Class Index Number

Name : ____________________________________

Class	Index Number

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2025
Secondary 4

Wednesday **ADDITIONAL MATHEMATICS** **4049/02**

27 August 2025 **PAPER 2** 2 hours 15 mins

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 A piece of raw meat is accidentally left out at room temperature, and bacteria begin to grow on it. Initially, there are 500 bacteria presents, and the number of bacteria doubles every 3 hours. The number of bacteria, N, after t hours can be modelled by the formula $N = 500(2)^{kt}$.

(i) Show that $k = \frac{1}{3}$. [2]

At $t = 3$,

$$1000 = 500(2)^{3k}$$
$$2 = (2)^{3k}$$
$$3k = 1$$
$$k = \frac{1}{3}$$

(ii) It is considered unsafe for consumption when the number of bacteria on the meat exceeds 3 000. Determine the maximum number of hours that it will still be safe to consume the meat.

[3]

$$N \leq 3000$$
$$500(2)^{\frac{t}{3}} \leq 3000$$
$$(2)^{\frac{t}{3}} \leq 6$$
$$\frac{t}{3} \leq \frac{\lg 6}{\lg 2}$$
$$t \leq 7.75$$

Maximum number of hours that it will still be safe to consume = 7 hours

2 **(a)** Given that $y = x^3 - x$, find the range of values of x such that $x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} > 0$. [4]

$$y = x^3 - x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 1$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x$$

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = x(6x) + 3x^2 - 1$$

$$= 9x^2 - 1$$

$$x\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} > 0$$

$$9x^2 - 1 > 0$$

$$(3x-1)(3x+1) > 0$$

$$x < -\frac{1}{3} \quad \text{or} \quad x > \frac{1}{3}$$

2 **(b)** A curve is such that $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{1}{3\sqrt{2x-1}}$. Given that the curve has a gradient of 3 at the point $(1,6)$, find the equation of the curve. [4]

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{1}{3\sqrt{2x-1}}$$
$$= \frac{1}{3}(2x-1)^{-\frac{1}{2}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \int \frac{1}{3}(2x-1)^{-\frac{1}{2}}\ \mathrm{d}x$$
$$= \frac{1}{3}\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2}(2)} + c \quad \text{, } c \text{ is an arbitrary constant}$$
$$= \frac{1}{3}\sqrt{(2x-1)} + c$$

When $x = 1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$,

$$3 = \frac{1}{3} + c$$
$$c = \frac{8}{3}$$

$$\therefore\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{3}\sqrt{(2x-1)} + \frac{8}{3}$$

$$y = \int \left[\frac{1}{3}\sqrt{(2x-1)} + \frac{8}{3}\right] \mathrm{d}x$$
$$= \frac{1}{3}\frac{(2x-1)^{\frac{3}{2}}}{\frac{3}{2}(2)} + \frac{8}{3}x + c_1 \quad c_1 \text{ is an arbitrary constant}$$
$$y = \frac{(2x-1)^{\frac{3}{2}}}{9} + \frac{8}{3}x + c_1$$

At $(1,6)$,

$$6 = \frac{1}{9} + \frac{8}{3} + c_1$$
$$c_1 = \frac{29}{9}$$

$$\therefore\ y = \frac{1}{9}(2x-1)^{\frac{3}{2}} + \frac{8}{3}x + \frac{29}{9}$$

3 A circle C_1 passes through $A(-1,-3)$. Its centre, B, lies on the line $4y+3x=10$. AB is perpendicular to $4y+3x=10$.

(a) Find the equation of the circle. [5]

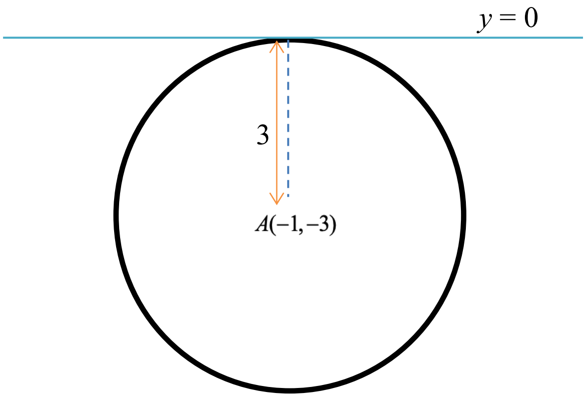
$$y+3=\frac{4}{3}(x+1)$$
$$y=\frac{4}{3}x-\frac{5}{3}$$

$$3y-4x=-5\cdots\text{(i)}$$
$$4y+3x=10\cdots\text{(ii)}$$
$$9y-12x=-15\cdots\text{(i)}$$
$$16y+12x=40\cdots\text{(ii)}$$
$$25y=25$$
$$y=1$$
$$x=2$$

Centre (2, 1) and radius $\sqrt{(2+1)^2+(1+3)^2}=5$

Equation of circle is $(x-2)^2+(y-1)^2=25$

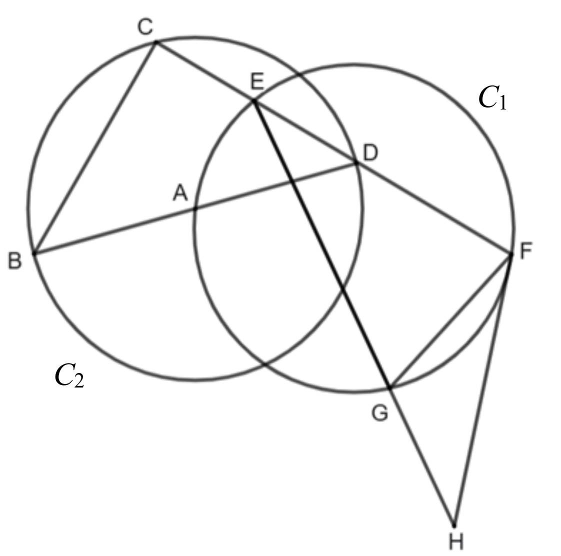
3 **(b)** A second circle C_2 has centre at $A(-1,-3)$. The line $y = 0$ is a tangent to circle C_2. Find the equation of the circle in the form of $x^2 + y^2 + px + qy + r = 0$, where p, q and r are integers. [2]



radius of $C_2 = 3$

$$(x+1)^2 + (y+3)^2 = 3^2$$
$$x^2 + 2x + 1 + y^2 + 6y + 9 = 9$$
$$x^2 + y^2 + 2x + 6y + 1 = 0$$

4



The diagram shows two intersecting circles C_1 and C_2. The line *CF* intersects C_1 at *E*, and C_2 at *D*, such that $CE = ED$. *BAD* is a straight line, where *A* is the centre of C_2. The tangent to C_1 at *F* meets *EG* produced at *H*.

Stating your reasons clearly, prove that

(i) ΔGFH and ΔFEH are similar, [2]

$\angle GHF = \angle FHE$ (Common)
$\angle HFG = \angle HEF$ (alternate segment or tangent-chord theorem)

Hence ΔGFH and ΔFEH are similar

4 **(ii)** $HF^2 = HE \times GH$, [2]

Since ΔGFH and ΔFEH are similar ,

$$\frac{HF}{HE} = \frac{GH}{HF}$$
$$(HF)^2 = HE \times GH$$

(iii) $AF^2 = AE^2 + EF^2$ [3]

In ΔAED and ΔBCD ,
A and E are midpoints of BD and CD respectively.

By mid-point theorem,

$$AE // BC \text{ and } AE = \frac{1}{2}BC$$
$$\angle BCD = 90^\circ (\text{right } \angle \text{ in semicircle})$$
$$\angle AEF = 90^\circ (\text{corresponding } \angle\text{s})$$

Therefore, by Pythagoras Theorem,

$$AF^2 = AE^2 + EF^2$$

Alternative method:

$$\frac{CE}{CD} = \frac{1}{2} (\text{given})$$
$$\frac{BA}{BD} = \frac{1}{2} (A \text{ is the centre})$$
$$\angle EDA = \angle CDB (\text{common})$$
$$\therefore \Delta EDA \text{ and } \Delta CDB \text{ are similar}$$
$$\angle BCD = 90^\circ (\text{rt } \measuredangle \text{in semicircle})$$
$$\angle AED = 90^\circ$$

Therefore, by Pythagoras Theorem,

$$AF^2 = AE^2 + EF^2$$

5 **(a)** Show that $6\log_4 x - \log_2 y = 3$ can be expressed as $y = ax^n$, where a and n are constants to be determined. [4]

$$6\log_4 x - \log_2 y = 3$$
$$6\frac{\log_2 x}{\log_2 4} - \log_2 y = 3$$
$$\frac{6}{2}\log_2 x - \log_2 y = 3$$
$$\log_2 x^3 - \log_2 y = 3$$
$$\log_2 \frac{x^3}{y} = 3$$
$$\frac{x^3}{y} = 8$$
$$y = \frac{x^3}{8}$$
$$a = \frac{1}{8}, n = 3$$

(b) Solve the equation $3e^y - 5 = 2e^{-y}$, giving values of y in logarithmic form. [4]

$$3e^y - 5 = 2e^{-y}$$
$$\text{let } p = e^y$$
$$3p - 5 = \frac{2}{p}$$
$$3p^2 - 5p - 2 = 0$$
$$(p-2)(3p+1) = 0$$

$$e^y = 2 \quad \text{or} \quad e^y = -\frac{1}{3} \text{ (NA)}$$
$$y = \ln 2$$

6 **(a)** Find the range of values of m such that $(m-6)x^2-8x+m>0$ for all values of x. [4]

$$m-6>0$$
$$m>6$$
$$b^2-4ac<0$$
$$(-8)^2-4(m-6)(m)<0$$
$$64-4m^2+24m<0$$
$$m^2-6m-16>0$$
$$(m-8)(m+2)>0$$
$$m>8 \text{ or } m<-2$$

Since $m>6$, $m>8$

(b) Given that the quadratic equation $px^2-2(p-q)x-(r-p)=0$ has equal roots, prove that $p=\dfrac{q^2}{2q-r}$. [4]

$$b^2-4ac=0$$
$$\left[-2(p-q)\right]^2-4p\left[-(r-p)\right]=0$$
$$4(p-q)^2+4p(r-p)=0$$
$$4\left(p^2-2pq+q^2\right)+4pr-4p^2=0$$
$$4p^2-8pq+4q^2+4pr-4p^2=0$$
$$pr-2pq+q^2=0$$
$$q^2=2pq-pr$$
$$=p(2q-r)$$
$$p=\frac{q^2}{2q-r}$$

7 A particle moves in a straight line so that, t seconds after passing through a fixed point O, its velocity, v m/s, is given by $v = k\cos 2t$, where k is a constant. When $t = \frac{\pi}{12}$, the acceleration is -6 m/s^2.

(a) Find the value of k. [2]

$$v = k\cos 2t$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t}$$

$$= -2k\sin 2t$$

$$t = \frac{\pi}{12}, \quad a = -6$$

$$-6 = -2k\sin 2\left(\frac{\pi}{12}\right)$$

When $-3 = -k\sin\left(\frac{\pi}{6}\right)$

$$3 = k\left(\frac{1}{2}\right)$$

$$k = 6$$

(b) The particle is first at rest at point P. Find the value of t when the particle is at point P. [2]

At rest $\Rightarrow v = 0$

$$v = 6\cos 2t$$

$$6\cos 2t = 0$$

$$\cos 2t = 0$$

$$2t = \frac{\pi}{2}$$

$$t = \frac{\pi}{4}$$

7 **(c)** Find the distance travelled by the particle from O to P. [3]

Distance from O to P =

$$\int_0^{\frac{\pi}{4}} 6\cos 2t \ \ \mathrm{d}t$$

$$= \left[\frac{6\sin 2t}{2}\right]_0^{\frac{\pi}{4}}$$

$$= 3\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$= 3\text{m}$$

Alternative:

$$s = \int 6\cos 2t \ \ \mathrm{d}t$$

$$= \frac{6\sin 2t}{2} + c$$

At $t = 0,\ s = 0$

$$\frac{6\sin 2(0)}{2} + c = 0$$

$$c = 0$$

At $t = \frac{\pi}{4}$,

$$s = \frac{6\sin 2\left(\frac{\pi}{4}\right)}{2} = 3$$

8 The equation of the curve is $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = (x+4)\sqrt{2x-3}$, $x > \dfrac{3}{2}$.

(a) Show that $\mathrm{f}'(x) = \dfrac{px+q}{\sqrt{2x-3}}$, where p and q are integers. [3]

$$\begin{aligned}
\mathrm{f}'(x) &= (x+4)\frac{1}{2}(2x-3)^{-\frac{1}{2}}(2) + \sqrt{2x-3} \\
&= \frac{x+4}{\sqrt{2x-3}} + \frac{2x-3}{\sqrt{2x-3}} \\
&= \frac{3x+1}{\sqrt{2x-3}}
\end{aligned}$$

8 **(b)** Find the x-coordinates of the points where the gradient of the normal is $-\frac{1}{7}$. [5]

gradient of the normal is $-\frac{1}{7}$

gradient of the tangent is 7

$$\frac{3x+1}{\sqrt{2x-3}}=7$$

$$3x+1=7\sqrt{2x-3}$$

$$(3x+1)^2=49(2x-3)$$

$$9x^2+6x+1=98x-147$$

$$9x^2-92x+148=0$$

$$x=\frac{92\pm\sqrt{(92)^2-4(9)(148)}}{2(9)}$$

$$x=\frac{74}{9} \quad \text{or} \quad x=2$$

(c) Determine whether f is an increasing or a decreasing function. Explain your answer. [2]

$$f'(x)=\frac{3x+1}{\sqrt{2x-3}}$$

$$x>\frac{3}{2}$$

$$3x+1>0$$

$$f'(x)>0$$

Hence, f is an increasing function.

9 **(a)** Prove that $\cot x + \dfrac{\sin x}{1+\cos x} = \operatorname{cosec} x$. [3]

$$
\begin{aligned}
\cot x + \frac{\sin x}{1+\cos x} &= \frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x} \\
&= \frac{\cos x\left(1+\cos x\right)+\sin^2 x}{\left(\sin x\right)\left(1+\cos x\right)} \\
&= \frac{\cos x+\cos^2 x+\sin^2 x}{\left(\sin x\right)\left(1+\cos x\right)} \\
&= \frac{\cos x+1}{\left(\sin x\right)\left(1+\cos x\right)} \\
&= \frac{1}{\left(\sin x\right)} \\
&= \cos ecx
\end{aligned}
$$

9 **(b)** Hence solve $\cot x + \dfrac{\sin x}{1+\cos x} = 4\cos x$, for $0 \le x \le 2\pi$. [4]

$$\cos ecx = 4\cos x$$
$$\frac{1}{\sin x} = 4\cos x$$
$$1 = 4\sin x\cos x \qquad 0 \le 2x \le 4\pi$$
$$2\sin x\cos x = \frac{1}{2}$$
$$\sin 2x = \frac{1}{2}$$

$$\alpha = \frac{\pi}{6}$$
$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}$$
$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

(c) Using the result in part **(a)**, find the set of values of the *k*, such that the equation $\cot x + \dfrac{\sin x}{1+\cos x} = \dfrac{1}{k}$, $k \ne 0$has solutions. [2]

$$\cos ecx = \frac{1}{k},\ k \ne 0$$
$$\sin x = k$$
$$-1 \le k \le 1$$

10 **(i)** The function $f(x)=ax^3+4x^2+bx-2$, where a and b are constants, is exactly divisible by $x+2$ and leaves a remainder of 4 when divided by $x+1$.

Find the value of a and of b. [4]

$$f(x)=ax^3+4x^2+bx-2$$
$$f(-2)=0$$
$$-8a+4(4)-2b-2=0$$
$$8a+2b=14$$
$$4a+b=7\cdots\cdots\cdots(1)$$
$$f(-1)=4$$
$$-a+4-b-2=4$$
$$-a-b=2\cdots\cdots\cdots(2)$$

$(1)+(2)$

$$3a=9$$
$$a=3$$
$$4(3)+b=7$$
$$b=-5$$

10 **(ii)** Factorise $\mathrm{f}(x)$ completely. [3]

$$\mathrm{f}(x) = 3x^3 + 4x^2 - 5x - 2$$
$$= (x+2)(3x^2 + bx - 1)$$

By comparing,

$$4 = 6 + b$$
$$b = -2$$

$$\mathrm{f}(x) = 3x^3 + 4x^2 - 5x - 2$$
$$= (x+2)(3x^2 - 2x - 1)$$
$$= (x+2)(3x+1)(x-1)$$

(iii) By substituting a suitable value of x, find the remainder when 3 003 994 998 is divided by 1002. [2]

$$x = 1000$$
$$\mathrm{f}(x) = 3(1000)^3 + 4(1000)^2 - 5(1000) - 2$$
$$= 3\ 003\ 994\ 998$$

Hence,

$$\mathrm{f}(1000) = (1000+2)(3000+1)(1000-1)$$

Remainder = 0

11 It is believed that the number of views, V, of a viral video online after t hours are related by the equation $V = ab^{2t}$, where a and b are constants.

The table below shows a set of survey results for the variables V and t.

Time **(hours)**	1	2	3	4	5
Views **(in thousands)**	2.16	3.11	4.48	6.45	9.29

(a)(i) Explain how a straight line graph can be drawn to represent the formula and draw it for the given data. [4]

$V = ab^{2t}$

$\lg V = \lg a + 2t \lg b$

Plot lg V against t

lg a = vertical intercept

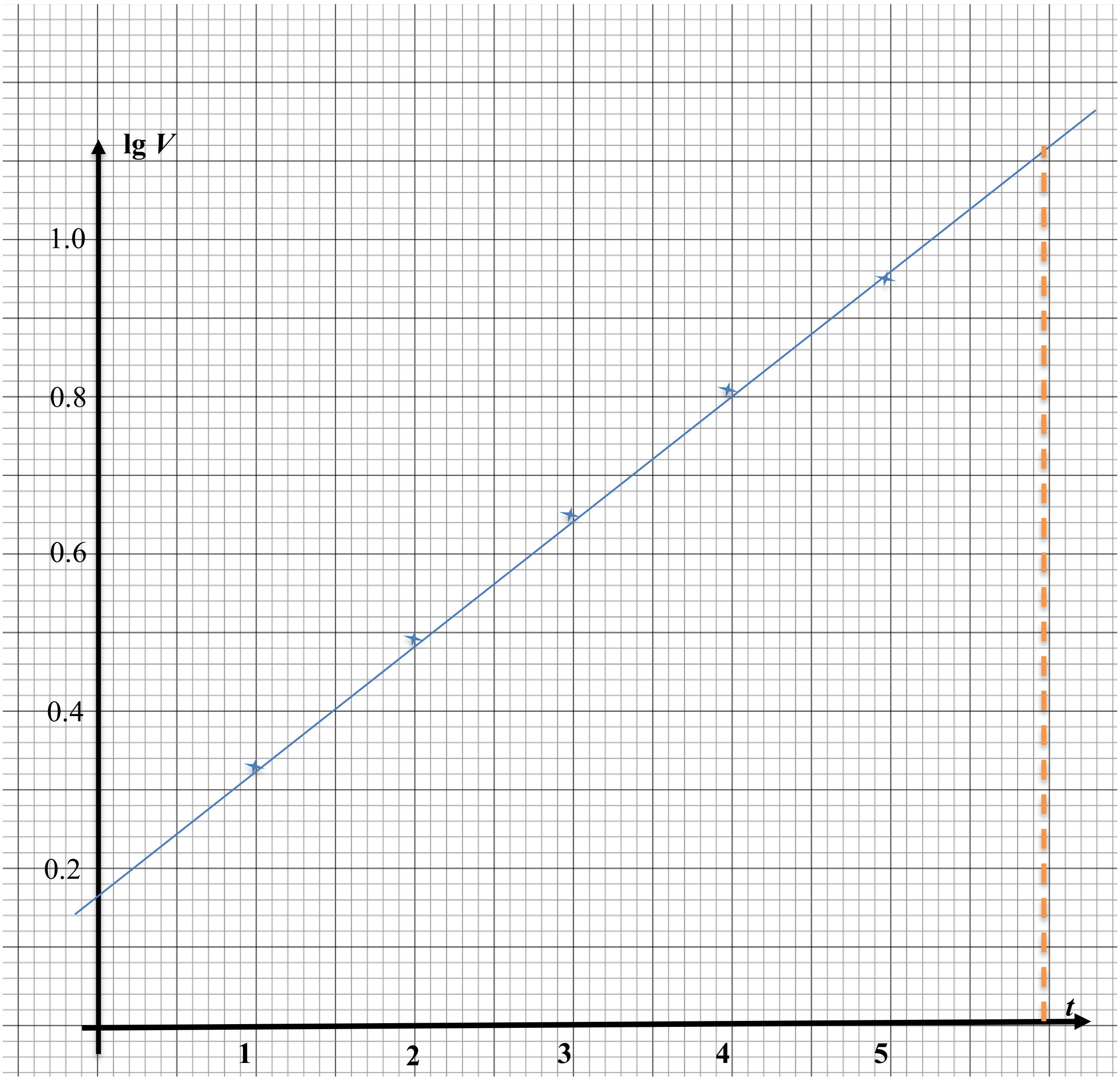
2lg b = gradient

Time **(hours)**	1	2	3	4	5
lg V **(in thousands)**	0.33	0.49	0.65	0.81	0.97

Alternative:

Time **(hours)**	1	2	3	4	5
ln V **(in thousands)**	0.77	1.13	1.5	1.86	2.23

(a)(ii) On the grid below, draw the graph and estimate the value of *a* and of *b*. [3]



$$\lg a = 0.17 \ \ [0.16 - 0.18]$$
$$a = 10^{0.17}$$
$$= 1.48 \ \ [1.445 \text{ to } 1.513]$$

$$2\lg b = \frac{1-0.24}{5.3-0.5} = \frac{19}{120} = 0.158 \ \ [0.14 \text{ to } 0.18]$$
$$\lg b = \frac{19}{240}$$
$$b = 1.20 \ \ [1.17 \text{ to } 1.23]$$

(iii) Using your graph to estimate the number of views when time is 6 hours. [2]

From the graph, at $t = 6$

$$\lg V = 1.12$$
$$V = 13.2 \ \ [13.2 \text{ to } 13.5]$$

When $t = 6$ hours, there are **13.2 thousands** of views

11 **(b)** The variables x and y are related in such a way that when a graph of xy is plotted against x^2, a straight line graph which passes through the points (1, 9) and (5, 1) is obtained. Express y in terms of x. [3]

$$Y = xy$$
$$X = x^2$$
$$m = \frac{9-1}{1-5} = -2$$

$$Y - 9 = -2(X - 1)$$
$$Y = -2X + 11$$

$$xy = -2x^2 + 11$$
$$y = \frac{-2x^2 + 11}{x}$$
$$= -2x + \frac{11}{x}$$

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