, i.e. $\angle APO = \angle BPO$.

Abbreviated as: tangent properties

Circle angle properties

Circle Angle Property 1: Angle at Centre

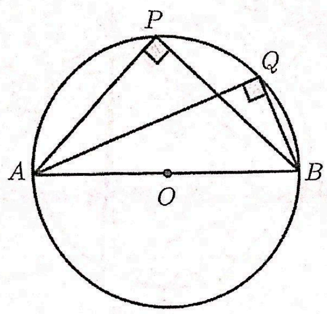


$\angle AOB = 2\angle APB$

An angle at the centre of a circle is **twice** that of any angle at the circumference subtended by the same arc.

Abbreviated as: $\angle$ at centre $= 2 \angle$ at $\odot^{ce}$

Circle Angle Property 2: Angle in Semicircle

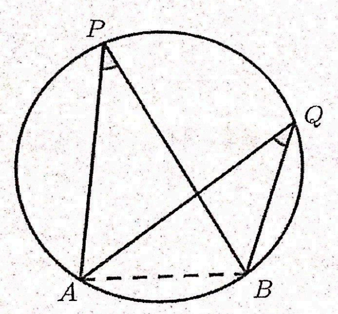


AOB is a diameter $\Leftrightarrow$ $\angle APB = 90°$, $\angle AQB = 90°$

An angle in a semicircle is always equal to $90°$.

Abbreviated as: rt. $\angle$ in a semicircle

Circle Angle Property 3: Angles in Same Segment

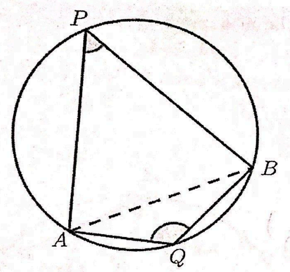


$\angle APB = \angle AQB$

Angles in the same segment are equal.

Abbreviated as: $\angle$s in same segment

Circle Angle Property 4: Angles in Opposite Segments



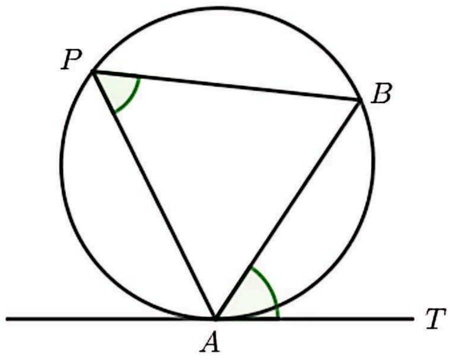
$\angle APB + \angle AQB = 180°$

$\angle PAQ + \angle PBQ = 180°$

Angles in opposite segments are supplementary.

Abbreviated as: $\angle$s in opp. segments OR
$\angle$s in a cyclic quad

Theorem 2: Alternate Segment Theorem (or **Tangent Chord Theorem**)

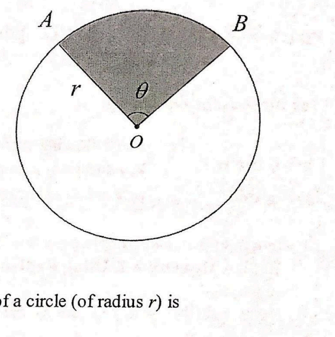


The angle between a tangent and a chord drawn at the point of contact is equal to the angle subtended by a chord in the alternate segment.

If AT is a tangent to the circle at A, then $\angle BAT = \angle APB$.

Abbreviated as: alt. seg. theorem

Arc Length (using Radian Measure)



In Part 2, we establish that:

Length of minor arc AB, $s = \frac{x°}{360°} \times 2\pi r$

Since $\theta = x° \times \frac{\pi}{180°}$, where θ is in radians.

$$s = \frac{2\pi r}{360°} \times x°$$
$$s = r \times \frac{\pi x°}{180°}$$
$$s = r\theta$$

Hence, when the angle subtended by the arc at the centre of a circle (of radius r) is θ rad,

$\boxed{\text{Arc Length} = r\theta}$

Does the formula work for both minor and major arcs?

Area of Sector (using Radian Measure)

In Part 2, we establish that:

Area of minor sector AOB, $A = \frac{x°}{360°} \times \pi r^2$

Since $\theta = x° \times \frac{\pi}{180°}$, where θ is in radians.

$$A = \frac{\pi r^2}{360°} \times x°$$
$$A = \frac{1}{2} \times r^2 \times \frac{\pi}{180°} \times x°$$
$$A = \frac{1}{2} r^2 \theta$$

Hence, when the angle subtended by the arc at the centre of a circle (of radius r) is θ rad,

$\boxed{\text{Area of sector} = \frac{1}{2} r^2 \theta}$

Do wo an

Proving circles (do not) intersect

1) Add radius of both circles
2) Find distance between centre of both circles
    a) (1) < (2), do not intersect
    b) (1) > (2), intersect at 2 points
    c) (1) = (2), intersect at 1 point

Trigonometric Functions and Graphs

Drawing parabola

1) Sub x=0 to find y-intercepts
2) Determine direction of graph
    a) $y^2 > 0$, $f(x) > 0$, $x > 0$ or $x < 0$
3) Sub y=0 to find x-intercept

Finding asinbx+c

1) y=asinbx+c or acosbx+c
    a) Period = $\frac{2\pi}{b}$ / b=$\frac{2\pi}{period}$

Transformation

Translation

Translation by a units in the positive/negative x/y direction.

1) y=f(x-a), increase x values by a
2) y=f(x+a), decrease x values by a
3) y-a=f(x), increase y values by a
4) y+a=f(x), decrease y values by a

Scalling //

Scaling parallel to x/y-axis by a factor of 1/a

1) y=f(a), multiply x values by 1/a
2) ay=f(x), multiply y values by 1/a

Reflection

Reflection in the x/y-axis.

1) y=f(-x), reverse sign for all x

2) -y=f(x), reverse sign for all y

(Further) Trigo identities and equations

Proving equations

General steps

1) Get rid of '1's using general methods
2) Combine 2 fractions into 1
3) Expand complicated expression (ax^2+bx+c)
    a) Usually complicated expressions will cancel out
    b) E.g ($\sin^3 x$, $\cot^4 x$, $\cos^3 x \sin x \cot^2 x$)
4) Simplify fraction

General methods

1) Addition formula
2) Double angle formula
3) Trigo identities
    a) $\sin^4 x + \cos^4 x \neq 1$
4) Multiply both numerator and denominator with same expression (introduce new expression)
5) Add and subtract same expression on numerator to match denominator (dividing into 2 fractions)

Finding cosb given cosa

1) Use triangle to find sin/cos/tan values
2) Use addition formula (or memorise)
    a) Take note of positive and negative

Diagrams

Solving

1) Pythagoras Theorem
2) Addition formula (given in formula sheet)

Bearings

Solving

1) Sine rule
2) Cosine rule

Modulus

Solving

1) Factorise out
    a) $|2x+2| - |x+1| = -5x$
    b) $2|x+1| - |x+1| = -5x$
    c) $|x+1| = -5x$
2) $|x^2| = x^2$
3) $|kx| = |k||x| = k|x|$
4) $|-kx| = |-k||x| = k|x|$

(Further) Differentiation and Applications

Solving

1) Use properties of log/ln to simplify.
    a) $\log(ab) = \log a + \log b$
    b) $\log(a/b) = \log a - \log b$
    c) $\log_a b = \lg b/\lg a$
    d) $a\log b = \log b^a$
2) Differentiation rules
    a) Addition/Subtraction → Split
    b) Power rule
    c) Product rule
    d) Quotient rule

Finding minimum/maximum area/volume

1) Represent area/volume with 1 expression with 1 unknown
    a) Combine 2 expressions with 2 unknowns together, make one unknown the subject, sub into second equation
2) dA/dx
3) Identify if it is maximum or minimum point

a) First Derivative Test

i) Most applications qns require use of concrete values.

x	value- / concrete value	value	value^{+} / concrete value
Value of dA/dx			
Shape of graph	/|\	—	/|\

b) 2nd Derivative test

i) $d^2A/dx^2 > 0$, 🙂, Minimum point

ii) $d^2A/dx^2 < 0$, ☹️, Maximum point

Finding change over time

1) dx/dt = dA/dt x dx/dA or dA/dt ÷ dA/dx

(Further) Integration and Applications

Solving

1) Use properties of log/ln to simplify.
   a) log (ab) = loga + logb
   b) log (a/b) = loga - logb
   c) $\log_a b$ = lgb/lga
   d) alogb = $\log b^a$
2) Integration rules
   a) Addition/Subtraction → Split
   b) Constants → Take out ($2\int f(x)dx$)
   c) Product: convert to $\int f'(x)[f(x)]^n dx$
   d) Fraction: convert to f(x) or use Ln
3) Integration of Trigo (take note f'(x) must be in front first)

dont forget + c

   a) sin → -cos

i) $\sin 2x \rightarrow -\frac{1}{2}\cos 2x$

b) cos → sin
c) tan → sin/cos → -lncos
d) sec → $\frac{secx\,(secx+tanx)}{secx+tanx} = \frac{sec^2x+secxtanx}{secx+tanx} = ln|secx\, +\, tanx|$
e) cosec → $\frac{cosecx\,(cosecx+cotx)}{cosecx+cotx} = \frac{cosec^2x+cosecxcotx}{cosecx+cotx} = ln|cosecx\, +\, cotx|$
f) cot → ln|sin|
g) sec2 → tan
h) coseccot → -cosec
i) cosec2 → -cot
j) seccosec → ln|tan| ($\frac{d}{dx}ln(tanx) = \frac{sec^2x}{tanx} = \frac{1}{sinxcosx}$)

Proving an integral is not defined (i.e has an asymptote)

1) Since (expression) is not defined when (denominator) = 0,
2) (expression) is not defined when (solve denominator).
3) Since (equation) is not continuous at (solved value in denominator),
4) It is not possible to evaluate (integral of expression).

Special methods

Qn	Suggested Solutions
(b)	$\int_0^7 3\mathrm{f}(x)\,\mathrm{d}x-3\int_9^7\left[\mathrm{f}(x)+\frac{x}{2x^2-2}\right]\mathrm{d}x$
	$=3\int_0^7 \mathrm{f}(x)\,\mathrm{d}x-3\int_9^7 \mathrm{f}(x)\mathrm{d}x-3\int_9^7\frac{x}{2x^2-2}\mathrm{d}x$
	$=3\int_0^7 \mathrm{f}(x)\,\mathrm{d}x+3\int_7^9 \mathrm{f}(x)\mathrm{d}x+\frac{3}{4}\int_7^9\frac{4x}{2x^2-2}\mathrm{d}x$
	$=3\int_0^9 \mathrm{f}(x)\,\mathrm{d}x+\frac{3}{4}\left[\ln\left\|2x^2-2\right\|\right]_7^9$
	$=3\left[\int_0^3 \mathrm{f}(x)\,\mathrm{d}x+\int_3^9 \mathrm{f}(x)\,\mathrm{d}x\right]+\frac{3}{4}(\ln 160-\ln 96)$
	$=3(4+11)+\frac{3}{4}\ln\left(\frac{160}{96}\right)$
	$=45+\frac{3}{4}\ln\left(\frac{5}{3}\right)$, where $a=45$, $b=\frac{3}{4}$, $c=\frac{5}{3}$

Tangents and Normals

Find equation of tangent at x=k

1) Find dy/dx
2) Sub x=k into dy/dx to find gradient
3) Sub x=k into y to find y coord
4) Sub in coord into $(y_1-y_2) = m(x_1-x_2)$

$M_{normal} \times M_{tangent} = -1$

Kinematics

1) Only use integration when converting from a → v → s
   a) Although v-t area is displacement and a-t area is velocity, DO NOT use integration to find change in displacement or change in v as integration is used solely to find area under graph.
2) Start fromr rest → t=0, v=0
3) To find *C*, alway sub in values (don't assume *C* is always the starting displacement/velocity given in the question)
4) To prove v always decreasing/increasing, prove a is always < 0 or > 0
5) To prove s always decreasing/increasing, prove v is always < 0 or > 0
   a) Use D < 0 if needed.

Linear Law

This honestly isnt that difficult

Binomial Theorem

9! = 9(8!)

n! = n(n-1)!

Notation	$\binom{n}{r}$ or ${}^nC_r$	**Definition**	$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$, for $r \leq n$.
Read as	n choose r	**Example**	$\binom{7}{3} = \frac{7\times6\times5}{3\times2\times1} = 35$

The General Term

1) $T_{r+1} = nCr\, a^{(n-r)} b^r$

Finding unknowns given first few terms

1) Manipulate expression (according to question requirement of ascending or descending order)
2) Find first 3 terms (remember one term can have multiple powers
    a) First term of binomial expansion ≠ First x term given

Vectors

1) CANNOT DIVIDE VECTORS
    a) When asked for fraction, form ratio first
    b) i = horizontal change, j = vertical change
2) Proving collinear
    a) Prove //
        i) Related by k|$\overline{AB}$|=|$\overline{BC}$|
    b) Have common point